

Exploring nontrivial topology at quantum criticality on a superconducting processor

Corresponding Author: Professor Chao Song

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

In this manuscript, the authors realize eigenstates of the cluster Ising model using variational quantum circuits on a superconducting quantum processor. The model analyzed is a paradigmatic model for exhibiting gapless symmetry-protected topological phases and is of high current interest.

Following the preparation of the eigenstates, the authors compare the overlaps of the states with classical computations. Then, they perform entanglement Hamiltonian tomography to capture signatures of the critical point.

The authors fabricate and conduct the experiments on their own superconducting chip of over a hundred transmon qubits – a nontrivial achievement that deserves special mention.

I have several questions about the methodologies and technical details.

1) The authors mention that they perform the optimization for system-sizes ≤ 16 and then use polynomial extrapolation to get the circuits for the 100 qubit system. The authors cite Refs. [45, 46] as works that also perform this extrapolation. Is it evident that this should always work and does this work for the model being analyzed? It is hard to imagine a situation where all $L = 100$ models can be analyzed by considering $L \leq 16$ system-sizes. Fig. 2d of the paper shows overlaps that are not perfect even for $L \leq 16$ and it gets even worse for bigger L -s. Can the authors comment on the discrepancy for small L -s and how one should improve the results for bigger L -s?

2) The authors use ZNE and certain circuit compression methods. Are these used in Fig. 2c and 2d? What is the percentage error in energy instead of the energy distance? I am puzzled since the error in the overlap is substantially larger than the energy distance. Also, how is the overlap computed in Fig. 2d?

3) Fig. 3 shows results for $L = 100$ with the extrapolated circuits. Is this correct? How are results modified if the authors showed only the results for $L \leq 16$ for the entropy and entanglement spectrum? Does one really need to resort to extrapolations to capture signatures of gapless SPT models?

4) ArXiv:2510.14817 also analyzes a gapless model related to the Ising model with very similar techniques. Could the authors comment on how their device parameters compare the IBM machine analyzed there?

5) The central charge obtained with DMRG is worse than the experimental data. Could the authors comment on the discrepancy?

The experiment performed in this manuscript is remarkable and the work deserves to be published. I am happy to recommend publication provided the authors adequately address these questions and comments. In particular, I urge the authors to consider clearly stating system-sizes and error techniques in the main text since these are crucial to get meaningful results for noisy hardware.

Reviewer #2

(Remarks to the Author)

This manuscript presents an experimental quantum simulation of the critical cluster Ising model on a superconducting processor with up to 100 qubits. By preparing low-lying states using variational circuits and applying entanglement Hamiltonian tomography, the authors reconstruct reduced density matrices and extract standard conformal and entanglement properties of the system.

The experimental implementation is technically competent, and the data are generally consistent with numerical benchmarks. The use of entanglement Hamiltonian tomography is careful, and the results demonstrate that known features

of the critical cluster Ising model (e.g., central charge $c=1/2$ and a two-fold degeneracy in the entanglement spectrum) can be reproduced from experimentally prepared low-lying states.

From a physics perspective, however, the results largely confirm well-established theoretical expectations for this model, and the conceptual novelty is rather limited. The physically relevant quantities are extracted from relatively small subsystems, and the analysis relies on a structured ansatz informed by prior knowledge of the target theory. As such, the work is best viewed as a proof-of-principle experimental demonstration rather than a source of new physical insight. Nevertheless, given the solid experimental execution, I recommend its publication in Communications Physics.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The authors have satisfactorily addressed my comments and questions. I am happy to recommend acceptance and publication.

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Responses to Referees' Comments for Manuscript COMMSPHYS-25-1678

Exploring nontrivial topology at quantum criticality on a superconducting processor

Addressed Comments for Publication in

Communications Physics

by

Ziqi Tan, Ke Wang, Sheng Yang, et al

January 17, 2026

Summary of Changes

All changes are marked in red in the main text and the Supplementary Information.

1. Based on Comment 1.2 and 1.6, we have clearly stated that the experimentally measured energies are obtained with error mitigation techniques combining zero noise extrapolation and circuit simplification, and give a direct reference to the Supplementary Information III.C for details. We have added these clarifications in Sec. *Preparing the low-lying states at QCPs* in the main text.
2. Based on Comment 1.3 and 1.6, we have clearly stated that all results in Fig. 3 are obtained at $L = 100$ in the caption of Fig. 3 in the main text.
3. Based on Comment 1.1, we have made further clarifications of the presented overlap data in Fig. 2d. We have added the clarification to the caption of Fig. 2d and Sec. *Preparing the low-lying states at QCPs* in the main text, and Sec. III.A in the Supplementary Information.
4. Based on Comment 1.5, we have added a brief discussion on the relatively smaller deviation of the experimentally fitted central charge in Sec. *Learning entanglement Hamiltonian* in the main text.
5. Based on Comment 1.3, we have conducted further numerical simulations at $L = 16$ to illustrate the finite-size effects in small systems to support the necessity of performing experiments at $L = 100$. We have added the simulation results to a new Sec. IV.C in the Supplementary Information.
6. Based on Comment 1.2, we have made further explanations to the discrepancy between ground state overlap error and energy error in Sec. III.A in the Supplementary Information.
7. We have made other necessary revisions throughout the whole manuscript to correct typos, improve the presentation, and address the Referees' comments/suggestions.

Authors' Response to Referee 1

General Comments of Referee 1

In this manuscript, the authors realize eigenstates of the cluster Ising model using variational quantum circuits on a superconducting quantum processor. The model analyzed is a paradigmatic model for exhibiting gapless symmetry-protected topological phases and is of high current interest.

Following the preparation of the eigenstates, the authors compare the overlaps of the states with classical computations. Then, they perform entanglement Hamiltonian tomography to capture signatures of the critical point.

The authors fabricate and conduct the experiments on their own superconducting chip of over a hundred transmon qubits – a nontrivial achievement that deserves special mention.

Response 1.0:

We sincerely thank the Referee for the nice summary of our work and for noting our experimental achievement on the fabrication and utilization of superconducting quantum processor with over 100 qubits. We also appreciate the Referee's comment that the cluster Ising model is "a paradigmatic system" for studying gapless symmetry-protected topological phases and "is of high current interest". We have carefully addressed the Referee's specific technical questions in the point-by-point responses below, and we believe the clarifications and additional analyses further strengthen the manuscript.

Comment 1.1

I have several questions about the methodologies and technical details.

1) The authors mention that they perform the optimization for system-sizes ≤ 16 and then use polynomial extrapolation to get the circuits for the 100 qubit system. The authors cite Refs. [45, 46] as works that also perform this extrapolation. Is it evident that this should always work and does this work for the model being analyzed? It is hard to imagine a situation where all $L = 100$ models can be analyzed by considering $L \leq 16$ system-sizes. Fig. 2d of the paper shows overlaps that are not perfect even for $L \leq 16$ and it gets even worse for bigger L -s. Can the authors comment on the discrepancy for small L -s and how one should improve the results for bigger L -s?

Response 1.1:

We thank the Referee for the insightful questions, which have prompted us to better clarify our extrapolation-based state preparation method.

In our work, we have validated our state preparation method across various phases of the cluster Ising model. It is important to note, however, that the approach is heuristic in nature. Its general applicability to other systems warrants further investigation. To facilitate extrapolation to larger system sizes, we constrain the parameter structure of the variational ansatz and employ a power-law extrapolation function for parameter scaling. The results demonstrate that this approach can effectively extrapolate low-lying states up to 100 qubits with considerable ground state overlap, albeit at the cost of limiting the expressivity of the ansatz at small system sizes.

The decline in ground state overlap with increasing system size is expected, since the circuit depth of the ansatz is fixed throughout the extrapolation and therefore limits the system from establishing long-range correlations that

are inherent to the critical ground state. To improve the results for larger system sizes, it is essential to enhance the device performances, such as extending coherence times and reducing gate errors, to support ansatz with deeper circuit depth and thus the ability to capture longer correlation lengths. Besides, ansatz with refined variational parameter structures could further improve expressivity and lead to larger ground state overlap.

Following the Referee's comment, we have incorporated the above discussion in the revised manuscript.

Comment 1.2

2) The authors use ZNE and certain circuit compression methods. Are these used in Fig. 2c and 2d? What is the percentage error in energy instead of the energy distance? I am puzzled since the error in the overlap is substantially larger than the energy distance. Also, how is the overlap computed in Fig. 2d?

Response 1.2:

We thank the Referee for the questions. The data in Fig. 2c is experimentally measured using ZNE error mitigation and circuit compression. In contrast, the data in Fig. 2d is obtained by numerical simulation without using ZNE and circuit compression.

The quantity plotted in Fig. 2c is the normalized energy distance, defined as $\varepsilon(E) = \frac{E - E_g}{E_{\max} - E_g}$, where E is the energy of the prepared state, $E_{\max}$ is the maximum energy of the target Hamiltonian and E_g is the energy of the ground state. This metric represents how close the energy of the prepared state is to the ground-state energy, expressed as a fraction of the entire spectral range. For example, the experimentally measured energy at $L = 100$ is -124.19 ± 0.11 , while the ground-state energy is -127.33 , giving an energy distance of 3.14 ± 0.11 . With $E_{\max} - E_g = 254.66$, we obtain the normalized energy distance of $1.2 \pm 0.1\%$, which is presented in Fig. 2c.

The small normalized energy distance despite a large overlap error is due to the fact that the cluster Ising model is gapless at the critical point. As a result, the energy gap closes rapidly with increasing L , and at $L = 100$, the normalized energy distance is only 0.01% between the ground state and the first excited state.

In Fig. 2d, the overlaps $|\langle \Psi_v | \psi_{\text{gs.}} \rangle|$ are analyzed numerically. To do this, the prepared states $|\Psi_v\rangle$ are generated by simulating the state-preparation circuits on the noiseless MPS simulator. These are then compared against the eigenstates $|\psi_{\text{gs.}}\rangle$ of the lowest four levels, which are obtained from Density Matrix Renormalization Group (DMRG) calculations.

Following the Referee's comment, we have incorporated the above clarifications in the revised manuscript.

Comment 1.3

3) Fig. 3 shows results for $L = 100$ with the extrapolated circuits. Is this correct? How are results modified if the authors showed only the results for $L \leq 16$ for the entropy and entanglement spectrum? Does one really need to resort to extrapolations to capture signatures of gapless SPT models?

Response 1.3:

We thank the Referee for raising these important questions regarding the role of system size in our results.

The Referee is correct that all results in Fig. 3 are obtained from the experimentally prepared state at $L = 100$. This state is prepared by the extrapolated circuit on our superconducting quantum device, with the subsequent EHT measurements performed directly on that state.

To make a comparison, Fig. R1 shows the results for $L = 16$ from exact diagonalization (ED). We first calculate the connected correlation function and entanglement entropy. By fitting the connected correlation function and entanglement entropy, we obtain a bulk scaling dimension of $\Delta_{16} = 0.205$ and a central charge of $c_{16} = 0.591$. Then we calculate the entanglement spectrum, where two-fold topological degeneracy can also be revealed.

We note that, despite the observed two-fold degeneracy, both the bulk scaling dimension and the central charge deviate noticeably from the thermodynamic-limit value of 0.25 and 0.5. In contrast, similar fit for the $L = 100$ DMRG ground state gives $\Delta_{100} = 0.244$ and $c_{100} = 0.543$, where finite-size effects are significantly inhibited. As a result, larger system sizes are essential for approaching the thermodynamic limit, which allows the universal critical signatures to be reliably extracted.

Following the Referee's comment, we have incorporated the above discussion in the revised manuscript.

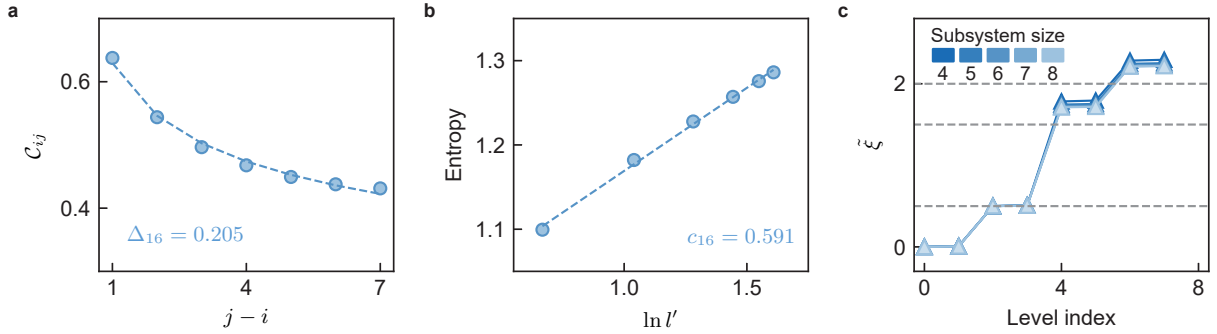


Figure R1: **Critical signatures of the cluster Ising model at $L = 16$ obtained from the ground state computed by exact diagonalization (ED).** **a**, Connected correlation functions $C_{ij} = \langle Z_i Z_j \rangle - \langle Z_i \rangle \langle Z_j \rangle$, where i and j are site index. A bulk scaling dimension of $\Delta_{16} = 0.205$ is obtained by fitting $C_{ij} = |j - i|^{-2\Delta_{16}}$ from ED. **b**, Entanglement entropy scaling. $l' = \ln \frac{L}{\pi} \sin \frac{\pi l}{L}$ where $L = 16$. A central charge of $c = 0.591$ is obtained by fitting $S_A = \frac{c}{3} \ln l' + \text{const.}$ according to conformal field theory (CFT) from ED. **c**, Normalized entanglement spectrum $\tilde{\xi}_n = 0.5 \times (\xi_n - \xi_0) / (\xi_2 - \xi_0)$ of subsystems with different sizes. The dashed lines indicate the low-lying part of the operator content expected from the Ising boundary CFT, which is 0.5, 1.5, 2.0, respectively. The lowest eight levels $\tilde{\xi}$ are displayed.

Comment 1.4

4) ArXiv:2510.14817 also analyzes a gapless model related to the Ising model with very similar techniques. Could the authors comment on how their device parameters compare the IBM machine analyzed there?

Response 1.4:

We thank the Referee for mentioning this relevant and excellent work (arXiv:2510.14817) conducted on an IBM quantum processor `ibm_kingston` (156 qubits). This device reports median fidelities of 99.95% for single-qubit gates, 99.56% for two-qubit gates, and 99.14% for measurements¹. For comparison, the corresponding median fidelities for the device used in our experiment are 99.95%, 99.57%, and 99.15%, respectively. Both processors exhibit similar gate and readout fidelities. Regarding the quantum simulation of one-dimensional spin chains such

¹https://quantum.cloud.ibm.com/computers?system=ibm_kingston. Time: 2026.01.07.

as the cluster Ising model, both the IBM device and our processor are capable platforms, as evidenced by the successful execution of the respective experiments.

While both works employ variational algorithms and error mitigation on state-of-the-art superconducting hardware, they address distinct physical questions that naturally lead to different experimental strategies. The referenced work provides a proof-of-principle demonstration of impurity Hamiltonians and loop operators associated with the topological symmetries at a small system size, utilizing 12 qubits. In contrast, our work aims to characterize the emergent critical properties of a gapless symmetry-protected topological (gSPT) phase in the 1D cluster Ising model. Our work aims to provide scalable observations of large systems, thereby suppressing finite-size effects and enabling reliable extraction of universal critical properties.

Comment 1.5

5) The central charge obtained with DMRG is worse than the experimental data. Could the authors comment on the discrepancy?

Response 1.5:

We thank the Referee for raising this point. Indeed, the central charge fitted from the experimental data ($c_{\text{exp}} = 0.518 \pm 0.063$) appears numerically closer to the exact thermodynamic-limit value ($c = 0.5$) than the value ($c_{\text{DMRG}} = 0.543$) obtained from the DMRG ground state at $L = 100$. The DMRG result (0.543) is a high-precision theoretical reference for a finite chain at $L = 100$ when the entanglement entropy is fitted over the limited subsystem sizes accessible in our experiment ($l \leq 12$). The deviation from 0.5 is mainly due to the finite-size effect. In addition to this influence, the experimental characterization is also affected by various error sources, including state preparation errors arising from the imperfect circuit ansatz and environmental noises. The fact that the mean value of our experiment (0.518) lies slightly closer to 0.5 than the DMRG value is likely due to the competing effects of the finite-size effects and the experimental errors.

Following the Referee's comment, we have incorporated the above discussion in the revised manuscript.

Comment 1.6

The experiment performed in this manuscript is remarkable and the work deserves to be published. I am happy to recommend publication provided the authors adequately address these questions and comments. In particular, I urge the authors to consider clearly stating system-sizes and error mitigation techniques in the main text since these are crucial to get meaningful results for noisy hardware.

Response 1.6:

We sincerely thank the Referee for considering our work as remarkable and for the recommendation for publication. We are grateful for the constructive and insightful comments, which have undoubtedly helped us improve the presentation of the manuscript. We have now clearly stated the system sizes used in the experiments and the specific error-mitigation techniques applied in the revised main text. We believe these clarifications ensure that readers can fully appreciate the scale of our experiment and our error mitigation techniques.

Authors' Response to Referee 2

General Comments of Referee 2

This manuscript presents an experimental quantum simulation of the critical cluster Ising model on a superconducting processor with up to 100 qubits. By preparing low-lying states using variational circuits and applying entanglement Hamiltonian tomography, the authors reconstruct reduced density matrices and extract standard conformal and entanglement properties of the system.

The experimental implementation is technically competent, and the data are generally consistent with numerical benchmarks. The use of entanglement Hamiltonian tomography is careful, and the results demonstrate that known features of the critical cluster Ising model (e.g., central charge $c=1/2$ and a two-fold degeneracy in the entanglement spectrum) can be reproduced from experimentally prepared low-lying states.

Response 2.0:

We sincerely thank the Referee for the nice summary and thoughtful evaluation of our work. Specifically, we appreciate the Referee for noting that our “experimental implementation is technically competent”. We hope that our work can be a meaningful contribution to this active field. We have carefully addressed the Referee’s additional comments in the responses below, and we believe the clarifications can further strengthen the manuscript.

Comment 2.1

From a physics perspective, however, the results largely confirm well-established theoretical expectations for this model, and the conceptual novelty is rather limited. The physically relevant quantities are extracted from relatively small subsystems, and the analysis relies on a structured ansatz informed by prior knowledge of the target theory. As such, the work is best viewed as a proof-of-principle experimental demonstration rather than a source of new physical insight. Nevertheless, given the solid experimental execution, I recommend its publication in Communications Physics.

Response 2.1:

We thank the Referee for this thoughtful comment and kind recommendation for publication. Beyond experimentally confirming established theoretical expectations, we would like to note the methodological contribution of our work. By integrating scalable low-lying state preparation with EHT-based reduced density matrix reconstruction, we introduce an experimental pathway to explore the gapless SPT phases. Indeed, the EH ansatz is constructed as the local deformation of the system Hamiltonian that exhibits the same $\mathbb{Z}_2$ symmetry. However, it is important to note that this ansatz alone cannot capture the characteristic features of the gapless SPT phase, such as the power-law scaling behaviors and the two-fold topological degeneracy. In fact, by applying EHT to low-lying states from different phases of the cluster Ising model, we observe distinct scaling behaviors despite using the same EH ansatz, underscoring the role of the experimentally prepared low-lying states. Our approach thus offers a potential route to circumvent long-standing challenges in studying large-scale critical systems, such as the notorious barren plateau problem and vanishing energy gap that hinder the variational optimization toward the ground state.