Supplementary Information: Laser-Driven Annular Shock Waves as Laboratory Analogues of wCDM Cosmologies and Cosmological Gravitational Waves

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Supplementary Note 1: ANALOGY WITH COSMOLOGICAL GRAVITATIONAL WAVES

A. First order perturbation

Consider the dynamical equation for the shock wave radius, and let us study the perturbation to the radius, in the form $r = r_u(1+\epsilon h)$, with $\epsilon \ll 1$ representing the small perturbation parameter, h is the first order perturbation function, and r_u the unperturbed solution found in such section. Using this, we can find an equation for h. At first order in h, we get

$$\ddot{h} + (2+\zeta)\frac{\dot{r}_u}{r_u}\dot{h} = 0 \tag{1}$$

Using the unperturbed solution r_u, we get

$$\ddot{h} + \frac{2}{t}\dot{h} = 0 \tag{2}$$

which is independent of ζ. The solution is

$$h = \frac{h_0}{t} \tag{3}$$

This is a general solution for the linear perturbation. Thus, the analogy between perturbation on the radius of the shock wave (in any dimension) and cosmological gravitational waves is only exact for $w = 0$, an Universe in a non-relativistic matter dominated case.

B. Second order perturbation

By repeating the same procedure to a second order in the perturbation, we find that a solution in the form $r = r_u(1+\epsilon h_1 + \epsilon^2 h_2)$ solve the shock wave dynamics for

$$h_1 = \frac{h_0}{t}, \tag{4}$$

$$h_2 = \alpha + \frac{\beta}{t} - \frac{\gamma h_0^2}{4t^2} \tag{5}$$

where α, β, and γ are constants.

In Supplementary Fig. 1, we display the evolution of the perturbation amplitude to the “bump" shock wave. In it, we show the second order perturbation fit in a dotdashed line, for $\alpha = -1.08$, $\beta = 15041.55$, and $\gamma = 0.53$. We can see that after the first $1/t$ decaying of the perturbation, analogously behaving as a cosmological gravitational wave, the wave decaying behavior shifted. That change is controlled by the second order perturbation in its decaying.

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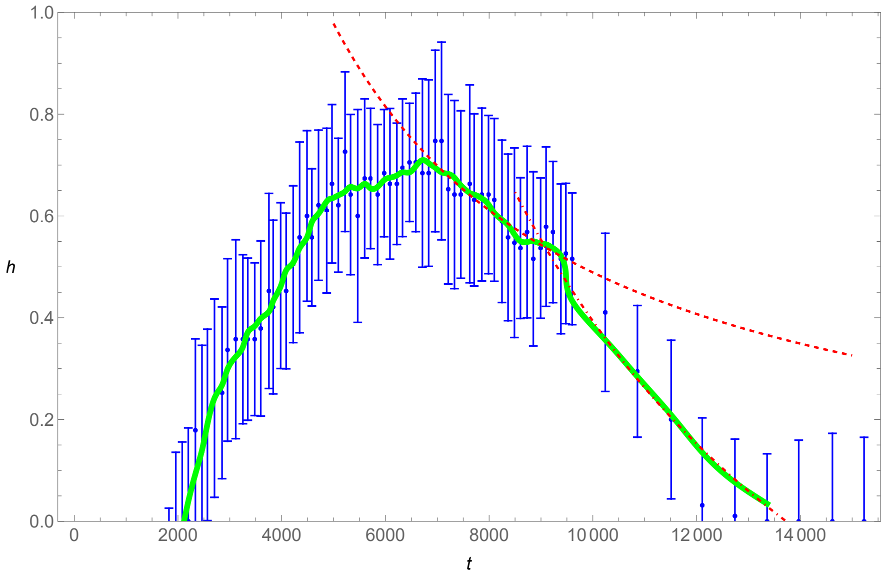

Supplementary Fig. 1: Temporal evolution of the distance radius of the perturbation amplitude. Blue points are data with error bars given by uncertainty in fitting shock fronts in schlieren images. In green solid line is shown the moving average of the data. In dashed red line is shown the first order perturbation fit for the decaying. In dotdashed red line is shown the second order perturbation fit for the decaying.