

Chiral damping with persistent edge states from the interplay of topologies in open quantum systems

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

This study explores the interplay between spectral topology and band topology in a dissipative Hofstadter model. The authors provide valuable insight into how the non-Hermitian skin effect and topological edge states can coexist and shape dynamics in open quantum systems. The discovery of decoupled time scales for chiral damping and edge-selective extremal damping is particularly intriguing. The identification of intermediate magnetic fields as a regime where chiral damping is partially restored adds further depth and practical relevance. I would suggest the authors specify the concrete influences of the spectrum winding number and the band topological number if their roles can be made clear. For example, whether the number of edge states is solely determined by the band topology, what is the meaning of the spectral winding number and its role in the chiral damping. Overall, the work makes a meaningful contribution to our understanding of dynamical behaviors in non-Hermitian topological systems and highlights the nuanced role of boundary-induced spectral topology.

Reviewer #2

(Remarks to the Author)

In this manuscript, the authors present a theoretical study on the coexistence and dynamical interplay of the non-Hermitian skin effect (NHSE) and topological edge states in a dissipative two-dimensional Hofstadter model. Using a combination of analytical arguments and numerical simulations, they demonstrate that the short-time dynamics is dominated by chiral damping—a signature of the NHSE—while the long-time dynamics is governed by edge-selective extremal damping due to topologically protected edge states. The role of magnetic flux in suppressing and restoring skin localization is also explored.

Overall, the topic is timely and fits well within the scope of Communications Physics. The manuscript is well written regarding the technical aspects, and the results appear sound to me. The observation of temporally separated dynamical regimes linked to different topological origins is interesting. However, I have a major concern regarding the motivation and positioning of this work relative to the existing literature, which the authors should address before I can fully support publication.

My main comment is as follows:

The authors should more clearly motivate the study of this interplay in a two-dimensional setting, particularly in light of several recent works that have already explored the coexistence or competition of NHSE and topological boundary modes in one-dimensional systems. For example, a study by Bo Li et al. (Phys. Rev. B 109, L201121 (2024)) investigated the dynamical consequences of a topological zero mode and NHSE in a 1D classical stochastic model, also revealing distinct transient and long-time damping behavior. While the present work studies a 2D quantum model, the introduction and discussion sections do not sufficiently clarify what qualitatively new insight or physical behavior is gained by moving to two dimensions. Is it the directional nature of the skin effect? The role of magnetic flux? Or the possibility of corner modes in fully open geometries? A clearer articulation of the added value of the 2D framework would significantly strengthen the motivation and impact of the manuscript.

In summary, this manuscript reports a relevant and potentially valuable contribution to the field of non-Hermitian topological physics. Provided the authors can convincingly address the issue of motivation and dimensionality-raised novelty relative to existing 1D studies, I would be pleased to recommend the manuscript for publication in Communications Physics.

Reviewer #3

(Remarks to the Author)

What are the major claims of the paper?

The article demonstrates the decoupling of the Liouvillian skin effect (referred to as chiral damping in the abstract) from the boundary modes (dubbed edge-selective extremal damping) in the dynamics of an open quantum system. The authors attribute this decoupling of Liouvillian skin states from the boundary modes to the interplay of different topologies.

Are they novel and will they be of interest to others in the community and the wider field?

The claims made by the authors showcase the individual contributions of boundary modes and the NHSE on the dynamics of fermionic open quantum system. While it was already known that skin states lead to the Liouvillian skin effect (chiral damping), the effect or contribution of individual boundary modes has been less explored. Hence, their conclusions provide new insights which are of interest to the specialists.

On a more subjective note, do you feel that the paper will influence thinking in the field?

The work might influence the field by providing further insight into the role of edge modes and the NHSE in the dynamics of open quantum systems.

Remarks

(1) It is claimed that "one edge mode is tightly localized, e^{-y/ξ_S} e^{-y/ξ_R} , while the other is stretched out, e^{-y/ξ_S} $e^{-(L-y)/\xi_L}$." It was not clear to me whether the two modes correspond to one left and one right eigenmode, or whether both are obtained from the right eigenvectors. Please clarify.

(2)

The authors state " Physically, the non-Hermiticity of X represents the tendency of the system to decay towards its steady state. " I am confused about what the authors were addressing within the statement. Please clarify.

(3)

The authors claim "This shows that the edge localization due to topological edge physics, and the edge localization due to the NHSE are essentially independent and superimposed."

I suggest that the authors make a short discussion about analogous independence of the edge modes and the NHSE in 1D systems.

The 1D analogy (removal of the NHSE does not affect the boundary mode) can be seen for instance in Ref. [10]. There, the NHSE can be viewed through transformation matrix S in Eq.2, where the first site is multiplied by factor of one. Perhaps even better demonstration of decoupling boundary and the NHSE states is in Ref. [26]. There, the extended mode comes from the boundary mode (in the Ref. [26] called topological zero mode (TzM)).

Hence, its 'tuning' is independent of the skin states (in certain sense). This 'independence', and different roles of NHSE and the boundary mode in 1D is well reported also in <https://pubs.aip.org/aip/apl/article/124/4/041105/3061555> (which might be of relevance also for the discussion in the introduction).

(4)

What do authors mean by the 'damping gap'? Please clarify.

(5)

I think that a short discussion (a couple of sentences) about the Liouvillian gap and how it affects the interplay of the phenomena might help to improve the clarity of the manuscript. Perhaps these references might be of relevance:

<https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.070402>,

<https://journals.aps.org/prl/abstract/10.1103/PhysRevA.98.042118>, or analogy from bosonic systems

<https://journals.aps.org/prl/abstract/10.1103/PhysRevA.110.032207>.

(6)

Authors claim "We find that an extensive skin effect is recovered at intermediate fluxes, rendering chiral damping observable again." Do authors have some intuition on why the intermediate fluxes again allow for the skin effect, however for the small fluxes, the NHSE is suppressed as the Fig. 3 (a) shows.

(6)

In the outlook, I would also suggest to mention ultracold atoms as a promising platform for simulating open quantum systems, and in which the signatures of the skin effect has been reported

<https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.129.070401>

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

I thank the authors for the further clarification and recommend the publication of the present version.

Reviewer #2

(Remarks to the Author)

The main issues have been satisfactorily addressed. The revised manuscript is now ready for publication.

Reviewer #3

(Remarks to the Author)

I am satisfied with the changes, and am pleased to recommend the manuscript for publication in the Communications Physics.

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Referee 1

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Referee: *This study explores the interplay between spectral topology and band topology in a dissipative Hofstadter model. The authors provide valuable insight into how the non-Hermitian skin effect and topological edge states can coexist and shape dynamics in open quantum systems. The discovery of decoupled time scales for chiral damping and edge-selective extremal damping is particularly intriguing. The identification of intermediate magnetic fields as a regime where chiral damping is partially restored adds further depth and practical relevance. I would suggest the authors specify the concrete influences of the spectrum winding number and the band topological number if their roles can be made clear. For example, whether the number of edge states is solely determined by the band topology, what is the meaning of the spectral winding number and its role in the chiral damping. Overall, the work makes a meaningful contribution to our understanding of dynamical behaviors in non-Hermitian topological systems and highlights the nuanced role of boundary-induced spectral topology.*

Reply: We thank the referee for their positive comments on our work and for this insightful suggestion. We clarify the point as follows: The number of edge states is indeed solely determined by the band topology. Specifically, the band topological number, known as the Chern number, defines both the number and chirality of the edge states that persist even during the time evolution of the dissipative model.

In contrast, the spectral topology is governed by the global structure of the complex eigenvalue spectrum of the damping matrix X , which is generally non-Hermitian. The spectral winding number is defined by the number of times the complex eigenvalues encircle a chosen reference energy point in the complex plane. A nontrivial spectral topology of X gives rise to characteristic dynamical phenomena, such as chiral damping, during time evolution.

Thus, while the robust edge modes originate from the band topology of the underlying Hamiltonian, the spectral topology governs how dissipation and non-Hermiticity shape dynamical features like directional (chiral) damping. We have now added a note regarding this point in the revised manuscript, highlighted in blue.

Changes to the Manuscript: “While the robust edge modes originate from the band topology of the underlying Hermitian Hamiltonian, the spectral topology governs how dissipation and non-Hermiticity shape dynamical features like directional (chiral) damping.”

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Referee 2

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Referee: *In this manuscript, the authors present a theoretical study on the coexistence and dynamical interplay of the non-Hermitian skin effect (NHSE) and topological edge states in a dissipative two-dimensional Hofstadter model. Using a combination of analytical arguments and numerical simulations, they demonstrate that the short-time dynamics is dominated by chiral damping—a signature of the NHSE—while the long-time dynamics is governed by edge-selective extremal damping due to topologically protected edge states. The role of magnetic flux in suppressing and restoring*

skin localization is also explored.

Overall, the topic is timely and fits well within the scope of Communications Physics. The manuscript is well written regarding the technical aspects, and the results appear sound to me. The observation of temporally separated dynamical regimes linked to different topological origins is interesting. However, I have a major concern regarding the motivation and positioning of this work relative to the existing literature, which the authors should address before I can fully support publication.

Reply: We thank the referee for their positive comments and appreciate the acknowledgment of our technical results. We are happy to clarify their concerns as addressed below.

Referee: *The authors should more clearly motivate the study of this interplay in a two-dimensional setting, particularly in light of several recent works that have already explored the coexistence or competition of NHSE and topological boundary modes in one-dimensional systems. For example, a study by Bo Li et al. (Phys. Rev. B 109, L201121 (2024)) investigated the dynamical consequences of a topological zero mode and NHSE in a 1D classical stochastic model, also revealing distinct transient and long-time damping behavior. While the present work studies a 2D quantum model, the introduction and discussion sections do not sufficiently clarify what qualitatively new insight or physical behavior is gained by moving to two dimensions. Is it the directional nature of the skin effect? The role of magnetic flux? Or the possibility of corner modes in fully open geometries? A clearer articulation of the added value of the 2D framework would significantly strengthen the motivation and impact of the manuscript.*

Reply: Thank you for highlighting the need to more clearly motivate our focus on the two-dimensional setting, and how our work advances beyond recent one-dimensional studies. One clear distinction of our work is that we begin with a purely Hermitian Hamiltonian and introduce dissipation to study the resulting open quantum system. As a result of these dissipation effects, the damping matrix which characterizes the time evolution turns out to be non-Hermitian, rather than non-Hermiticity being built into the static Hamiltonian from the outset. The choice of the two-dimensional Hofstadter model is motivated by its ability to support nontrivial band topology when subjected to a magnetic flux. This enables us to investigate the interplay of two independent types of topology: the band topology (e.g., Chern number) and the spectral topology of the non-Hermitian damping matrix, as well as how both are influenced by tunable magnetic fields. Such a framework cannot be realized in purely one-dimensional systems, where magnetic flux and its rich topological consequences are absent. Moreover, the 2D geometry gives rise to additional features, such as the emergence of nontrivial spectral topology at system edges along the direction of PBC, which is distinct from the bulk spectral topology. This gives rise to potential higher-order topological modes, including corner modes under fully open boundary conditions, as the referee has also highlighted. We also emphasize that it is the spectral topology along the edges that gives rise to a damping gap in the spectrum and thus the separation of times scales. These phenomena are exclusive to two dimensions and provide new avenues for exploring the impact of dissipation and topology in quantum systems.

We have added a brief discussion about this point in the revised manuscript, highlighted in blue.

Changes to the Manuscript: “Our work advances the study of non-Bloch dynamics and topological edge modes [1, 2] to a two-dimensional setting where new phenomena emerge. By introducing dissipation to an initially Hermitian Hamiltonian, we realize non-Hermitian spectral topology

originating from the system's open dynamics rather than from inherent non-Hermiticity. The 2D Hofstadter model enables the controlled interplay between band topology and non-Hermitian spectral topology through magnetic flux, giving rise to edge-localized spectral structures, damping gaps, and potential higher-order boundary modes, unattainable in one dimension."

Referee: *In summary, this manuscript reports a relevant and potentially valuable contribution to the field of non-Hermitian topological physics. Provided the authors can convincingly address the issue of motivation and dimensionality-raised novelty relative to existing 1D studies, I would be pleased to recommend the manuscript for publication in Communications Physics.*

Reply: Thank you for your positive evaluation and for recognizing the potential contribution of our manuscript. We appreciate your constructive feedback regarding the need to strengthen the motivation for our two-dimensional approach. We have revised our introduction and discussion sections to address this.

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Referee 3
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Referee: *The article demonstrates the decoupling of the Liouvillian skin effect (referred to as chiral damping in the abstract) from the boundary modes (dubbed edge-selective extremal damping) in the dynamics of an open quantum system. The authors attribute this decoupling of Liouvillian skin states from the boundary modes to the interplay of different topologies. The claims made by the authors showcase the individual contributions of boundary modes and the NHSE on the dynamics of fermionic open quantum system. While it was already known that skin states lead to the Liouvillian skin effect (chiral damping), the effect or contribution of individual boundary modes has been less explored. Hence, their conclusions provide new insights which are of interest to the specialists. The work might influence the field by providing further insight into the role of edge modes and the NHSE in the dynamics of open quantum systems.*

Reply: We thank the referee for their thoughtful assessment and for recognizing the new insights provided by our work. In the following we reply to all their concerns and summarize the changes made to the manuscript as a result. All modifications to the manuscript are highlighted in blue.

Referee: *(1) It is claimed that "one edge mode is tightly localized, $e^{-y/\xi_S}e^{-y/\xi_R}$, while the other is stretched out, $e^{-y/\xi_S}e^{-(L-y)/\xi_L}$." It was not clear to me whether the two modes correspond to one left and one right eigenmode, or whether both are obtained from the right eigenvectors. Please clarify.*

Reply: Thank you for drawing attention to this point and allowing us to clarify the nature of the two edge modes described in the manuscript.

The two edge-localized modes referenced in the statement are both obtained from the right eigenvectors of the non-Hermitian damping matrix governing the system's dynamics. These right eigenvectors correspond to states located at either the right or left edge (in space) of the sample. Specifically, one right eigenmode is tightly localized near one boundary, decaying as $e^{-y/\xi_S}e^{-y/\xi_R}$, while the other right eigenmode is more delocalized, exhibiting a stretched localization profile

given by $e^{-y/\xi_S}e^{-(L-y)/\xi_L}$. These distinct spatial profiles arise due to the interplay between the non-Hermitian skin effect (which pushes localization towards a single edge) and the underlying band topology, as encoded in the different localization lengths (ξ_S, ξ_R, ξ_L). We have clarified this point in the revised manuscript.

Changes to the Manuscript: “All eigenstates in our computations are obtained conventionally from the right eigenvectors of the non-Hermitian damping matrix X .”

Referee: (2) *The authors state “Physically, the non-Hermiticity of X represents the tendency of the system to decay towards its steady state.” I am confused about what the authors were addressing within the statement. Please clarify.*

Reply: We thank the referee for this comment and for allowing us to clarify this point. As highlighted in our manuscript, to study the dynamics, our main observable of interest is the covariance matrix C whose exact time evolution is governed by the Lyapunov equation

$$\frac{dC(t)}{dt} = XC(t) + C(t)X^\dagger + 2M^g. \quad (1)$$

The solution of Eq. (1) can be split into a constant steady-state value C_{ss} and the convergence towards the steady state as $C(t) = C_{ss} + \tilde{C}(t)$. The time evolution of $\tilde{C}(t)$ can be cast into an analytic expression in terms of the eigensystem of X ,

$$\tilde{C}(t) = \sum_{m,m'} e^{(\lambda_m + \lambda_{m'}^*)t} |\psi_{R,m}\rangle \langle \psi_{L,m}| \tilde{C}(0) |\psi_{L,m'}\rangle \langle \psi_{R,m'}|, \quad (2)$$

where λ_m ($\psi_{R/L,m}$) denotes the m -th eigenvalue (right/left eigenvector) of the damping matrix X . The matrix X arises from the dissipative processes in the open quantum system. The non-Hermiticity of X is solely due to the dissipative jump operators. These dissipation terms lead to the decay of the system towards its steady state, and this information is encoded into the non-Hermiticity of the damping matrix X . This is what we mean when we state that the non-Hermiticity of X represents the tendency of the system to decay towards its steady state.

The nonzero imaginary parts of the eigenvalues of X encode the decay rates at which different modes relax or are damped through interaction with the environment. The dissipative nature and resulting steady-state behavior of the system are thus fully captured by the non-Hermitian structure of X .

We have added a clarifying note in the revised manuscript

Changes to the Manuscript: “In particular, the imaginary nonzero parts of the eigenvalues of X encode the decay rates at which different modes relax or are damped through interaction with the environment. The dissipative nature and resulting steady-state behavior of the system are thus fully captured by the non-Hermitian structure of X .”

Referee: (3) *The authors claim “This shows that the edge localization due to topological edge physics, and the edge localization due to the NHSE are essentially independent and superimposed.” I suggest that the authors make a short discussion about analogous independence of the edge modes and the NHSE in 1D systems. The 1D analogy (removal of the NHSE does not affect the boundary*

mode) can be seen for instance in Ref. [10]. There, the NHSE can be viewed through transformation matrix S in Eq.2, where the first site is multiplied by factor of one. Perhaps even better demonstration of decoupling boundary and the NHSE states is in Ref. [26]. There, the extended mode comes from the boundary mode (in the Ref. [26] called topological zero mode (TZM)). Hence, its ‘tuning’ is independent of the skin states (in certain sense). This ‘independence’, and different roles of NHSE and the boundary mode in 1D is well reported also in <https://pubs.aip.org/aip/apl/article/124/4/041105/3061555> (which might be of relevance also for the discussion in the introduction).

Reply: Thank you for this thoughtful suggestion and for pointing out relevant works on the relationship between boundary modes and the NHSE in one-dimensional systems. We agree that the independence and distinct roles of the NHSE and topological boundary modes, while emphasized in our two-dimensional study, also have analogs in 1D systems. We have included a brief discussion of these analogies in the revised manuscript, citing the relevant literature to show that, the boundary and skin modes play independent roles in the dynamical and spectral properties in 1D non-Hermitian settings, as they do in our 2D open quantum system.

Changes to the Manuscript: “Our work advances the study of non-Bloch dynamics and topological edge modes [1, 2] to a two-dimensional setting where new phenomena emerge. By introducing dissipation to an initially Hermitian Hamiltonian, we realize non-Hermitian spectral topology originating from the system’s open dynamics rather than from inherent non-Hermiticity. The 2D Hofstadter model enables the controlled interplay between band topology and non-Hermitian spectral topology through magnetic flux, giving rise to edge-localized spectral structures, damping gaps, and potential higher-order boundary modes, unattainable in one dimension.”

Referee: (4) *What do authors mean by the ‘damping gap’? Please clarify.*

Reply: Thank you for this question. In our manuscript, we specify that the damping gap refers to a finite separation between the least-damped and more strongly damped modes in the spectrum of a dissipative quantum system.

More explicitly, since the eigenvalues of X encode the decay rates of different modes toward the steady state, the damping gap quantifies the slowest (i.e., least negative real part) nonzero decay rate in the system. This gap sets the asymptotic rate at which perturbations relax: a finite damping gap implies exponential and rapid convergence to steady state, whereas a closing (vanishing) damping gap signals slow relaxation or possible long-lived modes.

Changes to the Manuscript: We add a consolidated note addressing this point and the next, in the revised manuscript. You can find the changes after the next point.

Referee: (5) *I think that a short discussion (a couple of sentences) about the Liouvillian gap and how it affects the interplay of the phenomena might help to improve the clarity of the manuscript. Perhaps these references might be of relevance: <https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.070402>, <https://journals.aps.org/prl/abstract/10.1103/PhysRevA.98.042118>, or analogy from bosonic systems <https://journals.aps.org/prl/abstract/10.1103/PhysRevA.110.032207>.*

Reply: Thank you for directing us to these relevant references. The Liouvillian gap is, in fact, equivalent to the “damping gap”, referred to earlier. We have updated our manuscript to clarify this terminology and include these references to strengthen and consolidate our discussion.

To reinforce, the Liouvillian gap, defined as the smallest nonzero absolute value of the real part of the Liouvillian (or damping matrix) eigenvalues, sets the slowest relaxation rate toward the steady state in an open quantum system. A finite Liouvillian gap ensures that the system relaxes exponentially and rapidly, while a closing or vanishing gap can result in slow, non-exponential relaxation. In the context of the interplay between chiral damping and topological edge modes, the Liouvillian gap controls how quickly dynamical signatures of both phenomena appear and how robustly they persist before the final steady state is reached.

Changes to the Manuscript: “Since the eigenvalues of X encode the decay rates of different modes toward the steady state, the damping gap (or Liouvillian gap) [3, 4, 5] quantifies the slowest (i.e., least negative real part) nonzero decay rate in the system. This gap sets the asymptotic rate at which perturbations relax: a finite damping gap implies exponential and rapid convergence to steady state, whereas a closing (vanishing) damping gap signals slow relaxation or possible long-lived modes. In the context of the interplay between chiral damping and topological edge modes, the Liouvillian gap controls how quickly dynamical signatures of both phenomena appear and how robustly they persist before the final steady state is reached.”

Referee: (6) *Authors claim “We find that an extensive skin effect is recovered at intermediate fluxes, rendering chiral damping observable again.” Do authors have some intuition on why the intermediate fluxes again allow for the skin effect, however for the small fluxes, the NHSE is suppressed as the Fig. 3 (a) shows.*

Reply: We thank the referee for this insightful question. In small flux regimes, as shown in [6], applying a magnetic field can suppress the non-Hermitian skin effect by pushing skin modes from the boundary back into the bulk. This suppression occurs because the magnetic field restores the standard bulk description that is otherwise invalidated by non-Bloch skin modes. Intuitively, this suppression must eventually cease, particularly upon reaching a full flux quantum, at which point the global skin effect is reinstated. While the precise dynamics of skin localization restoration at intermediate fluxes require further theoretical investigation, our numerical calculations confirm the validity of this behavior.

Changes to the Manuscript: “In small flux regimes [6], applying a magnetic field can suppress the NHSE by pushing skin modes from the boundary back into the bulk. This suppression occurs because the magnetic field restores the standard bulk description that is otherwise invalidated by non-Bloch skin modes. Intuitively, this suppression must eventually cease, particularly upon reaching a full flux quantum, at which point the global skin effect is reinstated. While the precise dynamics of skin localization restoration at intermediate fluxes require further theoretical investigation, our numerical calculations confirm the validity of this behavior.”

Referee: (7) *In the outlook, I would also suggest to mention ultracold atoms as a promising platform for simulating open quantum systems, and in which the signatures of the skin effect has been reported <https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.129.070401>*

Reply: Thank you for this great suggestion! We have included this point and the reference in the outlook section of our revised manuscript.

Changes to the Manuscript: “Ultracold atoms [7], where signatures of skin effect have been reported is also a promising platform for simulating open quantum systems such as ours.”

References

- [1] Bo Li, He-Ran Wang, Fei Song, and Zhong Wang. Non-bloch dynamics and topology in a classical nonequilibrium process. *Physical Review B*, 109(20):L201121, 2024.
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- [3] Taiki Haga, Masaya Nakagawa, Ryusuke Hamazaki, and Masahito Ueda. Liouvillian skin effect: Slowing down of relaxation processes without gap closing. *Physical Review Letters*, 127(7):070402, 2021.
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- [6] Ming Lu, Xiao-Xiao Zhang, and Marcel Franz. Magnetic suppression of non-hermitian skin effects. *Phys. Rev. Lett.*, 127:256402, Dec 2021.
- [7] Qian Liang, Dizhou Xie, Zhaoli Dong, Haowei Li, Hang Li, Bryce Gadway, Wei Yi, and Bo Yan. Dynamic signatures of non-hermitian skin effect and topology in ultracold atoms. *Physical review letters*, 129(7):070401, 2022.