

Directional-dependent Berezinskii-Kosterlitz-Thouless transition at EuO/KTaO₃(111) interfaces

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

Z. Huang et al. investigated the anisotropic Berezinskii–Kosterlitz–Thouless (BKT) transition temperature at the interface between EuO and a KTaO₃ (KTO)(111) substrate. The BKT transition is a quasi-long-range ordered phase transition, and its coherence is generally expected to be isotropic. The results presented in this study could have a significant impact on the research community focused on low-dimensional superconductivity, provided that the data analysis is physically justified. However, the interpretation and analysis of the experimental data remain questionable. The central issue in this work is whether the observed anisotropy of the BKT phenomena is an intrinsic physical property. Therefore, the authors must convincingly rule out the possibility that the anisotropy arises from extrinsic factors such as sample inhomogeneity or structural imperfections.

Q1.

The authors prepared MBE-grown polycrystalline EuO films on KTO substrates. It appears crucial to define the direction of the electrical current (I) in this study, as they referred to the $[11\bar{2}]$ and $[1\bar{1}0]$ directions of KTO, for example. However, since the interface between a polycrystalline film and the substrate is typically disordered, the electrons in the two-dimensional electron gas (2DEG) cannot exhibit well-defined energy dispersion in momentum space. In such a case, any dependence of transport data on the current direction would merely reflect planar inhomogeneity in the EuO film. The authors should therefore clarify the interface quality of the film—STEM imaging would be particularly helpful in this regard. If the interface quality is insufficient to justify discussion of crystalline azimuthal dependence, the authors must explain why their analysis of anisotropy in the BKT phenomenon is still valid.

Q2.

In Figs. 1b and S3, the authors used the van der Pauw (VDP) method to determine the sheet resistance (R_s). It is unclear why the VDP method was chosen for this measurement and how R_s was estimated. The VDP method assumes a homogeneous and isotropic sample; hence, R_s derived from this method cannot exhibit current-direction dependence. The sheet resistance is estimated using $R_{12,34} = V_{34}/I_{12}$ and $R_{23,41} = V_{41}/I_{23}$, where the contacts are numbered counterclockwise from 1 to 4. This means R_s contains contributions from both I_{12} and I_{23} (presumably $I \parallel [1\bar{1}0]$ and $I \parallel [1\bar{1}2]$ in this experiment). If R_s was estimated simply as $(V_+ - V_-) / (I_+ - I_-)$, such an approach would also be incorrect, as the current path is not well defined. The authors should address this issue and explain why the VDP method is appropriate for their purpose.

Q3.

In Figs. 2a and 2b, the authors examined the rotational symmetry of the BKT phenomenon. To verify the reproducibility of the anisotropic TBKT, the same ρ – T measurements as shown in Figs. 2a and 2b should be performed on another device (Device 2) fabricated at a different position on the same sample. Such measurements would help confirm that the observed anisotropy in TBKT is not due to sample inhomogeneity, by comparing the corresponding ρ – T curves.

Q4.

In Fig. 2d, why does T_c (not ΔT_c) depend on the current I ? This suggests that island-like superconducting regions with different T_c values may couple or decouple as I changes, similar to Josephson-junction arrays. If this interpretation is correct, inhomogeneity would play a dominant role in the superconducting and BKT behaviors of this system.

Q5.

In Fig. 3, the authors investigated nonreciprocal charge transport in EuO/KTO. To convincingly demonstrate the existence of nonreciprocal transport, they must present the current-magnitude and current-polarity dependencies of $R_2\omega$ —specifically, verifying $R_2\omega \propto I$ and $R_2\omega(+I) = -R_2\omega(-I)$. (See Y. M. Itahashi et al., Sci. Adv. 6, eaay9120 (2020)).

Q6.

The authors propose that the superconducting stripe model can explain the anisotropic TBKT. According to their previous study (Nat. Phys. 20, 957–963 (2024)), stripe formation in EuO/KTO(110) occurs below approximately 450 mK. In the present work, however, the BKT transition appears around 1.0–1.4 K. If this temperature range lies outside the stripe-forming region, can the stripe model still be applied? Moreover, why does the stripe orientation change from [001] to [112] simply by altering the substrate orientation?

Reviewer #2

(Remarks to the Author)

The article titled “Directional-dependent Berezinskii-Kosterlitz-Thouless transition at EuO/KTaO₃(111) interfaces” by Zongyao Huang et al. investigates the BKT transition along two in-plane crystallographic directions, namely [1-10] and [11-2]. The results suggest that the BKT transition temperature is higher along the [11-2] direction compared to the [1-10] direction. This behavior has been correlated with the formation of elongated stripes along [11-2], which are typically observed in anisotropic superconducting transport in KTaO₃-based interfaces. Overall, the study is timely and relevant, with carefully executed experimental work. The results are convincing, and the manuscript is generally clear and accessible. However, I have outlined a few comments that the authors should address before reaching a conclusion regarding publication in Communications Physics.

1- On page 4, the authors discuss that the sample with the highest carrier density does not display anisotropy or an anomalous Hall effect, and they further correlate this with the requirement of ferromagnetism in the 2DEG for stripe formation. This explanation is not clear. Could the authors elaborate on:

- (a) the reason for the absence of the anomalous Hall effect in this sample, which is unusual, and
- (b) why ferromagnetism is required for stripe formation?

2- In Figures 2(a,b), the authors investigate the superconducting transition in a double-tri-beam geometry along three channels: [11-2] and [1-10]. I was expecting, particularly for the [1-10] channels, that the channel perpendicular to [11-2] would show a lower T_c (as observed here), while the other two channels would have similarly higher T_c values. However, this is not the case. The authors should clarify this discrepancy.

3- In Figure 2(c), the authors estimate the BKT coefficient from the current-voltage (I-V) curves shown in supplementary Figure S6. Upon closer inspection of Figures S6(c,d) and comparing them with the R-T curve in Figure S6(b), it appears that the coefficient is estimated from the I-V region where the resistance exceeds 1 kohm. This region cannot be attributed to BKT physics, which is only valid in the low-resistance regime (below approximately 1 ohm). The authors should clarify or justify this analysis.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The authors have responded to all the questions sincerely and have added new experimental data. They have provided convincing evidence that the anisotropic BKT phenomenon originates from intrinsic factors. Although I am not entirely convinced that the stripe model can fully describe the anisotropic behavior of the BKT transition, the intriguing data presented in this manuscript merit publication and will be of significant interest to readers. Therefore, I recommend that the manuscript be accepted for publication in Communications Physics.

I look forward to further theoretical and experimental developments in this line of research, particularly regarding the mechanism by which a specific crystalline orientation results in a higher T_{BKT} compared with other phases.

Reviewer #2

(Remarks to the Author)

The authors have successfully answered all my queries; therefore, I recommend acceptance for publication in Communication Physics.

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List of major changes

Main text:

- 1) We have added three authors, Huiyu Wang, Zhen Wang and Jing Tao, to the author list. They contributed to the scanning transmission electron microscopy (STEM) measurements.
- 2) In **Figure 1b**, we added the data of superconducting transition temperature T_c measured on three new samples (Samples No. 8, 9 and 10, on which Devices 4, 5 and 6 were fabricated, respectively). Contexts in the manuscript were revised correspondingly.
- 3) Page 4, paragraph 1, we added contents referring to results of the STEM measurements on the EuO/KTO(111) interface samples. Here we emphasized that the crystal orientation of KTO remains distinguishable at the interface, and that the lattice distortion and atomic intermixing only occur within the first three unit cells of KTO.
- 4) “van der Pauw method” has been changed to “van der Pauw configuration”/“van der Pauw geometry” over the manuscript, in order to maintain scientific accuracy.
- 5) Ref. [27], i.e., the “footnote” mentioning Pearl screening length and the applicability of the BKT theory to our devices, has been removed in the revised manuscript. Its contents have been inserted to the second paragraph on page 5 and the section **Device fabrication** in **Methods**.
- 6) Page 6, paragraph 2, we added a brief note regarding to results of the measurements on two new devices, Device 4 and Device 5 (the “radial” Hall-bar devices), which are presented in detail in the Supplementary Information.
- 7) At the end of the same paragraph, the original sentence “Aside from the specific directions with ... is validated among all six channels” has been deleted, as new data from Devices 4 and 5 revealed less distinct behavior.
- 8) Page 7, the last paragraph, we added one sentence “The amplitude of the applied AC current (1 μ A) is within the linear-response regime ($R^{2\omega} \propto I$), see Supplementary Fig. 10d”.
- 9) Page 9, paragraph 2, we added one sentence “in particular, the I - V characteristics lack evidence for an arbitrary array of Josephson junctions (Supplementary Note 5)”.
- 10) Page 9, paragraph 3, after mentioning the meandering feature of putative superconducting stripes, we have supplied “this is corroborated by the results measured on six-beam Device 4 (Supplementary Table 4)”.
- 11) Page 10, paragraph 2, we added a brief discussion about the superconducting stripe model: Its temperature scale and characteristic directions are naturally dependent on KTO orientation; the former relies on the pairing strength, whereas

the latter implicitly changes with the Fermi surface structure of interfacial 2DEG. One new reference (Ref. [40]) has been inserted into this part.

- 12) Page 10, the last paragraph, we added “unmasked for both (110)- and (111)-oriented EuO/KTO interfaces”.to stress the connection between anisotropic T_c and ferromagnetic proximity.
- 13) The last paragraph of the main text, we noted two recent works (Refs. [45] and [46]), which also report directional-dependent BKT transition.
- 14) Page 16, **Methods**, in the section “**Device fabrication**”, we have added information about new devices (Devices 4-6).

Supplementary Information:

We adjusted the order of original figures (notes) and added more figures (notes) in the updated Supplemental Information; their numbering has been changed accordingly.

- 15) Updated **Supplementary Table 1**. The information of Samples No. 8-10 has been added. The correspondence between sample numbers and device numbers was also listed in this Table.
- 16) New **Supplementary Note 1**. An introduction to the STEM imaging of the interfaces along different orientations. Corresponding data are summarized in new Supplementary Fig. 2.
- 17) In **Supplementary Note 2**, we added contents elucidating the reason why anomalous Hall is absent in the sample with highest carrier density.
- 18) Updated **Supplementary Note 5**, in which the scenario of Josephson junction array at the interface has been discussed (and generally excluded based on the results of I - V characteristics). New references have been inserted to this part.
- 19) Updated **Supplementary Note 6**, we added one sentence “note that its definition requires the nonreciprocal response to satisfy $R^{2\omega} \propto BI$, which is valid in the low-field, low-current region” and the current dependence of $R^{2\omega}$ has been discussed based on the results of Supplementary Fig. 10d.
- 20) STEM images of the EuO/KTO(111) interface are shown in the new **Supplementary Fig. 2**.
- 21) Updated **Supplementary Fig. 4**, includes data for Samples No. 8-No. 10 measured using the van der Pauw configuration.
- 22) The new data measured on two “radial” devices with six Hall bars (Devices 4 and 5) are shown in the new **Supplementary Fig. 7**.
- 23) New **Supplementary Table 4** summarizing the BKT transition temperature measured for each channel of Devices 4 and 5.
- 24) We have also revised **Supplementary Fig. 10**, the current I dependence of $R^{2\omega}$ has been tested on Device 6 and the data are shown in Supplementary Fig. 10d.

Reply to Reviewer #1

Z. Huang et al. investigated the anisotropic Berezinskii – Kosterlitz – Thouless (BKT) transition temperature at the interface between EuO and a KTaO_3 (KTO)(111) substrate. The BKT transition is a quasi-long-range ordered phase transition, and its coherence is generally expected to be isotropic. The results presented in this study could have a significant impact on the research community focused on low-dimensional superconductivity, provided that the data analysis is physically justified. However, the interpretation and analysis of the experimental data remain questionable. The central issue in this work is whether the observed anisotropy of the BKT phenomena is an intrinsic physical property. Therefore, the authors must convincingly rule out the possibility that the anisotropy arises from extrinsic factors such as sample inhomogeneity or structural imperfections.

Reply: We sincerely thank Reviewer #1 for endorsing the significant impact of our work on the research community focused on low-dimensional superconductivity. We also appreciate the very relevant questions raised by the reviewer, which help us to improve the quality of the manuscript. We have since performed additional experiments to further verify our interpretations and to “rule out the possibility that the anisotropy arises from extrinsic factors” as required by the reviewer. These include scanning transmission electron microscopy (STEM) measurements, carried out by our colleague Huiyu Wang, Dr. Zhen Wang and Prof. Jing Tao (added to the author list in the revised manuscript), and transport measurements exploiting devices with different configurations. All the supporting experiments confirm our conclusion that the anisotropy of the Berezinskii-Kosterlitz-Thouless (BKT) transition is intrinsic at the $\text{EuO}/\text{KTaO}_3(111)$ interface. Details are presented in the following point-to-point responses. We hope that the newly-supplied data and the improved analysis can address Reviewer #1’s concern about extrinsic influences.

Q1.

The authors prepared MBE-grown polycrystalline EuO films on KTO substrates. It appears crucial to define the direction of the electrical current (I) in this study, as they referred to the $[11-2]$ and $[1-10]$ directions of KTO, for example. However, since the interface between a polycrystalline film and the substrate is typically disordered, the electrons in the two-dimensional electron gas (2DEG) cannot exhibit well-defined energy dispersion in momentum space. In such a case, any dependence of transport data on the current direction would merely reflect planar inhomogeneity in the EuO film. The authors should therefore clarify the interface quality of the film—STEM imaging would be particularly helpful in this regard. If the interface quality is insufficient to justify discussion of crystalline azimuthal dependence, the authors must explain why their analysis of anisotropy in the BKT phenomenon is still valid.

Reply: We thank Reviewer #1 for bringing up this critical issue. First, we would like to point out that the interfacial two-dimensional electron gas (2DEG) studied in this work resides in KTaO_3 (KTO), owing to the oxygen vacancies created by EuO film deposition in the topmost atomic layers of the KTO substrate; the typical thickness of surface conduction layer is a few nanometers. As such, the 2DEG predominantly comprises Ta $5d$ electrons. Because the KTO substrate is a single crystal, the band structure of the interfacial 2DEG inherits its symmetry in the momentum (k -) space, despite that the EuO overlayer is polycrystalline. This has been elucidated in numerous earlier works [e.g., H. Zhang, et al., *Phys. Rev. Lett.* 121, 116803 (2018); C. Liu, et al., *Science* 371, 716 (2021); S. Mallik, et al., *APL Mater.* 11, 121108 (2023); C. Liu, et al., *Nat. Commun.* 14, 951 (2023)]. Notably, angle-resolved photoemission spectroscopy (ARPES) data clearly corroborate that (i) the mobile electrons (2DEG) accumulate at the KTO side of the interface, (ii) the symmetry of the Fermi surfaces of interfacial 2DEG is imposed by the orientation of KTO substrate, two facts hold regardless of the presence of polycrystalline [S. Mallik, et al., *APL Mater.* 11, 121108 (2023)] or even amorphous overlayers [E. A. Martínez, et al., *Adv. Electron. Mater.* 9, 2300267 (2023); X. Chen, et al., *Nat. Commun.* 15, 7704 (2024)]. Judging from the existing information, we shall expect the EuO/KTO(111) interfacial 2DEG to exhibit well-defined energy dispersion that follows the threefold rotational symmetry of the KTO (111) surface; the sixfold rotational symmetry of corresponding Fermi surfaces has also been verified by ARPES [S. Mallik, et al., *APL Mater.* 11, 121108 (2023); X. Chen, et al., *Nat. Commun.* 15, 7704 (2024)].

This notwithstanding, it is still worth examining the morphology of the interface --- as correctly pointed out by Reviewer #1, there is still a possibility that the strong disorder at the interface renders the interfacial conduction layer polycrystalline (to some extent), and hence the 2DEG loses distinct k -dependent dispersions. To clarify this issue, we conducted scanning transmission electron microscopy (STEM) to directly image the EuO/KTO(111) interface. In these experiments, the sample was prepared using focused ion beam (Gallium) in advance. STEM images were collected on a Thermo Scientific Themis Z microscope (equipped with a probe-forming spherical-aberration corrector) along two different azimuths of the KTO substrate ([11-2] and [1-10]). The results are presented in Fig. R1-1. A clearly resolved abrupt interface region with a width less than 1 nm on the KTO side (Figs. R1-1b and c) is in agreement with previous reports [C. Liu, et al., *Science* 371, 716 (2021); X. Hua, et al., *npj Quantum. Mater.* 7, 97 (2022)]. The interfacial disorder, manifested as lattice distortion and Eu substitution on the K sites (caused by interfacial ionic intermixing), occurs only within this region, which does not exceed the first three unit cells of KTO from the interface.

Therefore, we conclude that although the EuO layer is polycrystalline (as shown in Figs. R1-1d and e), the interfacial disorder is limited to a roughly 1-nm layer at the surface of KTO and is thus insufficient to destroy the well-defined E - k relation of the interfacial itinerant electrons. We emphasize that the conducting layer in KTO is appreciably thicker than this disordered region: A thickness of 5.7 nm has been estimated based on its conductance [H. Zhang, et al., *Phys. Rev. Lett.* 121, 116803

(2018)], whereas the superconducting layer thickness in KTO-based heterostructures has been given as 6.4 nm [EuO/KTO(110), X. Hua, et al., *npj Quantum. Mater.* 7, 97 (2022)], 6 nm [LaAlO₃/KTO(111), Z. Chen et al., *Science* 372, 721 (2021)], 8 nm [LaAlO₃/KTO(110), Z. Chen et al., *Phys. Rev. Lett.* 126, 026802 (2021)] and 7–9 nm (this work, Supplementary Table 2). Considering a conducting layer thickness of 5–9 nm, the STEM results shown in Fig. R1-1 demonstrate that the major part of it occurs in the crystalline state of KTO, which is characterized by the absence of element substitution, lattice distortion or ferroelectric-type ion displacement. The symmetry arguments presented in our manuscript are thus justified; the crystalline azimuthal dependence of physical properties (including the BKT transition) reflects the intrinsic behavior of interfacial 2DEG, instead of merely planar inhomogeneity. The STEM data and corresponding discussion have been inserted to the updated manuscript (Supplementary Fig. 2 and Supplementary Note 1, properly noted in main text).

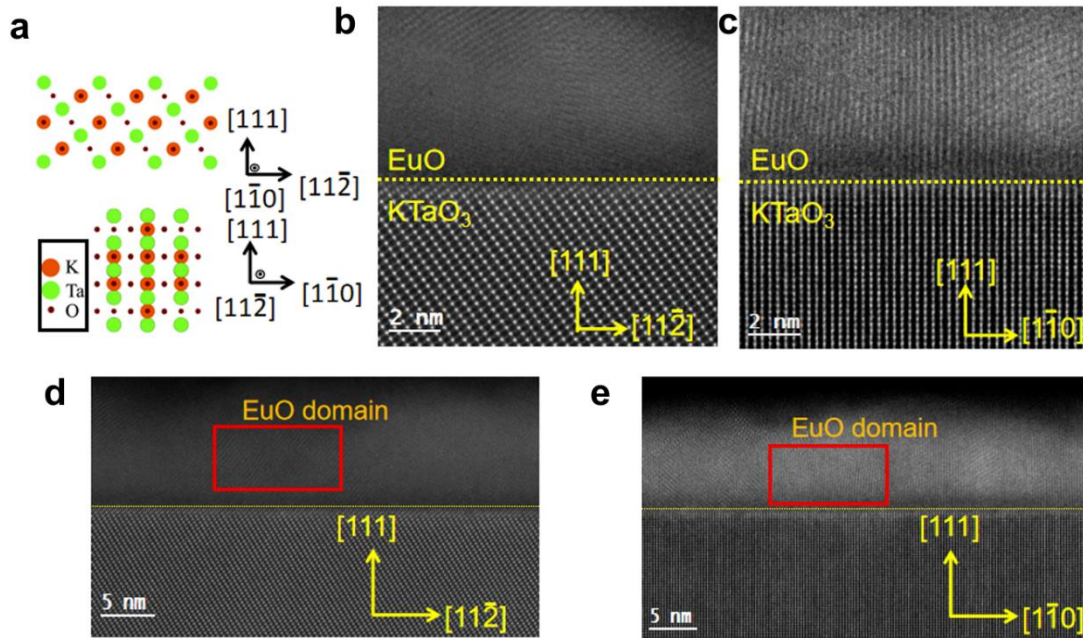


Fig. R1-1. **a** Schematics showing the atomic arrangement of KTO in the (1-10) and (11-2) planes. **b, c**, Cross-sectional atomic-resolution STEM images of the EuO/KTO(111) interface looking down the KTO [1-10] (**b**) and [11-2] (**c**) directions, respectively. Yellow dashed lines denote the position of interface. **d, e**, Low-magnification STEM images viewed along the KTO [1-10] (**d**) and [11-2] (**e**) directions, respectively. Red boxes roughly correspond to a polycrystalline grain of the EuO film.

In addition, we note that the in-plane directional dependence of the BKT transition temperature is highly systematic: Once the anisotropy sets in, the highest T_{BKT} always occurs along a specific [11-2] direction and the lowest along a [1-10], as observed in all prepared devices (Supplementary Tables 1, 3 and 4); the reversed result never happens. This can hardly be understood if only planar inhomogeneity plays a role, which inevitably leads to randomness in the distribution of T_{BKT} .

Q2.

In Figs. 1b and S3, the authors used the van der Pauw (VDP) method to determine the sheet resistance (R_s). It is unclear why the VDP method was chosen for this measurement and how R_s was estimated. The VDP method assumes a homogeneous and isotropic sample; hence, R_s derived from this method cannot exhibit current-direction dependence. The sheet resistance is estimated using $R_{12,34} = V_{34}/I_{12}$ and $R_{23,41} = V_{41}/I_{23}$, where the contacts are numbered counterclockwise from 1 to 4. This means R_s contains contributions from both I_{12} and I_{23} (presumably $I \parallel [1-10]$ and $I \parallel [11-2]$ in this experiment). If R_s was estimated simply as $(V_+ - V_-) / (I_+ - I_-)$, such an approach would also be incorrect, as the current path is not well defined. The authors should address this issue and explain why the VDP method is appropriate for their purpose.

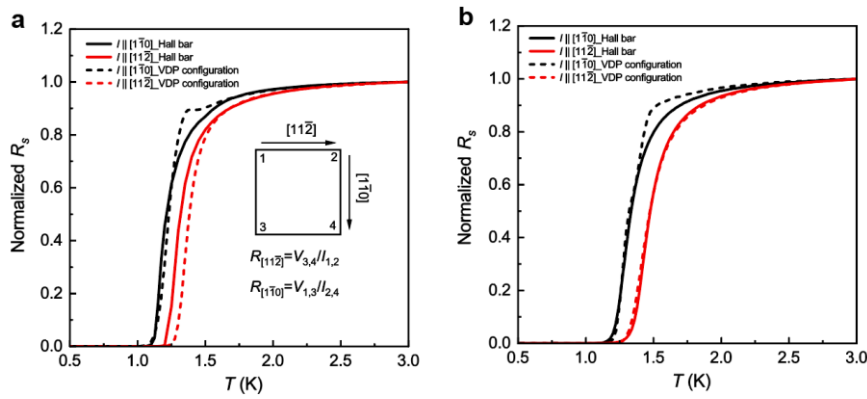


Fig. R1-2. Directional dependent superconducting transition in normalized sheet resistance measured in Sample No. 1 (a) and Sample No. 5 (b); data from the latter were not used in the manuscript. Dashed lines represent the results measured with the van der Pauw configuration, whereas solid lines are those measured on the standard Hall-bar devices.

Reply: We sincerely thank Reviewer #1 for raising this question. We realized that the term “van der Pauw (VDP) method” in the original version of the manuscript was inaccurate, and we have since replaced it with “van der Pauw (VDP) configuration” for scientific accuracy.

We agree with the reviewer that for a measurement employing the VDP method, one must switch the current direction along two orthogonal edges of a square-shaped sample; the resistance is consequently calculated based on the results of two rounds of voltage measurements. Rigorously, such a VDP measurement should not yield directional-dependent results as presented in our work. In practice, we mostly explore the directional-dependent properties of the EuO/KTO interfaces employing the fabricated devices with well-defined Hall-bar configurations, because we are aware of artificial features in the $R_{xx}(T)$ curves introduced by the VDP configuration [which has been elucidated in our previous work on EuO/KTO(110) heterostructures, X. Hua, et al., *Nat. Phys.* 20, 957 (2024)]. In the present work, all data shown in the main text figures were measured on Hall-bar devices, except for those displayed in Fig. 1b (raw

data in Supplementary Fig. 4). For the latter, we used the VDP configuration as illustrated in the inset of Fig. R1-2a: Four point contacts were made on the corners of the sample; current was applied along one edge (contacts 1 and 2), while the voltage signal was measured across the parallel edge (contacts 3 and 4). Strictly speaking, the calculated $R_s = R_{12,34} = V_{34}/I_{12}$ is not the “true” VDP sheet resistance of the sample. Hence, we avoid using this R_s in any quantitative analysis. Nevertheless, R_s obtained from such a process does capture the in-plane anisotropy (current-direction dependence) of the superconducting transition. As displayed in Fig. R1-2, the directional dependence of T_c shows good consistency between the data measured in the VDP and the standard Hall-bar configurations; there is almost no difference between the two data sets from Sample No. 5 (Fig. R1-2b). Therefore, we stress that the R_s data measured in the VDP configuration are still reliable for the purpose of determining (and quickly screening) the occurrence of anisotropic T_c among many samples. We did not go beyond this purpose when referring to the VDP configuration.

Q3.

In Figs. 2a and 2b, the authors examined the rotational symmetry of the BKT phenomenon. To verify the reproducibility of the anisotropic T_{BKT} , the same ρ - T measurements as shown in Figs. 2a and 2b should be performed on another device (Device 2) fabricated at a different position on the same sample. Such measurements would help confirm that the observed anisotropy in T_{BKT} is not due to sample inhomogeneity, by comparing the corresponding ρ - T curves.

Reply: We appreciate the reviewer’s suggestion to verify the reproducibility of the anisotropic T_{BKT} . Unfortunately, the two tri-beam Hall-bar devices almost occupy the entire available surface space of Sample No. 2, and hence we could not fabricate one more device on the same sample (see schematic in Supplementary Fig. 8a). In response to Reviewer #1’s comment, we selected two additional samples (No. 8, $n_{2D} = 7.46 \times 10^{13} \text{ cm}^{-2}$; No. 9, $n_{2D} = 7.40 \times 10^{13} \text{ cm}^{-2}$) and fabricated a new design of six-beam Hall bar device on each (Devices 4 and 5, respectively). In this new design (illustrated in the insets of Figs. R1-3a and b), three [11-2] arms and three [1-10] arms radiate from the same central point, which lies approximately at the center of the sample; we believe that by concentrating six Hall bars in a more compact area, we can reduce the potential influence of spatial inhomogeneity (which becomes significant over a large length scale). By performing measurements akin to those introduced in the main text, we obtained the directional dependent R_s curves for Sample No. 8 (Fig. R1-3a) and No. 9 (Fig. R1-3b); these outcomes allow us to draw the following conclusions:

- (i). For both devices, the highest T_{BKT} appears along one [11-2] channel, whereas the lowest T_{BKT} occurs for the [1-10] channel perpendicular to it. Such observation agrees with the results measured on Device 2 (Sample No. 2) shown in Figs. 2a and b, corroborating good data reproducibility.
- (ii). The main results of main text Fig. 2, i.e., the anisotropic T_{BKT} and the corresponding spontaneous rotational symmetry breaking, are verified in both

samples. The qualitative consistency strongly implies that these phenomena are intrinsic properties of our EuO/KTO interfaces and are not due to sample inhomogeneity.

(iii). Nevertheless, we found that whilst most [11-2] channels exhibit higher T_{BKT} than their [1-10] counterparts, this is not stringent. For instance, on Sample No. 8 (Fig. R1-3a), the [11-2] channel marked in green shows lower T_{BKT} than the two [1-10] channels that are not perpendicular to the [11-2] direction with the highest T_{BKT} . (Values of T_{BKT} of all channels are listed in Table R1-1).

The conclusion (iii) defies our original claim in the manuscript that “ $T_{\text{BKT}} (I \parallel [11-2]) \geq T_{\text{BKT}} (I \parallel [1-10])$ is validated among all six channels.” We have deleted this sentence according to the new data and added a brief note about the complicated behavior we observed. At this stage, we cannot give a clear description of the hierarchy for T_{BKT} among all six channels, apart from one [11-2] direction with the highest T_{BKT} and the orthogonal [1-10] direction which always exhibits the lowest T_{BKT} . Note that even when there are superconducting stripes as we have proposed, the stripes are likely to be meandering “rivers” instead of straight lines (see Z.-X. Li, et al., arXiv 2407.10269); therefore, T_{BKT} in a channel that is neither along nor perpendicular to the global stretching direction of the stripes may depend on the detailed condition of local disorder along the channel (which determines the paths along which superconducting stripes cross the channel). In this sense, here we should avoid making excessive comments on these channels. We will further scrutinize this issue by conducting more experimental tests in future works.

We added the new data presented in Fig. R1-3 and Table R1-1 to the Supplementary Information (the new Supplementary Fig. 7 and the newly-added Supplementary Table 4) and referred to it in the revised main text.

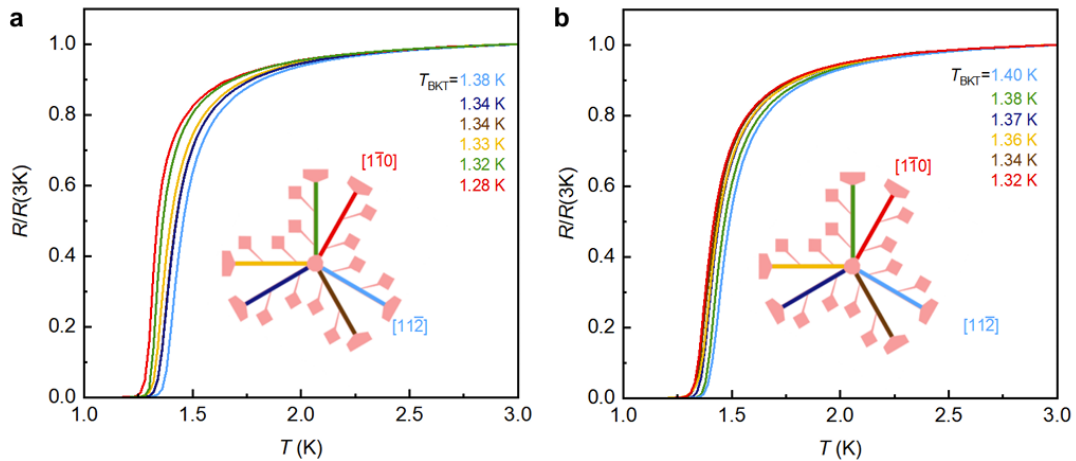


Fig. R1-3. Normalized R_s curves measured on radial devices composed of six Hall bars along three [11-2] and three [1-10] directions, fabricated on Sample No. 8 (a) and No. 9 (b), respectively. Patterns of the device are shown in the insets. Colors of the curves and the Hall-bar channels are set in correspondence. The BKT transition temperature for each channel is listed in the panels as well as in Table R1-1.

Table R1-1. T_{BKT} of the Hall-bar channels shown in Fig. R1-3 obtained by fitting the Halperin–Nelson formula to the resistivity data.

Sample	[11-2] channels			[1-10] channels		
	cyan	green	navy	red	amber gold	brown
No. 8	1.38 K	1.32 K	1.34 K	1.28 K	1.33 K	1.34 K
No. 9	1.40 K	1.38 K	1.37 K	1.32 K	1.34 K	1.36 K

Q4.

In Fig. 2d, why does T_c (not ΔT_c) depend on the current I ? This suggests that island-like superconducting regions with different T_c values may couple or decouple as I changes, similar to Josephson-junction arrays behaviors of this system.

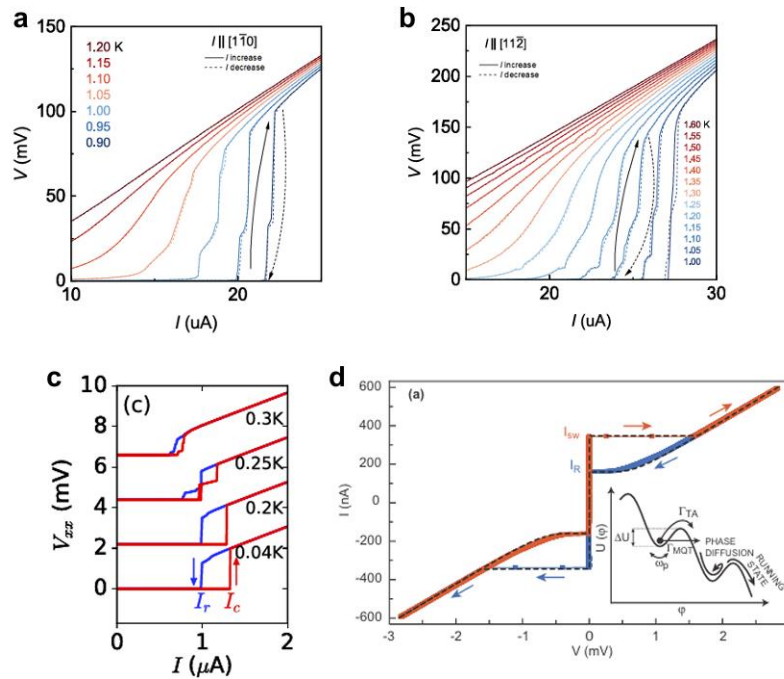


Fig. R1-4. **a, b** Current-voltage (I - V) characteristics of a EuO/KTO(111) interface, with current applied along the [1-10] direction (**a**) and the [11-2] direction (**b**). Same data plots as Supplementary Figs. 8e and f. **c, d** I - V characteristics of the LAO/STO interfaces, showing features indicative of Josephson-like dynamics [adapted from G. E. D. K. Prawiroatmodjo et al., *Phys. Rev. B* **93**, 184504 (2016) (**c**); S. Hurand et al., *Phys. Rev. B* **99**, 104515 (2019) (**d**)].

Reply: The superconducting transition of our EuO/KTO interface samples exhibits a weak current dependence, which becomes noticeable above $I \sim 1 \mu\text{A}$ (Supplementary Fig. 8b). We postulate that there are two most likely origins of this behavior. First, as the normal state resistance of the Hall bar devices is generally on the order of $1 \text{ k}\Omega$, a large current passing through it will cause Joule heating effect. As a result, the local temperature of the sample can be slightly higher than the reading of system

thermometer (mounted on the cryostat, not very close to the sample), rendering a lower “apparent” T_c for the case with larger I . Second, it is also possible that the spatial inhomogeneity of interfacial superconductivity comes into play. In a 2D superconductor with mesoscopic inhomogeneity, T_c can be suppressed with increasing currents; this is because a large current may break the fragile links between superconducting patches, subsequently hindering the establishment of a full superconducting path (which is described by a percolation process) [see, e.g., G. Venditti, et al., *Phys. Rev. B* **100**, 064506 (2019)]. In practice, such an effect also gives rise to a lowered apparent T_c with increasing I . Probably both mechanisms contribute to the observed current dependence in our samples.

However, here we would like to clarify that even the validity of the second mechanism mentioned above (i.e., the inhomogeneous superconductivity) does not necessarily mean the EuO/KTO samples can be seen as Josephson-junction arrays (JJAs). Whereas an inhomogeneous 2D superconductor is characterized by a distribution of local T_c , the Josephson-junction array scenario requires that the supercurrent is established by the Josephson tunneling of Cooper pairs across weak links --- which exist down to temperatures much lower than the apparent T_c . Interestingly, it has been argued that at the interface between LaAlO₃ and SrTiO₃ (LAO/STO), the 2D superconductivity occasionally behaves like a random array of Josephson-coupled superconducting islands [G. E. D. K. Prawiroatmodjo et al., *Phys. Rev. B* **93**, 184504 (2016); S. Hurand et al., *Phys. Rev. B* **99**, 104515 (2019)]. In such cases, the I - V characteristics usually exhibit a pronounced hysteresis upon reversal of current-sweep direction, persisting down to the lowest T . Here, we show the typical hysteretic I - V curves reproduced from works by Prawiroatmodjo et al. and Hurand et al. in Figs. R1-4c and d, respectively. Considering the large dielectric constants of both STO and KTO, we expect underdamped JJs characterized by a non-negligible capacitance in both cases, which inevitably gives rise to I - V hysteresis if the JJA scenario is valid. Such hysteresis, however, is absent in our EuO/KTO samples, as shown in Figs. R1-4a and b (which are identical to those in Supplementary Figs. 8e and f). Note that the narrow clockwise hysteresis loops observed for our samples stem from the Larkin-Ovchinnikov-type flux-flow instability as explained in the Supplementary Note 5; it is distinct from the anticlockwise hysteresis of critical current expected in an underdamped JJ. Hence, we conclude that there is no convincing evidence for Josephson-like dynamics at the EuO/KTO interfaces studied in this work. We have elucidated this in the main text and Supplementary Note 5.

Q5.

*In Fig. 3, the authors investigated nonreciprocal charge transport in EuO/KTO. To convincingly demonstrate the existence of nonreciprocal transport, they must present the current-magnitude and current-polarity dependencies of $R^{2\omega}$ — specifically, verifying $R^{2\omega} \propto I$ and $R^{2\omega}(+I) = -R^{2\omega}(-I)$. (See Y. M. Itahashi et al., *Sci. Adv.* **6**, eaay9120 (2020)).*

Reply: We are grateful to Reviewer #1 for this suggestion. We have since examined the current (I) dependence of the second-harmonic resistance ($R^{2\omega}$) signal on the driving current in a new sample (No. 10, $n_{2D} = 8.8 \times 10^{13} \text{ cm}^{-2}$). As shown in Fig. R1-5, at $T = 1.7 \text{ K}$, $R^{2\omega}$ increases linearly with the I in the low-current region ($|I| < 3 \text{ }\mu\text{A}$). Reversing the polarity of I induces a π -phase shift in the signal, which corresponds to a sign reversal of the second-harmonic voltage, thereby confirming the relationship $R^{2\omega}(+I) = -R^{2\omega}(-I)$.

The $R^{2\omega}(I)$ curve displayed in Fig. R1-5 not only convincingly demonstrates the existence of nonreciprocal transport at the EuO/KTO(111) interface (manifesting as $R^{2\omega}$), but also verifies that bias current employed in the $R^{2\omega}$ measurement discussed in the main text ($I = 1 \text{ }\mu\text{A}$, used for all data shown in Figs. 3 and 4) is in the linear response regime: $R^{2\omega} \propto I$, thus the parameter $\gamma = 2R^{2\omega}/(\mu_0 H R^\omega)$ can be safely defined at the low- H limit. We have included Fig. R1-5 in Supplementary Fig. 10 as a newly-added panel (d) and mentioned it in the main text. When $|I| > \sim 3 \text{ }\mu\text{A}$, $R^{2\omega}$ loses the I -linear dependence and develops a non-monotonic behavior at higher I , implying current suppression of superconductivity (which is also valid in the superconducting fluctuation regime). This observation is consistent with previous result measured in polar superconductor SrTiO₃ [Y. M. Itahashi et al., *Sci. Adv.* **6**, eaay9120 (2020)].

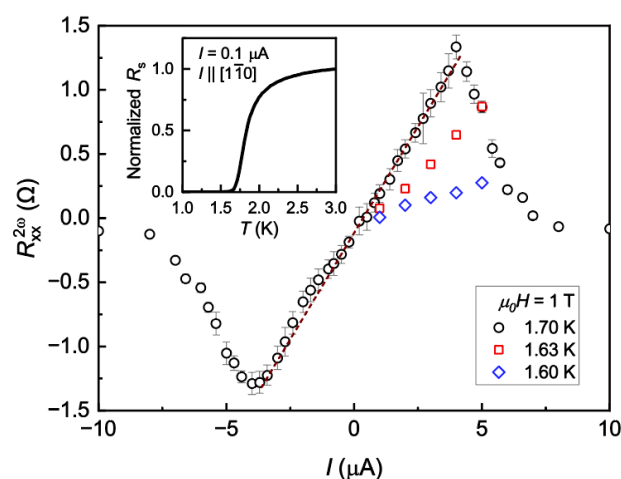


Fig. R1-5. Current dependence of $R^{2\omega}$ measured at 1.7 K (black circles), 1.63 K (red squares) and 1.6 K (blue diamonds) under a magnetic field of 1 T applied along the in-plane $[11-2]$ direction. Data were taken on a Hall-bar device patterned on Sample No. 10 ($n_{2D} = 8.8 \times 10^{13} \text{ cm}^{-2}$) with current applied along the $[1-10]$ direction. Inset: T -dependent resistance R_s measured on the same device. Note that I is an AC current and the sign on the transverse axis denotes its phase.

Q6.

The authors propose that the superconducting stripe model can explain the anisotropic TBKT. According to their previous study (Nat. Phys. 20, 957 – 963 (2024)), stripe formation in EuO/KTO(110) occurs below approximately 450 mK. In the present work, however, the BKT transition appears around 1.0 – 1.4 K. If this temperature range lies outside the stripe-forming region, can the stripe model still be

applied? Moreover, why does the stripe orientation change from [001] to [112] simply by altering the substrate orientation?

Reply: First of all, we would like to emphasize that the superconducting stripe model [proposed in our previous work, X. Hua, et al., *Nat. Phys.* 20, 957 (2024)] is a phenomenological and hypothetical picture aiming at interpreting the unexpected observation of directional dependent superconducting phase transition at the EuO/KTO(110) interfaces. In the present work, we mention that the same model may be able to explain the exotic BKT physics at EuO/KTO(111) interfaces as well. Here, in response to Reviewer #1's question, we briefly discuss the temperature scales and orientations of the stripe phase at EuO/KTO (110) and (111) interfaces within the framework of the superconducting stripe scenario.

If the anisotropic T_c (and T_{BKT}) originates from the formation of superconducting stripes, then we can tentatively align the onset temperature of such stripe phase (at which superconducting paths become fully developed along the stripe direction) with the highest value of the directional-dependent T_c at the interface; meanwhile, the stripes are assumed to run along the in-plane direction with the highest T_c . As such, we propose that:

- At EuO/KTO(110) interfaces, stripes stretch along [001], perpendicular to [1-10]; the onset temperature is approximately $T_c[001] \approx 0.37\text{-}0.9$ K (depending on n_{2D}).
- At EuO/KTO(111) interfaces, stripes stretch along [11-2], perpendicular to [1-10]; the onset temperature is approximately $T_c[11-2] \sim 1.1\text{-}1.6$ K (depending on n_{2D}).

Apparently, the superconducting stripe phase shows different onset temperatures and reference directions at superconducting EuO/KTO interfaces with different (KTO) orientations. However, this is not very surprising: Since the superconducting stripe phase is likely to be linked to emergent electronic instability, its properties must be delineated by the electronic structure of the interfacial 2DEG, which in turn depends on the orientation of the KTO substrate.

As we mention in the reply to Q1, ARPES measurements have revealed that the Fermi surface geometry for KTO-based interfacial 2DEGs largely emulates the projection of KTO bulk Fermi surface onto the corresponding surfaces [S. Varotto, et al., *Nat. Commun.* 13, 6165 (2022); E. A. Martínez, et al., *Adv. Electron. Mater.* 9, 2300267 (2023); S. Mallik, et al., *APL Mater.* 11, 121108 (2023); X. Chen, et al., *Nat. Commun.* 15, 7704 (2024)]. Therefore, we expect that the Fermi surfaces of 2DEGs at the EuO/KTO(110) and (111) interfaces have two-fold and six-fold rotational symmetry, respectively [see, the ARPES paper listed above and calculation results in X. Hua, et al., *Nat. Phys.* 20, 957 (2024)]. It is natural to see the change of direction of the superconducting stripes between these two orientations of interfaces, considering their distinct Fermi surface structures. We cannot explain at this stage why exact these two directions ([001] for the former and [11-2] for the latter) are special --- because the microscopic mechanism for the stripe formation remains unclear.

Moreover, it is well known that the KTO-based interfacial 2DEGs harbor

orientation-dependent superconductivity with its optimal T_c ubiquitously relying on the orientation of the KTO substrate [most typically, the optimal T_c is ~ 0.05 K, ~ 1 K, ~ 2 K for (001)-, (110)- and (111)-oriented KTO, see, e.g., C. Liu, et al., *Nat. Commun.* **14**, 951 (2023); Z. Chen, et al., *Phys. Rev. Lett.* **126**, 026802 (2021)]. This unique phenomenon is absent at superconducting LAO/STO interfaces. It has been elucidated that the orientation-dependent superconductivity originates from the combined influence of interfacial (Ta $5d t_{2g}$) orbital degeneracy and electron-phonon coupling strength, all of which depend on the KTO orientation; the (111)-oriented interface has the highest level of orbital degeneracy and the strongest electron-phonon coupling, thus showing the highest T_c [C. Liu, et al., *Nat. Commun.* **14**, 951 (2023); X. Chen, et al., *Nat. Commun.* **15**, 7704 (2024)]. Since the formation of superconducting stripes also requires pairing instability, it is not unexpected that its temperature scale has orientation dependence following the trend for normal superconducting pairing, which is consistent with our experimental results (i.e., the stripes emerge at a higher temperature at the EuO/KTO(111) interfaces compared with EuO/KTO(110)).

To conclude, we infer from the existing literature that the orientation-dependent (interfacial 2DEF's) electronic band structure and interaction strength may contribute to the apparent difference between the onset temperature and direction of superconducting stripes at EuO/KTO (111) and (110) surfaces. We have included some of the key points mentioned above in the Discussion section of the revised manuscript and in the Supplementary Information.

Again, we sincerely thank Reviewer #1 for all these constructive comments on our manuscript. To answer the most important question raised by the reviewer --- whether the anisotropic BKT transition is intrinsic --- we have conducted a series of additional experiments (including STEM characterization, transport measurements on new samples with specially-designed Hall-bar patterns, current dependence of second harmonic resistance etc). All those results point towards intrinsic origin of the directional-dependent BKT transition at the EuO/KTO(111) surfaces, thereby comprehensively validating our analysis and strengthen our original arguments. We also mention that directional-dependent superconducting transition has been reported for other KTO(111) interfaces that possess either magnetic proximity effect [e.g., LaSrMnO₃/KTO(111), E. G. Arnault, et al., *Sci. Adv.* **9**, eadf1414 (2023); Eu/KTO(111), S. Mallik, et al., *APL Mater.* **11**, 121108 (2023)] or itinerant ferromagnetism [a-CaZrO₃/KTO(111), H. Zhang, et al., *Nat. Commun.* **16**, 3035 (2025)]. These works congruently show higher (lower) T_c along [11-2] ([1-10]). Despite the lack of BKT analysis in these systems, we believe that such consistency also supports the intrinsic nature of the directional-dependent BKT transition reported herein. We hope that, with all the revisions and corrections, the update manuscript can match the reviewer's standard for publication in *Communications Physics*.

Reply to Reviewer #2

The article titled “Directional-dependent Berezinskii-Kosterlitz-Thouless transition at EuO/KTaO₃(111) interfaces” by Zongyao Huang et al. investigates the BKT transition along two in-plane crystallographic directions, namely [1-10] and [11-2]. The results suggest that the BKT transition temperature is higher along the [11-2] direction compared to the [1-10] direction. This behavior has been correlated with the formation of elongated stripes along [11-2], which are typically observed in anisotropic superconducting transport in KTaO₃-based interfaces. Overall, the study is timely and relevant, with carefully executed experimental work. The results are convincing, and the manuscript is generally clear and accessible. However, I have outlined a few comments that the authors should address before reaching a conclusion regarding publication in Communications Physics.

Reply: We are very thankful to Reviewer #2 for their positive evaluation of our work and, in particular, for kindly recommending publication. Meanwhile, we really appreciate Reviewer #2’s insightful comments and suggestions. We have addressed all questions in the following point-to-point response with the revisions made accordingly in the manuscript.

1- On page 4, the authors discuss that the sample with the highest carrier density does not display anisotropy or an anomalous Hall effect, and they further correlate this with the requirement of ferromagnetism in the 2DEG for stripe formation. This explanation is not clear. Could the authors elaborate on:

(a) the reason for the absence of the anomalous Hall effect in this sample, which is unusual, and

(b) why ferromagnetism is required for stripe formation?

Reply: We thank Reviewer #2 for raising these very relevant questions, which encourage us to clarify two important points. We have also elucidated the main points of the following contents in the revised manuscript.

(a) Absence of the anomalous Hall effect (AHE) in Sample No. 7

The AHE observed in our EuO/KTaO₃ (EuO/KTO) heterostructure samples exhibits characteristics (e.g., the onset temperature, the saturation field and the profile of its field dependence) consistent with the ferromagnetism in the EuO layer, as has been explicitly mentioned in our previous work [X. Hua, et al., *Nat. Phys.* 20, 957 (2024)] on EuO/KTO(110) interfaces. This strongly suggests that the AHE originates from a ferromagnetic proximity effect: The interfacial 2DEG (predominantly Ta 5d electrons) acquires spin polarization through hybridization with the Eu 5d electrons, which are strongly spin-polarized via *d-f* exchange interaction with the Eu 4f local moments that contribute to the ferromagnetic order [H. Zhang, et al., *Phys. Rev. Lett.* 121, 116803 (2018)]. The remarkable spin polarization on the Fermi surface implies that the interfacial 2DEG can be seen as a 2D itinerant ferromagnet. Earlier calculations for EuO/KTO(110) interface suggested that the spin splitting in the Ta 5d bands decreases

at higher band filling; when the Fermi level rises to a position sufficiently far from the band bottom (where the hybridization is the strongest), the spin polarization at the Fermi level becomes negligible [X. Hua, et al., *Nat. Phys.* 20, 957 (2024)]. Such a scenario provides qualitative interpretation for the vanishing AHE in the samples with higher carrier density (i.e., higher Fermi level). We believe that the same scenario can also be applied to the EuO/KTO(111) interfaces studied in this work. As Sample No. 7 has the highest carrier density n_{2D} ($= 9.79 \times 10^{13} \text{ cm}^{-2}$), its band filling is higher than other samples, and its Fermi level may be located at a position in the Ta $5d$ band where the ferromagnetic proximity is insignificant and thus spin splitting is negligible; thereby, the AHE is absent due to persistent spin degeneracy. We have added this elucidation to Supplementary Note 2.

(b) Connection between ferromagnetism and stripe formation

To date, the connection between ferromagnetism and stripe formation is mainly phenomenological. We stress that the superconducting stripe model is hypothetical: None of the available theories can unambiguously predict the emergence of superconducting stripes from a ferromagnetic 2DEG --- however, despite the lack of theoretical understanding, this is exactly the case implied by experimental results from several different groups. Apart from our earlier work mentioned above, directional-dependent superconducting transition has also been reported for other KTO-based interfaces that are likely to experience ferromagnetic proximity effect [E. G. Arnault, et al., *Sci. Adv.* 9, eadf1414 (2023); S. Mallik, et al., *APL Mater.* 11, 121108 (2023)] or possess Stoner-type itinerant ferromagnetism without a magnetic overlayer [α -CaZrO₃/KTO(111), H. Zhang, et al., *Nat. Commun.* 16, 3035 (2025)]. Whether such ubiquitous link between directional-dependent superconductivity and ferromagnetism at KTO-based interface is related to exotic pairing states that have been proposed for ferromagnetic superconductors --- e.g., Fulde-Ferrell states or p -wave superconductivity --- remains to be answered by further theoretical studies. Indeed, during the private communications with some prominent theorists, we have learned that modeling of the directional-dependent BKT transition is theoretically very challenging, even with the consideration of the interfacial ferromagnetism; hence, establishment of a clear picture how superconducting stripes develop from a ferromagnetic 2DEG is beyond the scope of our present work. Nevertheless, we can still say that all experimental evidence at hand points towards a scenario in which the putative formation of superconducting stripes (if we use these to explain the directional-dependent superconducting transition in all cases) relies on ferromagnetic interfacial 2DEGs with spin-polarized Fermi surfaces, as has been stressed in the last paragraph of the manuscript.

2- In Figures 2(a,b), the authors investigate the superconducting transition in a double-tri-beam geometry along three channels: [11-2] and [1-10]. I was expecting, particularly for the [1-10] channels, that the channel perpendicular to [11-2] would show a lower T_c (as observed here), while the other two channels would have similarly higher T_c values. However, this is not the case. The authors should clarify this discrepancy.

Reply: We thank Reviewer #2 for pointing out this issue. To verify the results of such tri-beam Hall-bar measurement, we have since conducted additional measurements on two more samples (No. 8, $n_{2D} = 7.46 \times 10^{13} \text{ cm}^{-2}$; No. 9, $n_{2D} = 7.40 \times 10^{13} \text{ cm}^{-2}$) and fabricated a new design of six-beam “radial” Hall bar device on each. The pattern of the six-beam device is illustrated in the insets of Fig. R2-1; it is composed of three [11-2] channels and three [1-10] channels, all of which having one end merging on one same spot on the sample, in order to minimize the influence of spatial inhomogeneity. As shown in Fig. R2-1, the measurement results on these two new devices further clarify the hierarchy of T_{BKT} among these channels:

- (i). For both devices, the highest T_{BKT} appears along one [11-2] channel. Meanwhile, the lowest T_{BKT} occurs for the [1-10] channel perpendicular to it. Such observations consistently reinforce the results measured on the tri-beam Device 2 (Sample No. 2) shown in Figs. 2a and b, corroborating good data reproducibility.
- (ii). The main results of main text Figs. 2, i.e., the anisotropic T_{BKT} and the corresponding spontaneous rotational symmetry breaking, are verified in both samples. The qualitative consistency strongly implies that these phenomena are intrinsic properties of our EuO/KTO interfaces and are not due to sample inhomogeneity.
- (iii). Nonetheless, we found that apart from the [11-2] direction with the highest T_{BKT} and the [1-10] direction perpendicular to it with the lowest T_{BKT} , the other four channels do not show stringent hierarchy of T_{BKT} . For instance, the [11-2] channel on Sample No. 8 marked by green color (Figs. R2-1a) shows the second lowest T_{BKT} that is lower than the [1-10] channels colored by orange and brown, respectively.

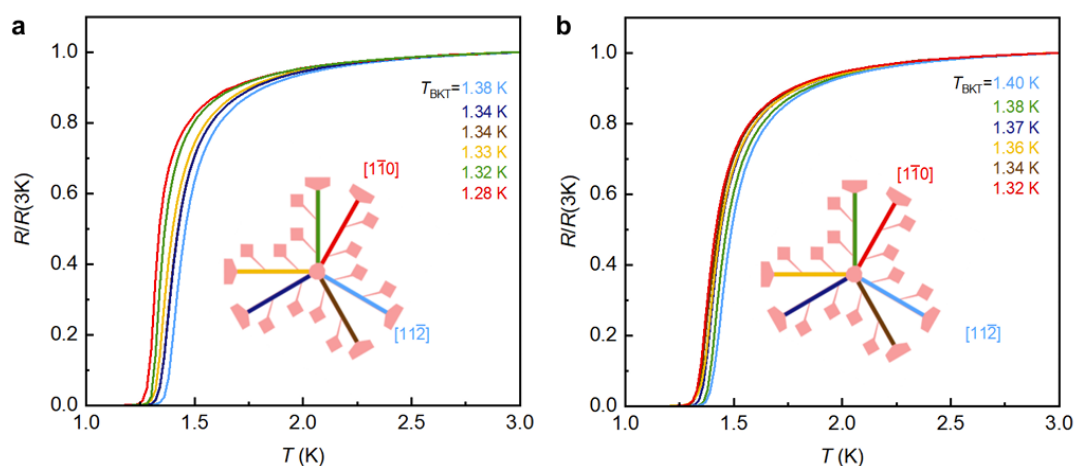


Fig. R2-1. Normalized R_s curves measured on devices composed of six Hall bars along three [11-2] and three [1-10] directions, fabricated on Sample No. 8 (a) and No. 9 (b), respectively. Patterns of the devices are shown in the insets. Colors of the curves and the Hall-bar channels are set in correspondence. The BKT transition temperature for each channel is listed in the panels.

The observation (iii) invalidates our original claim in the manuscript that “ $T_{\text{BKT}} (I \parallel [11-2]) \geq T_{\text{BKT}} (I \parallel [1-10])$ is validated among all six channels.” We have deleted this sentence according to the new results and added a brief note about the complicated behavior we observed. At this stage, we cannot give a clear description of the T_{BKT} among all six channels, apart from one $[11-2]$ direction with the highest T_{BKT} and the orthogonal $[1-10]$ direction which always exhibits the lowest T_{BKT} . Note that even when there are superconducting stripes as we have proposed, the stripes are likely to be meandering “rivers” instead of straight lines (see Z.-X. Li, S. A. Kivelson, D.-H. Lee, arXiv 2407.10269); therefore, T_{BKT} in a channel that is neither along nor perpendicular to the global stretching direction of the stripes may depend on the detailed condition of local disorder along the channel (which determines the paths where superconducting stripes cross the channel). In this sense, here we should avoid making excessive comments on these channels. We will further scrutinize this issue by conducting more experimental tests in future works.

We have added the new data presented in Fig. R2-1 and the corresponding results to the Supplementary Information (new Supplementary Fig. 7 and the newly-added Supplementary Table 4) and referred to these in the revised main text.

3- In Figure 2(c), the authors estimate the BKT coefficient from the current-voltage (I - V) curves shown in supplementary Figure S8. Upon closer inspection of Figures S6(c,d) and comparing them with the R - T curve in Figure S6(b), it appears that the coefficient is estimated from the I - V region where the resistance exceeds 1 kohm. This region cannot be attributed to BKT physics, which is only valid in the low-resistance regime (below approximately 1 ohm). The authors should clarify or justify this analysis.

Reply: We highly appreciate the reviewer’s careful examination of our data. The current-voltage (I - V) characteristics displayed in Supplementary Figs. 8c and d were measured on two Hall-bar channels in Device 2. Considering the dimensions of the channel ($600 \times 100 \mu\text{m}^2$), the resistance of $\sim 1 \text{ k}\Omega$ (determined at the onset of $V \propto I^3$ behavior on the transverse axis) at $T \approx T_{\text{BKT}}$ reflected in the I - V curves converts to a sheet resistance R_s of approximately $150\text{-}200 \Omega/\text{sq}$, which corresponds to less than 20% of the normal-state R_s measured on the same Hall bars (Supplementary Fig. 8b). This seems to be still a reasonable value for us. According to the Halperin-Nelson formula:

$$\frac{R(T)}{R_N} = [1 + \frac{4}{A^2} \sinh^2(2a \sqrt{\frac{T_c^{\text{MF}} - T_{\text{BKT}}}{T_{\text{BKT}} - T}})]^{-1}$$

(R_N is the normal state resistance, A is a dimensionless constant of order 1, a is related to the vortex core energy), the resistance should reach zero at T_{BKT} . However, this formula can only be applied at the low-current limit; increasing I will provoke a finite (non-ohmic) resistance even below T_{BKT} , owing to the current-induced dissociation of vortex-antivortex pairs. Although the case of large current (as in the I - V measurements) has been rarely touched in BKT analysis, it has been established by B.

I. Halperin and D. R. Nelson that such nonlinear resistance scales with the normal state resistance R_N [J. Low Temp. Phys. 36, 599 (1979)]; they provide a complicated form of the current-dependent nonlinear resistivity $\rho(I, T)$ which, unfortunately, cannot be employed in our analysis --- due to the insufficient knowledge of certain parameters such as the coherence length $\xi(T)$. This notwithstanding, we are unaware of any mechanism that leads to an upper limit of resistance for the BKT physics.

Here we note that the determination of BKT exponent $\alpha(T)$ (Fig. 2c) of the nonlinear I - V characteristics was forced to be performed in the high-current regime ($I > 5 \mu\text{A}$) of the I - V curves (dashed lines in Supplementary Figs. 8c and d). This is because the $V \propto I^\alpha$ relation cannot be extended down to the low-current limit; the low-current I - V characteristics are contaminated by the semi-ohmic behavior stemming from finite-size effect [N. Reyren et al., *Science* 317, 1196 (2007) and the references therein]. As such, the relatively large applied currents, combined with a high normal-state resistance on the order of 1 k Ω (Supplementary Fig. 8b), may naturally give rise to the appreciable $R_s \sim 150\text{--}200 \Omega/\text{sq}$ at $T \approx T_{\text{BKT}}$ in our devices.

We further point out that a considerably large resistance value can be derived from the nonlinear I - V characteristics measured in numerous 2D superconducting systems, especially those with higher normal-state resistance (mostly due to reduced thicknesses). These results are summarized in Table R2-1. In practice, this observation is quite common among the 2D superconductors, while BKT analysis has been routinely performed in these systems regardless of it. We thus suggest that the large resistance value yielded from the nonlinear I - V characteristics does not invalidate the BKT analysis in our manuscript.

Table R2-1 Resistance $R_{I-V}(T_{\text{BKT}})$ calculated from the nonlinear I - V characteristics in various 2D superconducting systems at $T \approx T_{\text{BKT}}$. Here we took the resistance value corresponding to the low-current end of the $V \propto I^3$ behavior on the I - V curves.

Sample	$R_{I-V}(T_{\text{BKT}})$ (Ω)	Normal-state R_s (Ω/sq)	Reference
LaAlO ₃ /SrTiO ₃ (001) interface (8 uc)	~ 250	~ 400	[1]
Ultrathin NbN films	~ 1000	unknown	[2]
LaAlO ₃ /KTaO ₃ (110) interface	~ 1000	~ 1200	[3]
Nd _{0.8} Sr _{0.2} NiO ₂ /SrTiO ₃ superlattice	~ 800	~ 1000	[4]
CaZrO ₃ /KTaO ₃ (111) interface	~ 120	~ 1000	[5]
Ion-liquid-gated KTaO ₃ (111) surface	~ 100	~ 220	[6]

- [1] N. Reyren et al., *Science* 317, 1196 (2007)
- [2] G. Venditti, et al., *Phys. Rev. B.* 100, 064506 (2019)
- [3] Z. Chen, et al., *Phys. Rev. Lett.* 126, 026802 (2021)
- [4] W. Xiao, et al., *Nat. Commun.* 15,10215(2024)
- [5] H. Zhang et al., *Nat. Commun.* 16, 3035 (2025)
- [6] T. Ren, et al., *Sci. Adv.* 8, eabn4273 (2022)

We thank Reviewer #2 again for approving the quality and significance of our work. The constructive comments and suggestions provided by the reviewer are very helpful for the improvement of our manuscript. We hope our responses can adequately address the reviewer's questions.

Response to Reviewer #1:

The authors have responded to all the questions sincerely and have added new experimental data. They have provided convincing evidence that the anisotropic BKT phenomenon originates from intrinsic factors. Although I am not entirely convinced that the stripe model can fully describe the anisotropic behavior of the BKT transition, the intriguing data presented in this manuscript merit publication and will be of significant interest to readers. Therefore, I recommend that the manuscript be accepted for publication in Communications Physics.

I look forward to further theoretical and experimental developments in this line of research, particularly regarding the mechanism by which a specific crystalline orientation results in a higher T_{BKT} compared with other phases.

Reply: We sincerely thank Reviewer #1 for recommending publication. We are working hard to design new experimental setups to further reveal the mechanism of the anisotropic BKT transition. We hope that our results can inspire new theoretical understandings.

Response to Reviewer #2:

The authors have successfully answered all my queries; therefore, I recommend acceptance for publication in Communication Physics.

Reply: We are grateful Reviewer #2 for recommending publication. We also thank the reviewer for providing valuable comments in the last round of review.