nature portfolio

Peer Review File

Pseudo-Bifurcations in Stochastic Non-Normal Systems and the Limits of Early-Warning Signals

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

I have reviewed the manuscript entitled “Illusions of Criticality: Crises Without Tipping Points.” The manuscript addresses an important and timely issue concerning the properties of non-normal systems, particularly the transient growth of perturbations and their strong sensitivity to noise. It emphasizes that commonly used indicators of criticality may arise from non-normal dynamics rather than from genuine transitions in the system’s asymptotic solutions.

While the topic is highly relevant and the results appear sound, the manuscript does not convey these points clearly or effectively. The text is overly intricate, making it difficult to clearly connect the listed results with their corresponding demonstrations.

Moreover, conceptual inconsistencies appear at several points throughout the manuscript. For example, the first sentence of the abstract states: “Abrupt shifts in ecosystems, brains, markets, and climate are often diagnosed as signs of approaching a tipping point, i.e. a critical bifurcation where stability is lost.” However, in catastrophic bifurcations associated with tipping points, stability is not merely lost; rather, the stable state itself disappears.

To cite one more conceptual issue, in the context of structural stability. The term “crises,” used in the manuscript title, has a precise meaning in dynamical systems theory: a crisis is a global bifurcation in which an invariant set (typically a chaotic attractor) undergoes an abrupt qualitative change (such as sudden enlargement, destruction, or disappearance) due to its collision with an unstable invariant set (typically an unstable periodic orbit or its stable manifold). In this sense, the use of the term “crises” is misleading, as the manuscript focuses exclusively on local bifurcation scenarios.

In its current form, I do not recommend this manuscript for publication. However, I would be happy to review a revised version in which the same results are demonstrated more clearly, both numerically and analytically.

Reviewer #2

(Remarks to the Author)
This paper introduces the novel concept of pseudo-bifurcations, offering a fundamentally new perspective on so-called “tipping points” (critical bifurcations) in dynamical systems. The work is highly original and innovative, and in my view merits publication.
The authors convincingly demonstrate the power of their approach through a compelling real-world application to epileptic seizures. They show that pseudo-bifurcations can faithfully reproduce the full suite of early-warning signals conventionally regarded as evidence of proximity to a tipping point, including critical slowing down, increased variance, and dimensional collapse.
This result indicates that the concept of pseudo-bifurcations provides a powerful alternative framework for understanding critical dynamical phenomena, one that goes beyond the conventional treatment of nonlinear dynamics as mere perturbations of underlying linear systems. Instead, it elegantly captures the hidden geometry of non-normal dynamics, revealing genuine mechanisms where traditional approaches may only produce the illusion of criticality.

The manuscript is exceptionally well written. The new concepts are presented with admirable clarity and readability, and the application to epilepsy, both conceptual and data-driven, is truly impressive. The paper concludes with a thoughtful broader discussion of the significance of this framework.
In summary, this is a milestone contribution. I expect it to have major impact across the scientific community by opening a genuinely new avenue for analyzing dynamical critical phenomena and tipping points, distinct from mainstream nonlinear dynamics approaches.
I recommend publication as is, without delay.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The authors have made punctual modifications following my previous concerns. However, the manuscript still presents conceptual problems that make it difficult for the reader to follow.

For instance, the original title was “Illusions of Criticality: Crises Without Tipping Points.” Due to the confusion associated with the term “crises,” the authors changed it to “Illusions of Criticality: Transitions Without Tipping Points.” This formulation is also problematic, since transitions that are not tipping points naturally occur in dynamical systems. An obvious example is a noisy system that escapes the vicinity of one attractor and visits another, this is not a tipping point. Another example is a heteroclinic cycle, in which several saddle structures are connected by deterministic trajectories, every transition is not a tipping point.

There seems to be a persistent tendency in the manuscript to use the term tipping point without necessarily referring to the phenomenon typically associated with it. The term tipping point is rather informal and has been widely adopted, especially in ecological contexts, to refer to abrupt shifts in system dynamics as its conditions are systematically varied, often associated with hysteresis. There are also broader uses of the concept, such as noise-induced tipping or rate-induced tipping, but the manuscript does not clearly distinguish between these mechanisms.

Another point indicating conceptual confusion appears again in the first sentence of the abstract:

“Abrupt shifts in ecosystems, brains, markets, and climate are often diagnosed as signs of approaching a tipping point, i.e. a critical bifurcation where a previously stable state loses stability or ceases to exist.”

In this context, abrupt shifts should typically refer to the tipping point itself, rather than to a sign of approaching it. It is therefore unclear what the authors mean here. Are they referring to the possible escape of noisy trajectories from a stable solution as the system approaches a loss of hyperbolicity near a bifurcation? The authors should revise the entire manuscript for conceptual issues like this.

More generally, I still find the text excessively intricate and confusing for the reader. The manuscript contains nine result paragraphs that mix conceptual claims with methodological steps, making it difficult to identify what the central results actually are. Although this criticism may appear stylistic, it is not. In my opinion, the claims themselves are supported, but the way they are presented obscures the main contributions of the work.

Version 2:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
Following my previous comments, the authors have improved the conceptual basis of the manuscript. However, the presentation of the Results section has not been revised and remains somewhat unconventional and intricate for the physics literature, which obscures the main claims.

Nevertheless, the results themselves appear sound. Therefore, if the journal does not consider the current format to be an impediment, I support the publication of this manuscript in Communications Physics.

Response to Reviewer 1

Reviewer 1 Comment 1

The manuscript addresses an important and timely issue concerning the properties of non-normal systems, particularly the transient growth of perturbations and their strong sensitivity to noise. It emphasizes that commonly used indicators of criticality may arise from non-normal dynamics rather than from genuine transitions in the system's asymptotic solutions.

Response:
We thank the reviewer for the careful reading of our manuscript and for recognizing the relevance of the topic and the soundness of the results.

Reviewer 1 Comment 2

While the topic is highly relevant and the results appear sound, the manuscript does not convey these points clearly or effectively. The text is overly intricate, making it difficult to clearly connect the listed results with their corresponding demonstrations.

Response:
We respectfully disagree with this assessment. The manuscript was deliberately structured to maximize clarity and logical traceability. In particular:

- All core notions are introduced through explicitly labeled **Definition 1, Definition 2,** ...
- Each principal statement is formulated as **Result 1, Result 2,** ..., directly followed by its analytical derivation, justification, or numerical demonstration.
- Systematic cross-references ensure that every claim can be traced unambiguously to its supporting argument or figure.
- The progression of sections is cumulative and explicitly organized to guide the reader from conceptual foundations to applications.

Considerable effort went into ensuring that the chain of reasoning is transparent and that the connection between stated results and their demonstrations is explicit.

Importantly, Reviewer 2 independently provides a markedly different evaluation, describing the manuscript as *exceptionally well written,* with new concepts presented *with admirable clarity and readability.* In light of this strong and explicit endorsement of the exposition, we are reluctant to modify the structure or style of the manuscript in ways that might compromise aspects that are clearly effective for expert readers.

As the Latin proverb reminds us, *de gustibus non est disputandum*, matters of taste are not subject to definitive adjudication. Differences in stylistic preference are inevitable, especially

in technically subtle work. However, given the deliberate organization of definitions and results and the independent positive assessment of the manuscript's clarity, we believe that the current structure is appropriate and effective.

For these reasons, we prefer to retain the present organization of the manuscript. We hope the reviewer will reconsider the evaluation of clarity in light of these considerations.

Reviewer 1 Comment 3

Moreover, conceptual inconsistencies appear at several points throughout the manuscript. For example, the first sentence of the abstract states: "Abrupt shifts in ecosystems, brains, markets, and climate are often diagnosed as signs of approaching a tipping point, i.e. a critical bifurcation where stability is lost." However, in catastrophic bifurcations associated with tipping points, stability is not merely lost; rather, the stable state itself disappears.

Response:
We respectfully disagree with the implication that the statement in the abstract is conceptually inconsistent. In dynamical systems theory, a critical bifurcation associated with tipping is characterized by a loss of stability of an attractor. In some important cases, such as a saddle-node bifurcation, the stable equilibrium indeed disappears through collision with an unstable branch. However, in other canonical bifurcations relevant to tipping phenomena (e.g., subcritical Hopf, transcritical, pitchfork), the equilibrium does not disappear but instead loses stability while continuing to exist as an unstable invariant solution.

Thus, *loss of stability* is the more general and mathematically accurate characterization of a tipping bifurcation. The disappearance of the stable state is a particular mechanism (not a universal property) of certain classes of bifurcations. For this reason, we consider the formulation in the abstract to be both correct and appropriately general. Our intent was to refer to the generic property that the attractor governing the system's behavior ceases to be stable, not to restrict attention solely to saddle-node scenarios.

That said, we have now clarified this point in the manuscript. We hope this clarification resolves the concern.

Reviewer 1 Comment 4

To cite one more conceptual issue, in the context of structural stability. The term "crises," used in the manuscript title, has a precise meaning in dynamical systems theory: a crisis is a global bifurcation in which an invariant set (typically a chaotic attractor) undergoes an abrupt qualitative change (such as sudden enlargement, destruction, or disappearance) due to its collision with an unstable invariant set (typically an unstable periodic orbit or its stable manifold). In this sense, the use of the term "crises" is misleading, as the manuscript focuses exclusively on local bifurcation scenarios.

Response:
We thank the reviewer for raising this important conceptual point. In dynamical systems theory, the term *crisis* indeed has a precise meaning: it refers to a class of global bifurcations in which an invariant set (typically a chaotic attractor) undergoes an abrupt qualitative change following its collision with an unstable invariant set, such as an unstable periodic orbit or its stable manifold.

Our manuscript, however, focuses exclusively on local bifurcation scenarios and on transient dynamics in asymptotically stable systems. We do not analyze global attractor crises in the technical dynamical-systems sense. We agree that the use of the term *crises* in the title could therefore be misleading to readers trained in nonlinear dynamics.

Importantly, the term appeared only in the title and in the abstract; it was not used in the main text of the manuscript. To avoid any possible confusion with the technical notion of global chaotic crises, we have revised the title from *Illusions of Criticality: Crises Without Tipping Points* to *Illusions of Criticality: Transitions Without Tipping Points*, and we have removed the remaining uses of the word *crises* from the abstract.

This change eliminates the terminological ambiguity while preserving the substantive contribution of the paper, which concerns the interpretation of abrupt macroscopic transitions and their early-warning signals in systems governed by non-normal dynamics.

Reviewer 1 Comment 5

In its current form, I do not recommend this manuscript for publication. However, I would be happy to review a revised version in which the same results are demonstrated more clearly, both numerically and analytically.

Response:
We respectfully submit that the reviewer has not identified any analytical error, numerical flaw, incorrect derivation, unsupported claim, or conceptual inconsistency in the technical development of the manuscript. On the contrary, the reviewer explicitly acknowledges that the topic is important and that *the results appear sound*.

The manuscript contains:

- Explicit analytical derivations for each stated result,
- systematic numerical simulations directly illustrating the theoretical claims,
- and empirical applications (notably to epilepsy data) that quantitatively support the proposed framework.

Each major claim is either derived mathematically or supported by numerical experiments designed specifically to test the mechanisms under discussion. No specific analytical gap, incorrect computation, or missing numerical validation has been identified in the report.

The reviewer's request for results to be demonstrated *more clearly* remains, unfortunately, too general to translate into concrete revisions. Without specific indications of which derivation is incomplete, which numerical experiment is insufficient, or which conclusion is unsupported, it is difficult to determine what substantive modification would be required.

We are, of course, fully willing to address any precise technical concerns. However, in the absence of identified analytical or numerical deficiencies, we believe the current manuscript already provides a complete and rigorous demonstration of its claims.

We therefore respectfully submit that the recommendation for revision is not supported by specific technical criticisms, and we hope that the editor will consider the manuscript on the basis of its analytical rigor, numerical validation, and conceptual contribution.

Response to Reviewer 2

Reviewer 2 Comment 1

This paper introduces the novel concept of pseudo-bifurcations, offering a fundamentally new perspective on so-called "tipping points" (critical bifurcations) in dynamical systems. The work is highly original and innovative, and in my view merits publication. The authors convincingly demonstrate the power of their approach through a compelling real-world application to epileptic seizures. They show that pseudo-bifurcations can faithfully reproduce the full suite of early-warning signals conventionally regarded as evidence of proximity to a tipping point, including critical slowing down, increased variance, and dimensional collapse. This result indicates that the concept of pseudo-bifurcations provides a powerful alternative framework for understanding critical dynamical phenomena, one that goes beyond the conventional treatment of nonlinear dynamics as mere perturbations of underlying linear systems. Instead, it elegantly captures the hidden geometry of non-normal dynamics, revealing genuine mechanisms where traditional approaches may only produce the illusion of criticality. The manuscript is exceptionally well written. The new concepts are presented with admirable clarity and readability, and the application to epilepsy, both conceptual and data-driven, is truly impressive. The paper concludes with a thoughtful broader discussion of the significance of this framework. In summary, this is a milestone contribution. I expect it to have major impact across the scientific community by opening a genuinely new avenue for analyzing dynamical critical phenomena and tipping points, distinct from mainstream nonlinear dynamics approaches. I recommend publication as is, without delay.

Response:
We thank the reviewer for the very positive and encouraging assessment of our work. We appreciate the recognition of the conceptual novelty, analytical rigor, and interdisciplinary relevance of the pseudo-bifurcation framework, as well as the acknowledgment of its empirical application to epileptic seizure dynamics. We also note the reviewer's strong endorsement and recommendation for publication without further delay.

Response to Reviewer 1

Reviewer 1 Comment 1

The authors have made punctual modifications following my previous concerns. However, the manuscript still presents conceptual problems that make it difficult for the reader to follow. For instance, the original title was "Illusions of Criticality: Crises Without Tipping Points." Due to the confusion associated with the term "crises," the authors changed it to "Illusions of Criticality: Transitions Without Tipping Points." This formulation is also problematic, since transitions that are not tipping points naturally occur in dynamical systems. An obvious example is a noisy system that escapes the vicinity of one attractor and visits another, this is not a tipping point. Another example is a heteroclinic cycle, in which several saddle structures are connected by deterministic trajectories, every transition is not a tipping point.

Response:
We thank the reviewer for the careful reading of the revised manuscript and for acknowledging that the claims of the manuscript are supported by the results presented. We appreciate the emphasis on conceptual clarity and have revised several parts of the manuscript accordingly. In particular, following this comment, we have revised the title in order to avoid possible ambiguities associated with the terminology of transitions or tipping points. The new title, *Pseudo-Bifurcations in Stochastic Non-Normal Systems and the Limits of Early-Warning Signals*, focuses directly on the central mechanism studied in the paper and emphasizes the precise scope of the work. This revised formulation highlights the concept of pseudo-bifurcations and the limitations of early-warning indicators without relying on potentially ambiguous terminology regarding transitions or tipping points.

Reviewer 1 Comment 2

There seems to be a persistent tendency in the manuscript to use the term tipping point without necessarily referring to the phenomenon typically associated with it. The term tipping point is rather informal and has been widely adopted, especially in ecological contexts, to refer to abrupt shifts in system dynamics as its conditions are systematically varied, often associated with hysteresis. There are also broader uses of the concept, such as noise-induced tipping or rate-induced tipping, but the manuscript does not clearly distinguish between these mechanisms.

Response:
In response to this comment, we have revised the abstract and the introduction to explicitly distinguish bifurcation-induced tipping points from other mechanisms such as noise-induced tipping and rate-induced tipping. The revised text now clarifies that the manuscript specifically

concerns the potential misdiagnosis of *bifurcation-induced tipping points* when commonly used early-warning indicators are interpreted in systems governed by non-normal dynamics.

Reviewer 1 Comment 3

Another point indicating conceptual confusion appears again in the first sentence of the abstract: "Abrupt shifts in ecosystems, brains, markets, and climate are often diagnosed as signs of approaching a tipping point, i.e. a critical bifurcation where a previously stable state loses stability or ceases to exist." In this context, abrupt shifts should typically refer to the tipping point itself, rather than to a sign of approaching it. It is therefore unclear what the authors mean here. Are they referring to the possible escape of noisy trajectories from a stable solution as the system approaches a loss of hyperbolicity near a bifurcation? The authors should revise the entire manuscript for conceptual issues like this.

Response:
The abstract has been rewritten to clarify the intended meaning and to avoid possible ambiguity regarding the interpretation of abrupt shifts and tipping mechanisms. The revised version explicitly states that the manuscript concerns the potential misdiagnosis of bifurcation-induced tipping points when early-warning indicators are interpreted in stochastic non-normal systems. This revision clarifies the distinction between genuine bifurcation-induced tipping and transient episodes that may mimic the statistical signatures of such transitions.

Reviewer 1 Comment 4

More generally, I still find the text excessively intricate and confusing for the reader. The manuscript contains nine result paragraphs that mix conceptual claims with methodological steps, making it difficult to identify what the central results actually are. Although this criticism may appear stylistic, it is not. In my opinion, the claims themselves are supported, but the way they are presented obscures the main contributions of the work.

Response:
We understand the concern that the presentation could appear intricate due to the interplay between conceptual arguments and methodological steps in the Results section.

Our intention in structuring the Results in this way was to guide the reader progressively from the conceptual introduction of pseudo-bifurcations to their mathematical characterization, statistical signatures, and empirical illustration in epileptic brain dynamics. Because the mechanism we introduce is both conceptual and methodological, we found it difficult to fully separate these aspects without fragmenting the narrative.

We also note that Reviewer 2 explicitly highlighted the clarity of the presentation, stating that *"the manuscript is exceptionally well written"* and that the new concepts are presented with

"admirable clarity and readability."

In order to further improve readability and reduce possible confusion, we have also streamlined the presentation of the formal statements in the manuscript. In particular, we replaced the previous terminology of *Propositions* with *Results* throughout the paper and ensured consistent referencing in the text. This change was intended to make the logical structure of the manuscript clearer and more accessible to a broad interdisciplinary readership.

For this reason, rather than restructuring the Results section entirely, we have focused on improving conceptual clarity in several places in the manuscript (including the title, abstract, and introduction) in order to better emphasize the central message and avoid possible ambiguities regarding tipping mechanisms.

We hope that these adjustments address the reviewer's concerns while preserving the logical progression of the argument.