

Supplementary Information for

Single Frame Absolute Angle Metrology with Dual Pancharatnam Berry Metasurfaces

Xiaoqi Jiang^{1,2}, Maoxiong Zhao^{2,*}, Shaohua Yang^{1,2}, Lan Lan^{1,2}, Chao Song^{1,2}, Fang Wu^{2,*}, Yang Bu^{1,2,*}

¹ School of Integrated Circuits, Shanghai Jiao Tong University, Shanghai 202240, China

² Zhangjiang Laboratory, Shanghai 201210, China

*Corresponding authors: MX. Z. (zhaomx@zjlab.ac.cn); F.W. (wufang@zjlab.ac.cn);

Y. B. (buyang@zjlab.ac.cn)

Supplementary Note 1 Retrieval of the metasurface phase distribution

When there are no samples in the object optical path. We can write the complex amplitude of the object beam as

$$\tilde{U}_1 = A_1 \exp [i(\mathbf{k}_1 \cdot \mathbf{r} + k_{1z}z_1 + \psi_1(x, y))] \quad (1)$$

where $\mathbf{k}_1 = (k_{1x}, k_{1y})$, $\mathbf{r} = (x, y)$, k_{1z} is the z component of wave vector, $\psi_1(x, y)$ phase distribution of the object beam.

Put the metasurface in the optical path of the object beam, leading to

$$\tilde{U}_2 = A_2 \exp [i(\mathbf{k}_1 \cdot \mathbf{r} + k_{1z}z_1 + \psi_1(x, y) + \varphi_{metasurfaces}(x, y))] \quad (2)$$

where $\varphi_{metasurfaces}(x, y)$ is phase distribution of the metalens. For the reference beam, the complex amplitude is given by

$$\tilde{U}_3 = A_3 \exp [i(\mathbf{k}_3 \cdot \mathbf{r} + k_{3z}z_3 + \psi_3(x, y))] \quad (3)$$

where $\mathbf{k}_3 = (k_{3x}, k_{3y})$, $\psi_3(x, y)$ is the phase distribution of the optical path of the reference beam.

Then the interference fringe intensity pattern was produced at the CCD, namely,

$$\begin{aligned} I(x, y) &= (\tilde{U}_2 + \tilde{U}_3)^* \cdot (\tilde{U}_2 + \tilde{U}_3) \\ &= |A_2|^2 + |A_3|^2 \\ &\quad + A_2^* A_3 \exp \left[i \left((\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r} + k_{3z}z_3 - k_{1z}z_1 + \psi_3(x, y) - \psi_1(x, y) \right. \right. \\ &\quad \left. \left. - \varphi_{metasurfaces}(x, y) \right) \right] \\ &\quad + A_3^* A_2 \exp \left[i \left((\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r} + k_{3z}z_3 - k_{1z}z_1 + \psi_3(x, y) - \psi_1(x, y) \right. \right. \\ &\quad \left. \left. - \varphi_{metasurfaces}(x, y) \right) \right] \end{aligned} \quad (4)$$

By operating Fourier-transformation on the fringe pattern, we can get

$$\tilde{I}(k_x, k_y) = \tilde{I}(0) + \tilde{I}(\Delta k) + \tilde{I}(-\Delta k) \quad (5)$$

where $\Delta k = \mathbf{k}_3 - \mathbf{k}_1$.

We retain the term of

$$\begin{aligned} \tilde{I}(\Delta k) &= A_2^* A_3 \exp \left[i \left((\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r} + k_{3z}z_3 - k_{1z}z_1 + \psi_3(x, y) - \psi_1(x, y) - \right. \right. \\ &\quad \left. \left. \varphi_{metasurfaces}(x, y) \right) \right] \end{aligned} \quad (6)$$

Then by inverse Fourier transformation and taking its argument, the difference of phase distribution can get,

$$\varphi_3 - \varphi_2 = (\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r} + k_{3z}z_3 - k_{1z}z_1 + \psi_3(x, y) - \psi_1(x, y) - \varphi_{\text{metasurfaces}}(x, y) \quad (7)$$

Remove metalens out of the object beam, we can easily obtain the phase contributions

from the measurement system,

$$\varphi_3 - \varphi_1 = (\mathbf{k}_3 - \mathbf{k}_1) \cdot \mathbf{r} + k_{3z}z_3 - k_{1z}z_1 + \psi_3(x, y) - \psi_1(x, y) \quad (8)$$

Combining the above two eq (7) and (8), the phase distribution of metalens $\varphi_{\text{metalens}}(x, y)$ can be obtained.

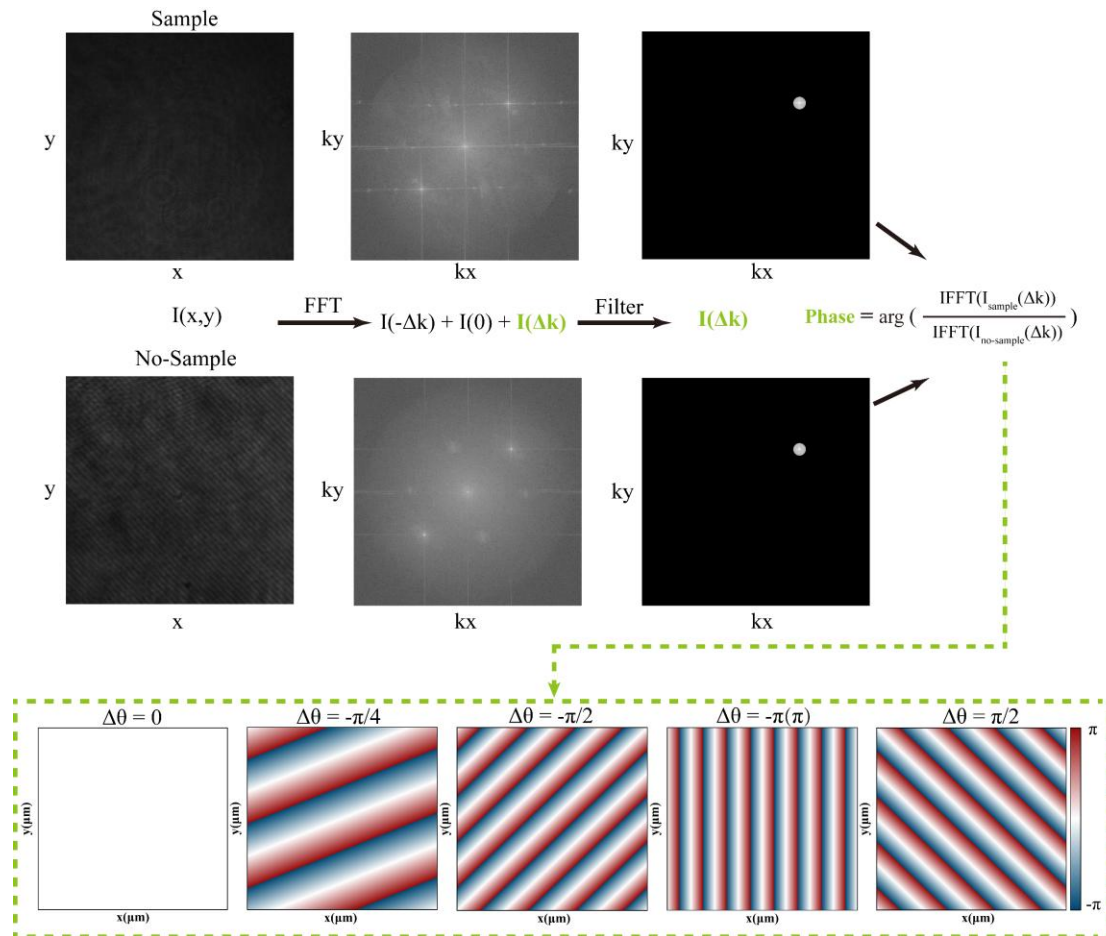


Fig. S1 A schematic of retrieving the phase distribution of metasurface

Supplementary Note 2 Simulation and Fabrication

Finite-difference time-domain (FDTD) simulations were performed to guide the metasurface design. The refractive indices of Si_3N_4 and the SiO_2 substrate were set to 2.06 and 1.46, respectively, at the design wavelength of 532 nm. To optimize the transmission efficiency, we systematically varied the long and short axes (a and b) of the elliptical nanopillars within the range of 50–250 nm, and evaluated the cross-polarized transmission under right-handed circularly polarized (RCP) illumination. In

the simulations, an RCP plane wave source was placed one wavelength below the nanopillars, and a monitor was positioned one wavelength above to capture the transmitted field. The Si_3N_4 nanopillars were 400 nm in height. The FDTD region length in x and y direction is set to 500 nm with periodic boundary conditions. The cross-polarized transmission collected by monitor are shown in Fig.S2.

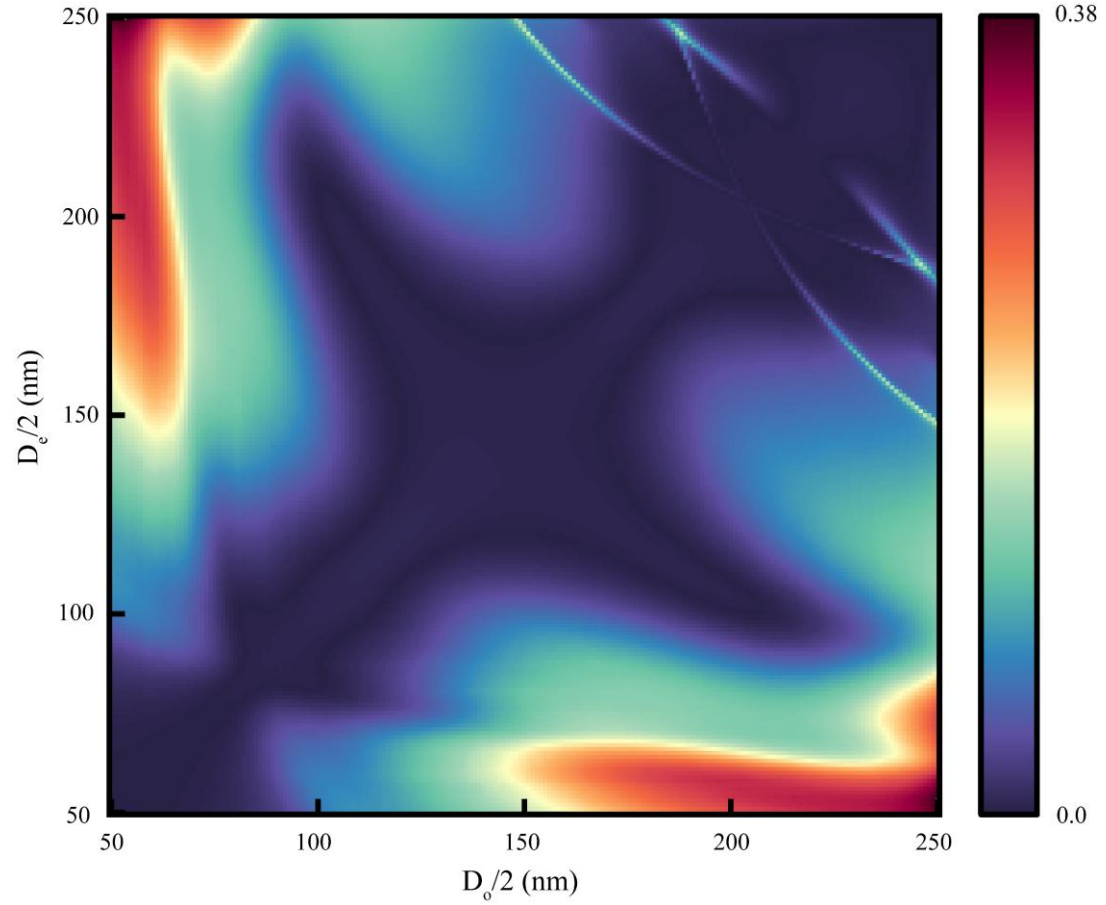


Fig. S2 The library of Si_3N_4 pillars simulated by FDTD. The transmission of the Si_3N_4 pillars in d1 axis and in d2 axis. The length of the pillars varies in 50-250 nm.

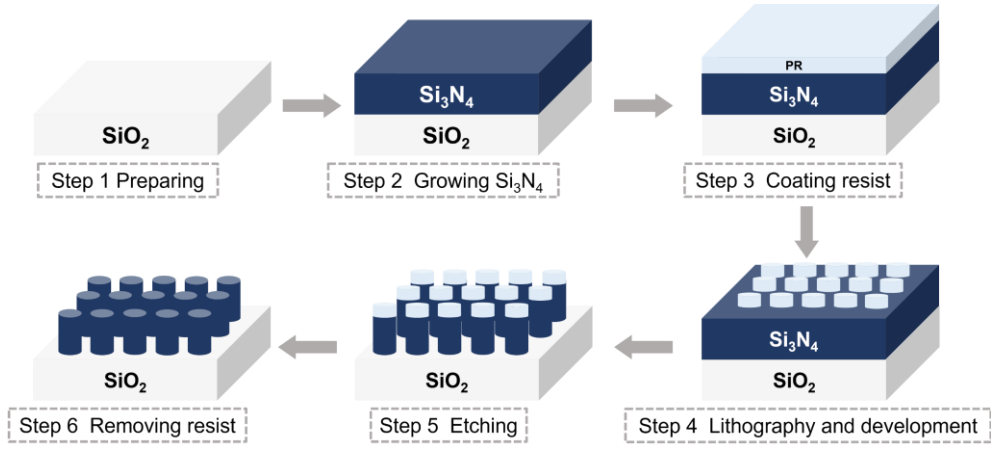


Fig. S3 The diagrammatic drawing of fabrication process. (1) The process starts with a fused quartz substrate. (2) Growing a 400-nm-thick Si_3N_4 film on the substrate. (3) Coating a 120-nm-thick negative electron-beam resist layer. (4) Patterning the metasurface array into resist and developing resist to reveal the pattern. (5) Etching to transfer the pattern to Si_3N_4 film. (6) Removing residual resist by buffered oxide etch (BOE) solutions.

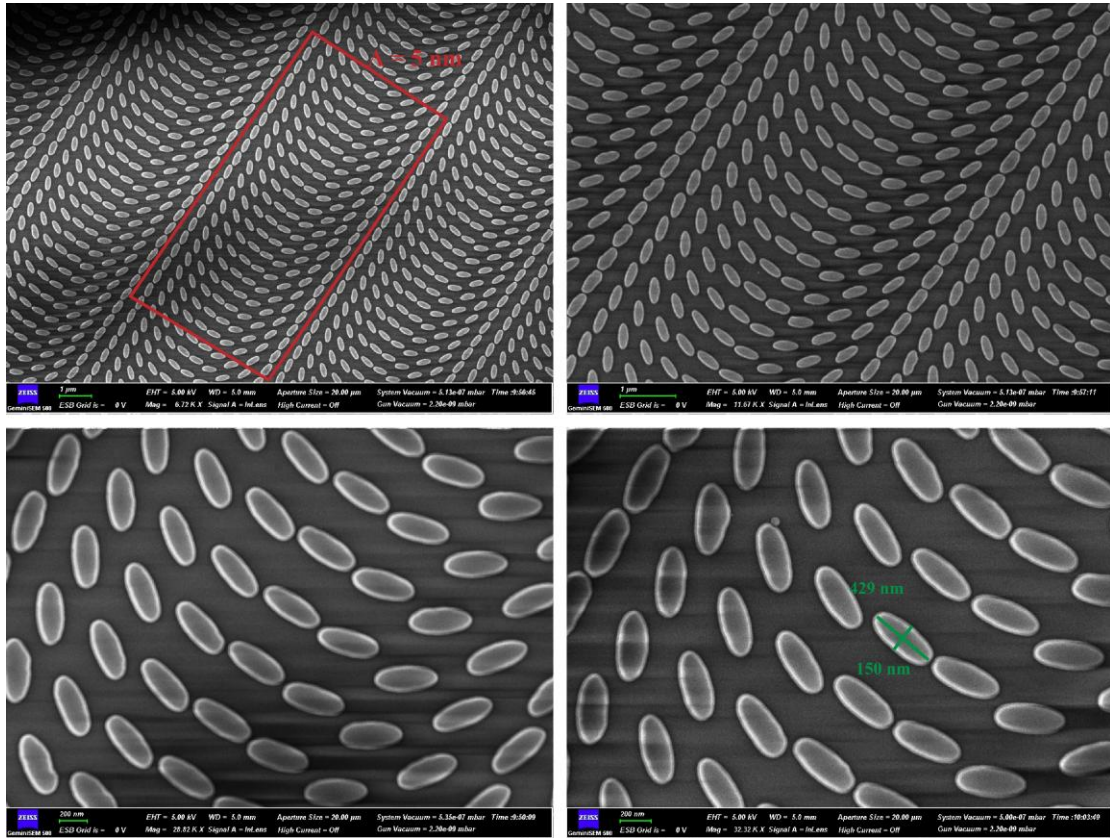


Fig. S4 The scanning electron microscope (SEM) images of the metasurface $\Lambda = 5 \mu\text{m}$, the red box in the upper left picture shows the periodic array with a period of $5 \mu\text{m}$, and the lower right picture shows the diameters of the long and short axes of a single nanopillar, which are $a = 429 \text{ nm}$ and $b = 150 \text{ nm}$, respectively

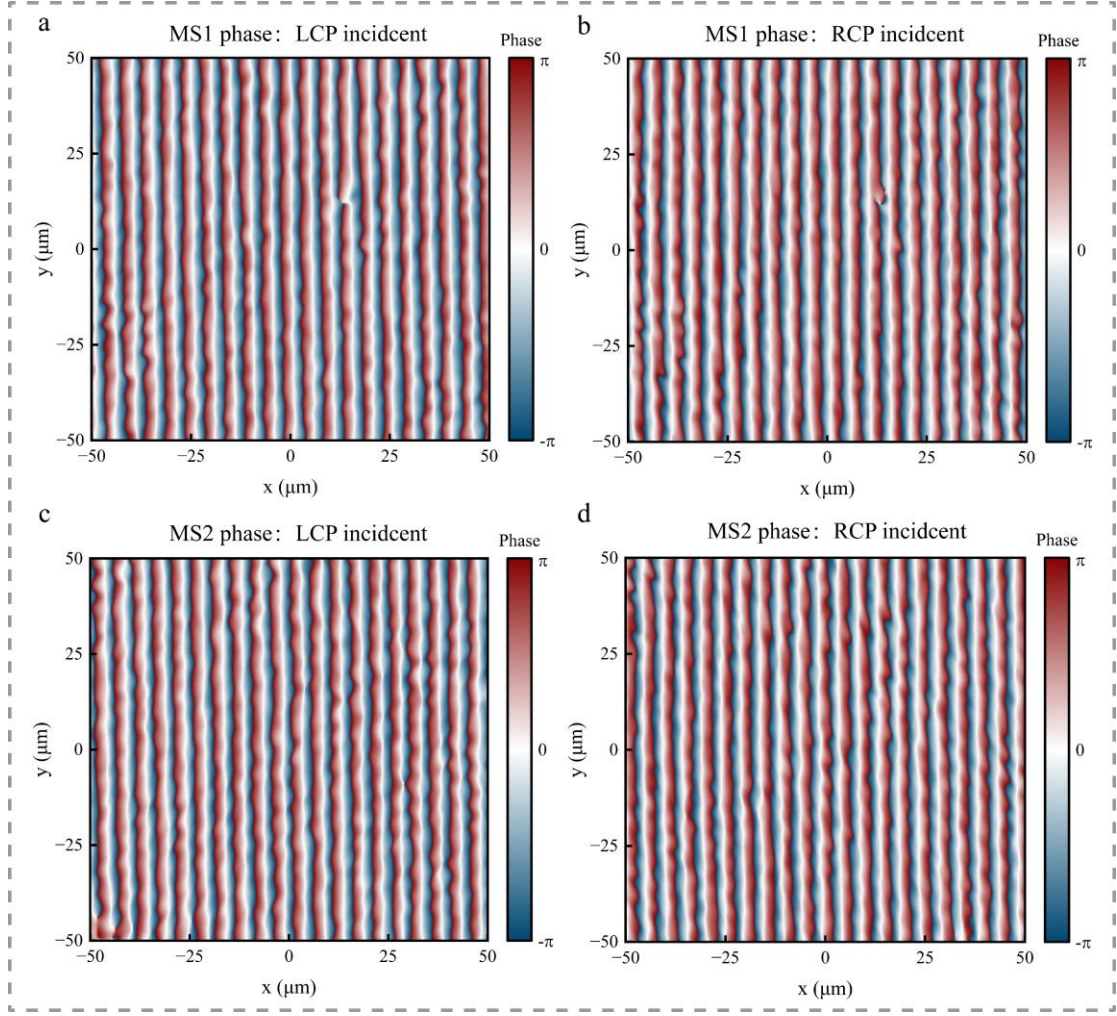


Fig. S5 Phase maps of Metasurface 1 and Metasurface 2 within a $100 \times 100 \mu\text{m}$ field of view under different incident polarizations. **a Metasurface 1 under left-circularly polarized (LCP) illumination. **b** Metasurface 1 under right-circularly polarized (RCP) illumination. **c** Metasurface 2 under LCP illumination. **d** Metasurface 2 under RCP illumination.**

Supplementary Note 3 Additional angular displacement measurements

To make our conclusion more convincing, we additionally verified the system's predictive ability within the range of $0 \sim 1.75 \times 10^{-3}$ rad. Fig. S6 demonstrates a further tightening of the angular increment to 1.75×10^{-4} rad. Even at this finer sampling, the experimental data agree closely with the simulation, while the uncertainty remains at $\sim 10^{-5}$ rad/ μm , still an order of magnitude below the measured gradient. The right panels compare simulated and experimental phase maps at $\Delta\theta = 1.75 \times 10^{-3}$ rad, showing excellent visual consistency. A few sporadic outliers appear in the experimental. At this level, as expected, but the estimator's robustness (outlier rejection and spatial

consistency constraints) minimizes their influence on the fit, underscoring the reliability of the readout.

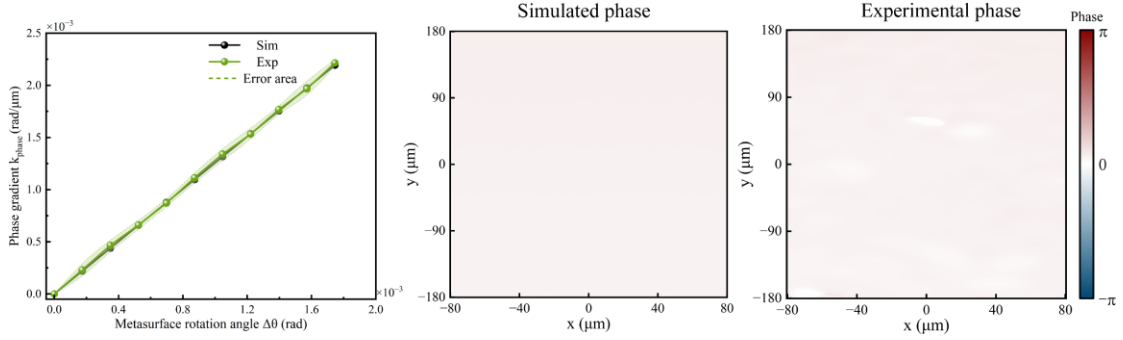


Fig. S6 The phase gradient varies with the rotation angle of the metasurface within the range of $0 \sim 1.75 \times 10^{-3}$ rad, with a step size of 1.75×10^{-4} rad. On the right are the simulated phase and the experimental phase distribution when rotating by 1.75×10^{-4} rad respectively.

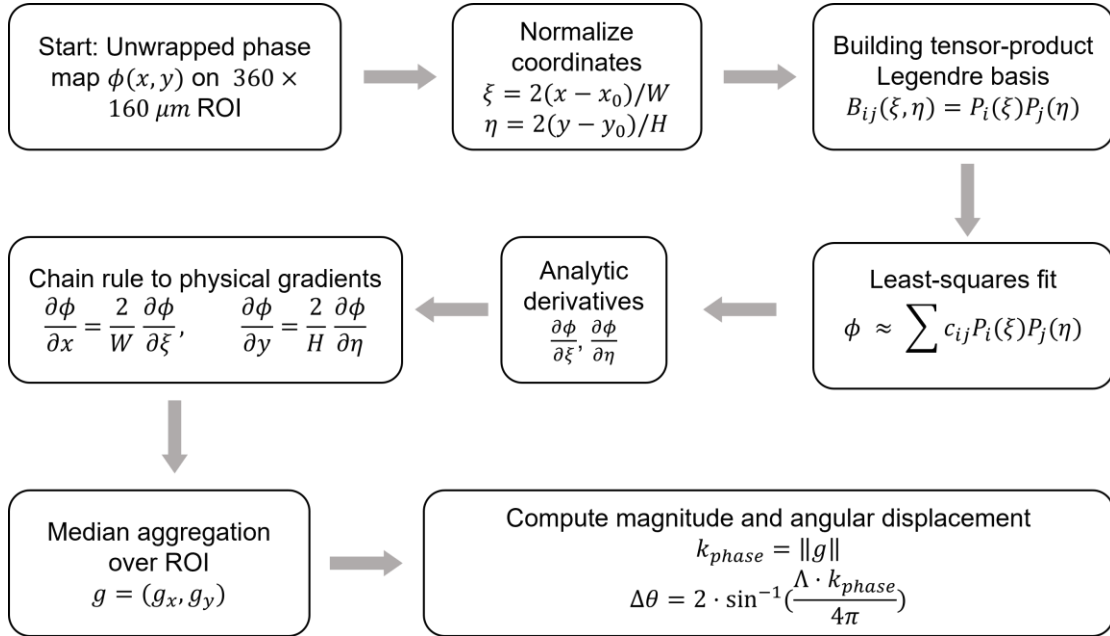


Fig. S7 Processing pipeline for phase-gradient estimation. Starting from the reconstructed, unwrapped phase, a rectangular ROI ($360 \times 160 \mu\text{m}$) aligned with the vertical ramp is selected and mapped to normalized coordinates $(\xi, \eta) \in [-1, 1]^2$. The phase is fitted on a tensor-product Legendre basis by least squares, analytic derivatives $\partial\phi/\xi, \partial\phi/\eta$ are computed and converted to physical gradients via the chain rule. After outlier rejection, per-pixel gradients are aggregated by the component-wise median to give $g = (g_x, g_y)$, and the scalar metric $k_{\text{phase}} = \|g\|$ (with sign from the dominant component) is reported along with basic residual/uncertainty diagnostics.