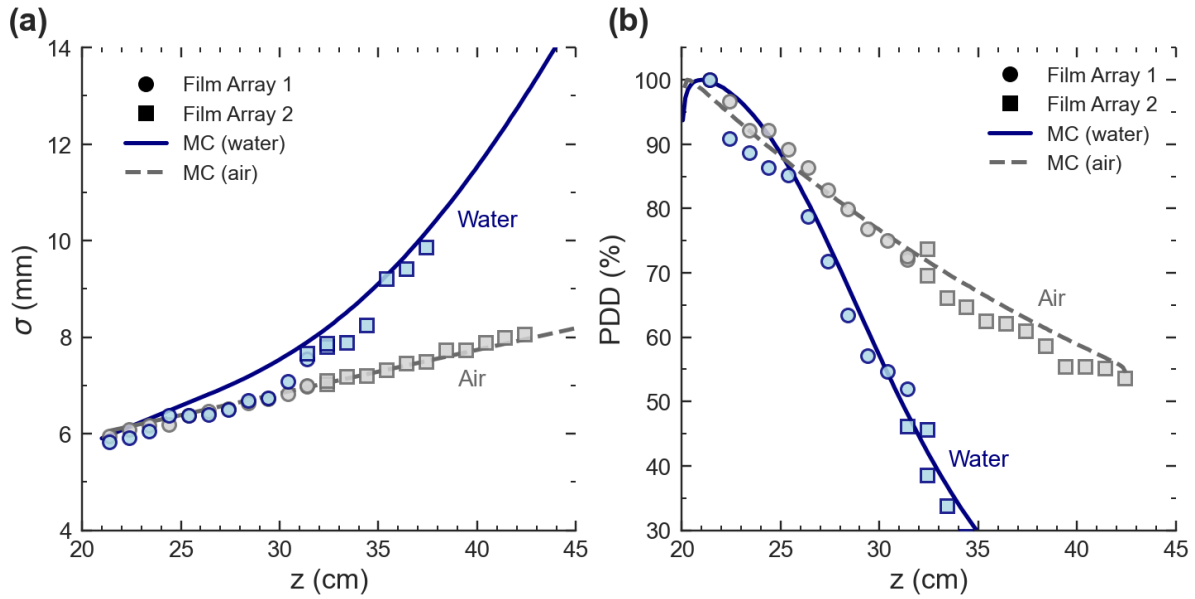


Supplementary



Supplementary Fig. 1: Monte Carlo model validation at 150 MeV. **a**, Validation of the Monte Carlo (MC) model (lines) against experimental film measurements (markers) for the 150 MeV scattered beam's spot-size evolution in air (grey) and a water phantom (blue), demonstrating accurate prediction of beam broadening. **b**, Validation of the central-axis percentage depth-dose (PDD) for the 150 MeV scattered beam. MC simulations (lines) are compared with film data (markers) in air and water. All results are normalised to the maximum dose in water, enabling a direct comparison of relative attenuation in the two media. Each marker represents one film measurement at the stated position ($n=1$ film per position); the error bars are assigned measurement uncertainties and do not represent variability across replicate films.

Supplementary Table 1: Details of beam energy, scattering mode, phantom insert, number of spots, spot spacing, and delivered charge for each experimental run performed in the MARVIN anthropomorphic phantom.

Run	Energy (MeV)	Mode	Insert	Spots (n)	Spot Spacing (cm)	Individual Charges (nC)	Total Charge (nC)
Run 1	200	Unscattered	Water Insert	1	—	1.0	1.0
Run 2	200	Unscattered	Foam Insert	1	—	1.0	1.0
Run 3	200	Scattered	Water Insert	1	—	7.8	7.8
Run 4	200	Scattered	Foam Insert	1	—	8.7	8.7
Run 5	200	Scattered	Water Insert	7	0.5	8.5, 8.6, 8.2, 8.6, 8.2, 8.5, 8.6	59.2
Run 6	200	Scattered	Foam Insert	7	0.5	8.9, 8.5, 9.3, 9.1, 8.5, 9.0, 8.7	62.0
Run 7	150	Scattered	Water Insert	1	—	9.2	9.2
Run 8	150	Scattered	Foam Insert	1	—	9.5	9.5

Supplementary Note 1. Reduced chi-squared analysis

The goodness of fit for main-text Eq. (2) was quantified using the reduced chi-squared (χ^2/ν) statistic. For each colour channel, a relative uncertainty was assigned to the $N = 9$ calibration points, yielding an absolute uncertainty (σ_i) and corresponding statistical weight ($w_i = 1 / \sigma_i^2$). These weights were applied in the non-linear least-square fit of main-text Eq. (2). The reduced chi-squared was calculated as:

$$\chi_v^2 = \frac{1}{v} \sum_{i=1}^N \frac{(D_{i,known} - D_{i,fit})^2}{\sigma_i^2} \quad (S1)$$

where $v = N - p$ is the number of degrees of freedom ($N = 9$ data points, $p = 3$ fit parameters).

Supplementary Table 2: Gafchromic EBT-4 film calibration fit parameters. Best-fit parameters (p_1 , p_2 , p_3) with standard errors (s.e.) for the calibration function (main-text Eq. 2), derived from weighted non-linear least-squares fits to the $N = 9$ -point dataset (main-text Fig. 8a). The channel-specific weighting was optimised to yield reduced χ^2 values ≈ 1 . Fit quality was further assessed using R^2 and root-mean-square error (RMSE) metrics.

Channel	p_1 ($\pm$ s.e.) [Gy]	p_2 ($\pm$ s.e.) [unitless]	p_3 ($\pm$ s.e.) [Gy]	Optimised Weighting Error (%)	χ^2	R^2	RMSE (%)
Red	2.045 ± 0.209	72.973 ± 13.106	-1.968 ± 0.247	5.1	0.998	0.9975	4.03
Green	8.776 ± 0.394	6.412 ± 0.327	-8.801 ± 0.407	1.95	0.998	0.9999	1.54
Blue	16.217 ± 3.264	20.485 ± 8.876	-16.139 ± 3.303	7.45	1.011	0.9964	5.92

Supplementary Table 3: Hyperbola beam model fit parameters for the MC source. Best-fit parameters (A , B , $C \pm$ standard error) for the hyperbolic fit (main-text Eq. 3) to the experimental in-air beam spot-size data (main-text Fig. 1a). These parameters were used to define the optical source properties in the MC simulations.

Beam Condition	A (mm^2)	B (mm)	C
150 MeV Scattered	$3.234 \times 10^1 \pm 4.134 \times 10^{-1}$	$6.424 \times 10^{-2} \pm 8.586 \times 10^{-4}$	$2.429 \times 10^{-4} \pm 7.275 \times 10^{-6}$
200 MeV Scattered	$1.566 \times 10^1 \pm 3.137 \times 10^{-1}$	$4.132 \times 10^{-2} \pm 6.100 \times 10^{-4}$	$1.587 \times 10^{-4} \pm 5.686 \times 10^{-6}$
200 MeV Unscattered	$6.243 \times 10^{-1} \pm 3.953 \times 10^{-2}$	$2.076 \times 10^{-3} \pm 3.913 \times 10^{-5}$	$1.115 \times 10^{-5} \pm 5.108 \times 10^{-7}$
150 MeV Unscattered	$6.646 \times 10^{-1} \pm 1.289 \times 10^{-1}$	$2.565 \times 10^{-4} \pm 1.352 \times 10^{-5}$	$1.419 \times 10^{-5} \pm 1.696 \times 10^{-6}$

Supplementary Table 4: Consolidated Monte Carlo simulation settings

Parameter	Specification
GATE Version	v8.1
Geant4 Version	v10.3.3
Physics List	QGSP_BIC_EMZ
Production Cuts	0.05mm
Voxel Size for Dose Scoring	$0.5 \times 0.5 \times 0.5 \text{ mm}^3$
Statistical Uncertainty Goal (in Dmax region)	$\leq 0.2\%$ in voxels within the 90–100% isodose region
Primary Particle	Electrons
Source Plane Position (relative to isocentre)	$z = -30 \text{ cm}$ (beam-line exit window)
Energy Spectrum (150 MeV)	Gaussian; mean 150.9 MeV; FWHM $\approx 1.0 \text{ MeV}$ (dipole spectrometer)
Energy Spectrum (200 MeV)	Gaussian; mean 200.35 MeV; FWHM $\approx 1.5 \text{ MeV}$ (dipole spectrometer)

Supplementary Note 2. Extended film dosimetry protocol for Gafchromic EBT-4 films

This section provides full methodological details of the film handling, irradiation, calibration, image-acquisition, and data-analysis procedures used for quantitative dosimetry with Gafchromic EBT-4 radiochromic films (Ashland Inc., Bridgewater, NJ, USA), supplementing the "Film dosimetry" section of the main manuscript. All procedures were designed to adhere to established best-practice guidelines [1] to ensure accuracy and reproducibility.

Film handling: Films were manipulated only by their edges using gloves to prevent damage. Laser cutting was used to minimise mechanical stress to the active layer. During water exposure, immersion time was minimised, and films were dried with lint-free paper, avoiding inter-layer moisture ingress.

Orientation and storage: Each film was marked outside the irradiation region to preserve consistent orientation across irradiation and scanning. All films were stored flat, in light-proof packaging at room temperature, shielded from ambient and UV light.

Control films: Unirradiated films from the same batch were processed and scanned with experimental films to provide PV_0 values and monitor batch uniformity. No significant drift was observed during the study period.

Scanning protocol:

- Scanner: Epson Expression 10000XL flat-bed scanner
- Warm-up: ≥ 5 dummy scans for lamp stability
- Post-irradiation delay: ≥ 24 h to ensure optical-density equilibrium ($< 2\%$ dose variation after 1 week)
- Mode: Transmission
- Resolution: 300 dpi
- Bit depth: 48-bit RGB (16 bits per channel)
- Format: TIFF

Rationale: High bit depth provided fine gradation of optical-density response, particularly in low- and high-dose regions, minimising quantisation error.

Positioning: Films were centrally placed and aligned consistently using the orientation mark.

Flat-field correction: A spatial-uniformity (flat-field) correction was applied to normalise light-source intensity, detector sensitivity, and optical-path variation, ensuring position-independent OD and dose accuracy.

Supplementary Note 3. Effect of spot-charge variation in the seven-spot plans

An additional Monte Carlo calculation used the measured spot-specific charges reported in Supplementary Table 1. Relative to a uniformly weighted seven-spot plan, the calculated dose differed by less than 0.2% in the high-dose region.

Reference

[1] Niroomand-Rad A, Chiu-Tsao ST, Grams MP, Lewis DF, Marinello G, Micke A, et al. Radiochromic film dosimetry: Recommendations of AAPM Radiation Therapy Committee Task Group 235. *Med Phys.* 2020;47:e1186–e1252. <https://doi.org/10.1002/mp.14497>