

Supplementary information.

In Situ Tunable Superconducting Diode Enabling Field Free Operation with Infinite Nonreciprocity

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SUPPLEMENTARY NOTE 1. ADDITIONAL INFORMATION ON EXPERIMENTAL PROCEDURES

Transport measurements

Transport measurements were carried out using a quasi-current-bias scheme based on a programmable voltage source with low internal resistance, connected in series with a large load resistor ($R > 1 \text{ k}\Omega$), whose resistance was much greater than the junction resistance, R_n .

The voltage was measured using low-noise preamplifiers and a high-resolution ADC.

The critical current was determined using an adjustable voltage threshold of $\sim 1 \text{ }\mu\text{V}$.

Lock-in measurements were performed to improve the measurement resolution and to directly measure the differential resistance.

A more detailed description of the noise characteristics and measurement procedures can be found in the Supplementary Information to Ref. [S1].

Field compensation

To cancel out the Earth's and environmental residual magnetic fields, we compensate for the residual static field by applying an offset current through the superconducting solenoid, using the junction itself as a sensitive magnetic-field detector [S2]. The ultimate field uncertainty of this active compensation is limited by the small magnetic-field inhomogeneity in the cryostat, caused by stray fields from ferromagnetic materials in the cryostat and by trapped Abrikosov vortices, either in the device or in the superconducting solenoid itself.

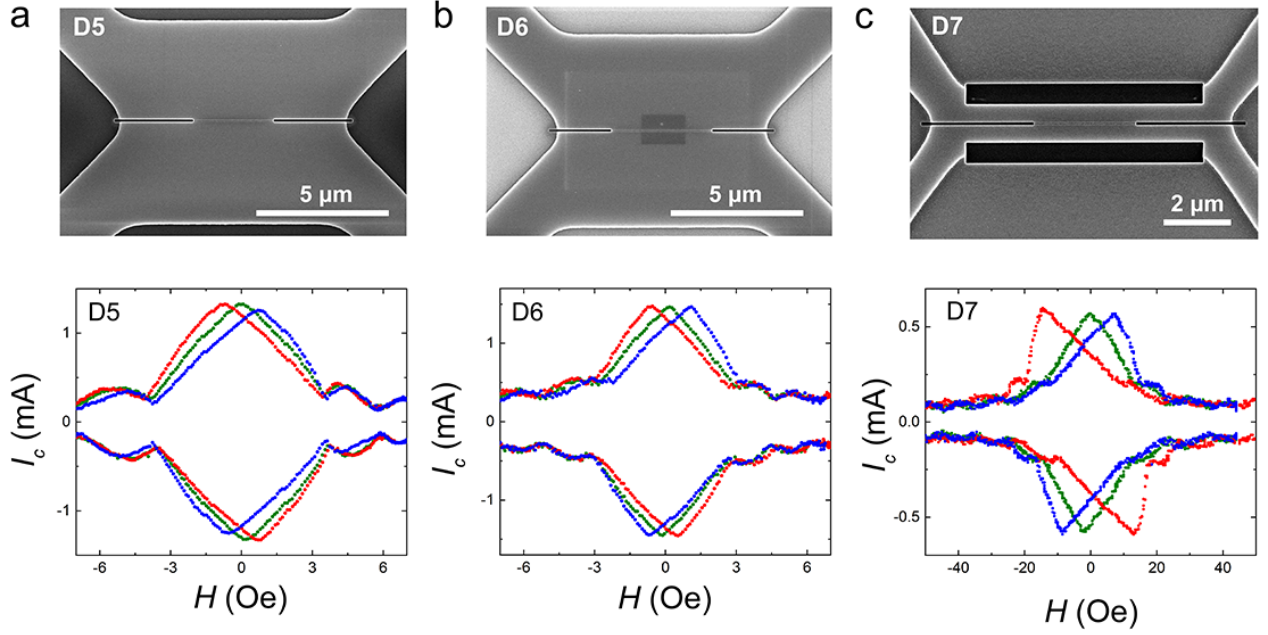
The finite uncertainty in field compensation due to field inhomogeneity may lead to incomplete reciprocity of the I - V characteristics at nominally zero field. This is the most likely reason for the small (approximately 2–3 %) difference between the positive and negative rectified signals in Fig. 2(d). Since optimal rectification, with $\eta \simeq 1$, for this ScD occurs at $H \simeq 5.5 \text{ Oe}$, the residual field corresponding to a 2–3 % deviation is approximately 0.1–0.15 Oe, which represents the accuracy of our active field-compensation technique.

SUPPLEMENTARY NOTE 2. CHARACTERISTICS OF DIODES, NOT INCLUDED IN THE MAIN TEXT

Fig. S1 shows SEM images and $I_c(H)$ patterns for three devices—D5, D6, and D7—not included in the main text. D5 and D6 have electrode widths similar to that of D1 ($W_z \simeq 3.86 \text{ }\mu\text{m}$) but differ in length: $W_x(\text{D5}) \simeq 3 \text{ }\mu\text{m}$ and $W_x(\text{D6}) \simeq 3.9 \text{ }\mu\text{m}$. D7 is similar to D2 but has a shorter junction. The total critical current increases proportionally with W_x , while the linear current density, $J_c = I_c/W_x$, remains approximately the same for D1, D5, and D6, reflecting the reproducibility of the fabrication process. Furthermore, H_{sf} —defined as twice the magnetic field at which the maximum critical current occurs—remains nearly constant across these devices. However, device D7, with $W_x(\text{D7}) \simeq 3 \text{ }\mu\text{m}$ and a width similar to that of D2, as shown in Fig. S1(c), exhibits a slightly reduced H_{sf} compared to D2.

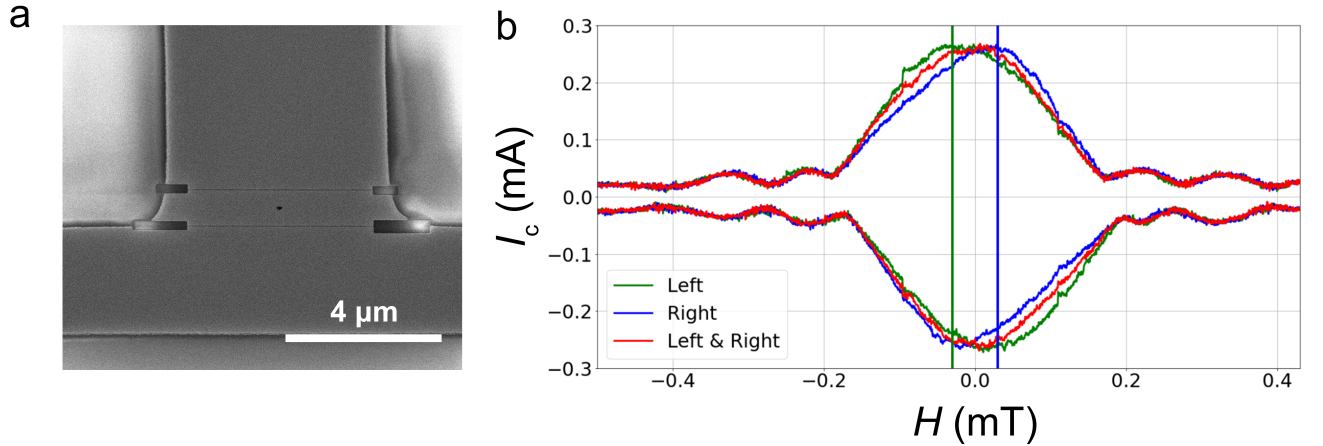
Table S1 summarizes the characteristics of the studied diodes from different sample batches. First, it can be observed that decreasing the length of the junction while maintaining a similar width W_z has only a minor effect on L_{sf} (compare D1, D5, D6, and D8). Furthermore, the values of L_{sf} for devices D10 and D2 differ by a factor of two, despite having nearly identical geometrical parameters. This difference can be attributed to the fact that L_{sf} depends not directly on L_x in the long-junction limit, but on the Josephson penetration depth λ_J (see the corresponding equation in the main text). Since λ_J is inversely proportional to I_c in planar geometries [S3], the observed difference in I_c leads to a factor of approximately 1.9 in λ_J , thereby explaining the variation in L_{sf} .

Second, in contrast to the effect of length, increasing the width while keeping other parameters similar substantially decreases the self-field inductance (compare D7 and D8), indicating that $L_{sf} \propto 1/W_z$.



Supplementary Figure S1. **Additional investigated devices.** SEM images and critical current as a function of applied magnetic field measured for different devices at $T = 5$ K. (a) Device D5 with the same electrode width as D1, $W_z \simeq 3.86 \mu\text{m}$, and junction length $W_x \simeq 3 \mu\text{m}$. (b) Device D6 with the same width, $W_z \simeq 3.86 \mu\text{m}$, but a longer junction length, $W_x \simeq 3.9 \mu\text{m}$. (c) Device D7 with a narrower width, $W_z \simeq 0.54 \mu\text{m}$, and $W_x = 3 \mu\text{m}$.

Figure S2 (a) shows an example of an earlier generation of T-shaped three-terminal diodes with significantly wider notches (~ 300 nm) and smaller self-field effect, as can be seen from the small tilt of $I_c(H)$ patterns presented in Fig. S2 (b).



Supplementary Figure S2. **Characteristics of a three-terminal diode.** (a) SEM image of a T-shaped diode containing two junctions with wider notches. (b) Measured $I_c(H)$ modulation for the three bias configurations. Data from Ref. [S4].

Figure S3 summarizes the magnetotransport response of device D9 in the conventional left-bias configuration with zero split current ratio (a), (c) and (d), and in the split-bias configurations with the split ratio one (b), (e) and (f). At $T = 6.5$ K, the differential-resistance maps in Figs. S3(a,b) exhibit a pronounced tilting of the central lobe. The tilt is

Device	W_x (μm)	W_z (μm)	I_{c0} (mA)	H_{sf} (Oe)	L_{sf} (pH)
D1	5	3.85	1.78	1.65	0.21
D2	5	0.58	0.73	40.8	2.37
D3	5.4	0.73	0.21	2.4	1.76
D4	4.97	1.06	0.32	6	0.74
D5	3.02	3.85	1.31	1.48	0.11
D6	3.9	3.85	1.45	1.24	0.16
D7	3.03	0.55	0.59	27.5	1.41
D8	3.04	3.85	1.58	2.49	0.19
D9	3.07	0.29	0.53	70	2.15
D10	5	0.47	1.3	52	1.02

Supplementary Table S1. **Characteristics of studied diodes.** Characteristics of devices analysed in this work. Here W_x is the junction length, W_z is the electrode width, I_{c0} is the maximum critical current at the central lobe of $I_c(H)$, H_{sf} is the effective self-field at the $I = I_{c0}$ representing the maximal tilt of the $I_c(H)$ pattern, and L_{sf} is the total self-field inductance. Data for D3 is obtained at $T = 6.85\text{K}$, the rest of the devices are characterised at $T = 5\text{ K}$.

substantially larger in the split-bias configuration (b), by approximately a factor of two, consistent with the behavior observed for device D2. The corresponding current–voltage characteristic measured in the left-bias configuration at $H = 8\text{ Oe}$ is shown in Fig. S3(c). A well-defined critical current is observed for positive bias, whereas no discernible critical current is detected in the negative-current direction, indicating a nearly perfect superconducting diode effect. Reentrant superconductivity does not occur in this conventional bias mode for any of the studied devices at any experimental conditions.

At $T = 5\text{ K}$, the differential-resistance maps shown in Figs. S3(d,e) reveal a broader Meissner region in both bias configurations. In the split-bias configuration (e), a distinct region of negative differential resistance appears in vicinity of the central lobe. The current–voltage characteristic measured at $H = 57\text{ Oe}$, presented in Fig. S3(f), displays a clear reentrant superconducting behavior, similar to that reported for device D2 in Fig. 3(e,f). These observations demonstrate that both the superconducting diode effect and reentrant superconductivity are reproducible in devices with different geometrical parameters.

SUPPLEMENTARY NOTE 3. SELF-FIELD INDUCTANCE

As mentioned in the manuscript, the self-field effect is induced by the Josephson phase gradient associated with an asymmetric current flow. The corresponding phase gradients at the junction edges are clearly visible in the numerical simulations presented in Figures S7(b–d) below.

The self-field inductance has two main contributions: the geometric inductance, L_g , and the kinetic inductance, L_k . Both contributions originate from the counterflow of currents in the junction electrodes, which, upon integration of the second London equation (1) over a contour enclosing the junction weak link, results in a finite phase difference. In particular, the kinetic contribution is proportional to the integral $\int (J_x(x)[\text{bottom}] - J_x(x)[\text{top}])dx$, which is maximized in the case of the counterflow of currents at the top and bottom sides of the junction interface (weak link), $J_x(x)[\text{top}] = -J_x(x)[\text{bottom}]$.

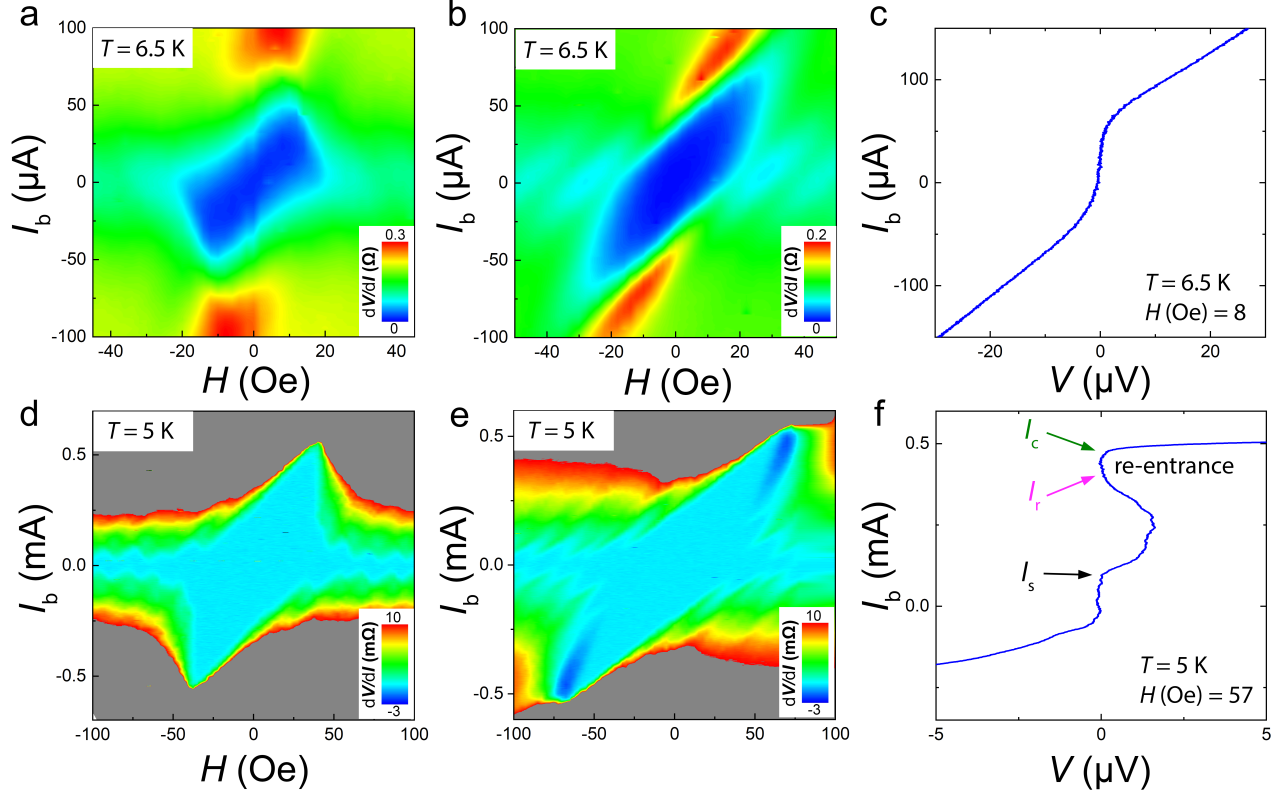
L_{sf} can be evaluated using the theoretical analysis of superconducting microstrip lines. For film thickness much smaller than the London penetration depth, $d \ll \lambda_L$, L_k and L_g are given by [S5],

$$L_k \simeq \frac{2\mu_0\lambda_L^2\lambda_J}{dW_zk'^2K^2(k')} \left\{ \frac{1}{S'} \ln \left[\frac{4S'}{d'(1+S')} \right] + \frac{1}{2+S'} \ln \left[\frac{4(2+S')}{d'(1+S')} \right] \right\} \quad (\text{S1})$$

$$L_g \simeq 2\mu_0 \frac{K(k)}{K(k')} \frac{\lambda_J\lambda_L^2}{Sd} \quad (\text{S2})$$

where $K(k)$ is the complete elliptic integral of the first kind with modulus $k = S/(S+2W_z)$, $k' = \sqrt{1-k^2}$, $S' = S/W_z$, $d' = d/W_z$, and S is the separation between the two strips (i.e., in our case between junction electrodes).

As follows from Eqs. (S1) and (S2), both the kinetic and geometric inductances increase as the separation between the electrodes, S , decreases. For the kinetic self-field inductance, S is determined by the width of the junction interface ($W_{JJ} \sim 30\text{--}50\text{ nm}$). For the geometric inductance, the definition of S is less straightforward because the self-field is generated not only inside the JJ but also in the notch region at the junction edge. Therefore, L_g increases as the notch



Supplementary Figure S3. **Demonstration of reentrant superconductivity and the near-perfect diode effect in device D9.** (a,b) Color-coded maps of the differential resistance, dV/dI , as a function of the applied magnetic field, H , and bias current, I_b , measured at $T = 6.5$ K (a) in the left-bias configuration without CL current, and (b) in the split-bias configurations with $I_{CL} = I_b$. The tilt of the main lobe in the split-bias configuration (b) is almost twice as large as that observed in the left-bias configuration (a). (c) Current-voltage characteristic measured at $T = 6.5$ K and $H = 8$ Oe in the left-bias configuration. No discernible critical current is observed in the negative-current direction, demonstrating a nearly perfect superconducting diode effect. Reentrant superconductivity does not occur in this conventional bias mode. (d,e) Differential-resistance maps of the same device as functions of the applied magnetic field and bias current, measured at $T = 5$ K (d) in the left-bias $I_{CL} = 0$, and (e) in the split-bias mode with $I_{CL}/I_b = 1$. The blue regions in (e) correspond to the region of negative differential resistance. (f) Current-voltage characteristic measured at $T = 5$ K and $H = 57$ Oe in the split-bias configuration. The curve exhibits reentrant behavior similar to that observed for device D2 in Fig. 3(e,f).

width, W_{Notch} , decreases. This trend is consistent with the observation that the T-shaped three-terminal JJs with a wider notch ($W_{\text{Notch}} \sim 300$ nm; Fig. S2) exhibit a significantly weaker self-field effect than the X-shaped four-terminal JJs with $W_{\text{Notch}} \sim 100$ nm studied in this manuscript. However, not all of this difference can be attributed to the notch width because the kinetic self-field inductance in the T-shaped JJs is also significantly reduced (by approximately a factor of two) owing to the absence, or only a weak presence, of counterflow in the single-terminal electrode.

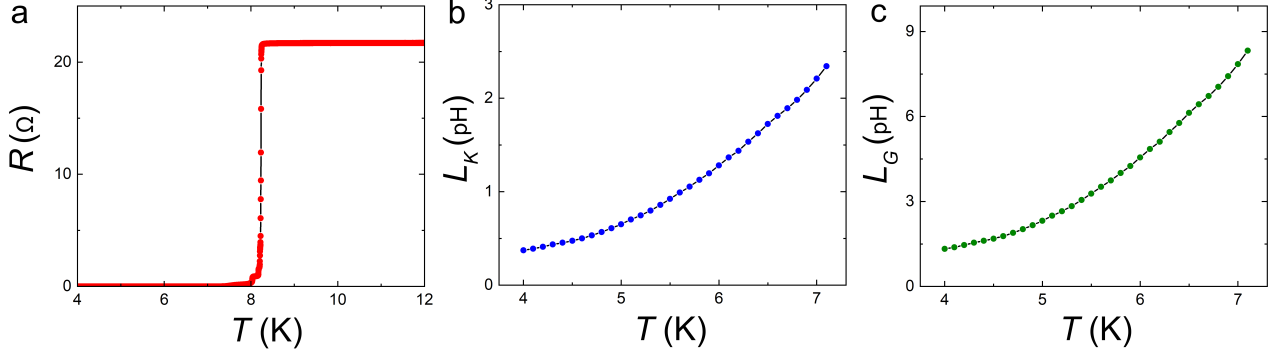
Note that in Eq. S2, the multiplication by the factor $2\lambda^2/Sd$ arises from the fact that the original work [S2] considered a conductor with infinite magnetic permeability. However, in planar junctions, the magnetic field penetrates into the superconductor to a depth given by the Pearl length, λ^2/d , which effectively accounts for flux focusing in planar geometry [S3].

Fig. S4(b) and (c) show the kinetic and geometrical inductances, calculated using Eq. S1 and Eq. S2, respectively, for D2 based on the temperature-dependent London penetration depth:

$$\lambda_L = \lambda_L(0)/\sqrt{1 - (T/T_c)^4} \quad (\text{S3})$$

where $\lambda_L(0) = 120$ nm is the zero-temperature penetration depth of Nb, and $T_c = 8.5$ K is determined experimentally from Fig. S4(a).

As can be seen, both inductances increase with increasing T . Note that near the critical temperature of Nb, the geometrical inductance does not diverge. This is because the effective magnetic field penetration length λ^2/d becomes



Supplementary Figure S4. **Temperature dependent characteristics of D2.** (a) Temperature dependence of the electrode resistance $R(T)$, showing a sharp superconducting transition at $T_c \simeq 8.5$ K. (b) and (c) Calculated temperature dependencies of kinetic (b) and geometric (c) inductances.

limited by the width W_z . Additionally, in planar geometry, the Josephson penetration depth λ_J does not diverge as $T \rightarrow T_c$ [S3], as shown in Fig. S5(e).

SUPPLEMENTARY NOTE 4. TEMPERATURE DEPENDENCE OF DIODE EFFECT

For further investigation of the self-field effect, we conducted an additional set of experiments in which the critical current of D2 was measured as a function of temperature and applied magnetic field. The results of these measurements are shown in Fig. S5(a). As can be seen, both the maximum critical current, I_{c0} [Fig. S5(b)], and H_{sf} [Fig. S5(c)] decrease with increasing temperature. Furthermore, it can be observed that the critical current in (b) exhibits a concave-like shape, with a gradual decay of I_c to zero at $T \simeq 7.2$ K, which is typical for proximity-coupled JJs, while H_{sf} in (c) shows a quasi-linear decay over the same temperature range.

To experimentally determine the self-field inductance, $L_{sf}I_{c0} \simeq \frac{\lambda_J}{W_x} \frac{\Phi_0}{H_0} H_{sf}$, of the asymmetrically biased JJ, it is necessary to know λ_J . We determine it using the relation [S3]:

$$\lambda_J = \frac{2\Phi_0}{\pi^2 H_{c1} W_{\text{eff}}}, \quad (\text{S4})$$

where $W_{\text{eff}} = \Phi_0/(H_0 W_x)$ represents the effective magnetic thickness, and H_{c1} , shown in Fig. S5(d), is the first critical field for Josephson vortex penetration [S3]. The resulting $\lambda_J(T)$ is shown in Fig. S5(e). As can be seen, λ_J increases monotonically but saturates near the critical temperature, in contrast to the behavior of λ_J in conventional overlapped junctions. This behavior results from the nonlocal electrodynamics present in planar Josephson junctions [S3].

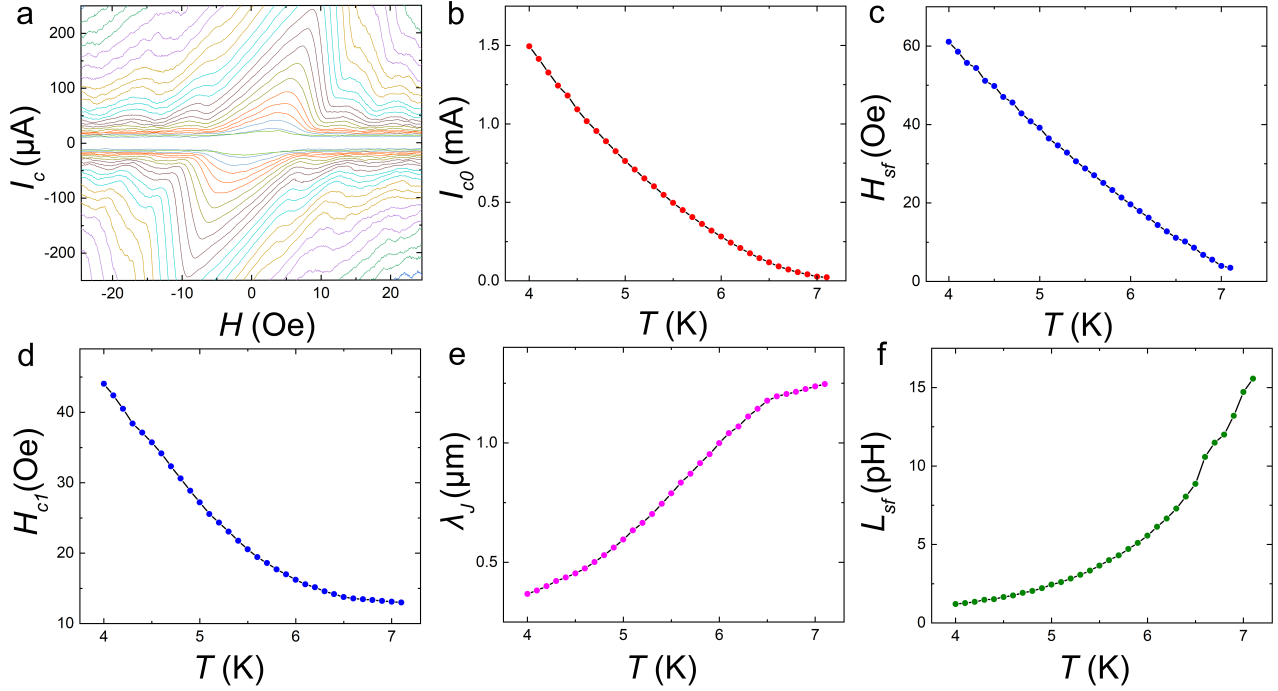
Given λ_J , H_{sf} , and I_c , one can calculate L_{sf} . Figure S5(f) shows the temperature dependence of L_{sf} , which exhibits a monotonic increase with a tendency to diverge near the critical temperature. The deduced L_{sf} is in a semi-quantitative agreement with the theoretically approximated sum of L_K and L_G , shown in Figs. S4(b) and (c).

SUPPLEMENTARY NOTE 5. ADDITIONAL DATA ON TRANSPORT-CURRENT RECTIFICATION

Rectification of the transport current, shown in Figs. 2(e,f), is caused by the nonreciprocity of the current-voltage characteristics, as shown in Fig. 2(b). Therefore, the rectification efficiency can be obtained by a straightforward integration of the dc I - V curve.

To reproduce the experimental conditions in the numerical simulations, we used the same parameters as in Fig. 2a of the main text, with a damping parameter $\eta = 1$ and a low driving frequency $\omega/\omega_p = 0.01$, corresponding to a quasi-dc regime for the Josephson junction. The voltage was averaged over a sufficiently large number of oscillation periods to ensure accuracy, and was normalized by the quantity $V_p = \Phi_0\omega_p$. The simulation results are shown in Fig. S6(a).

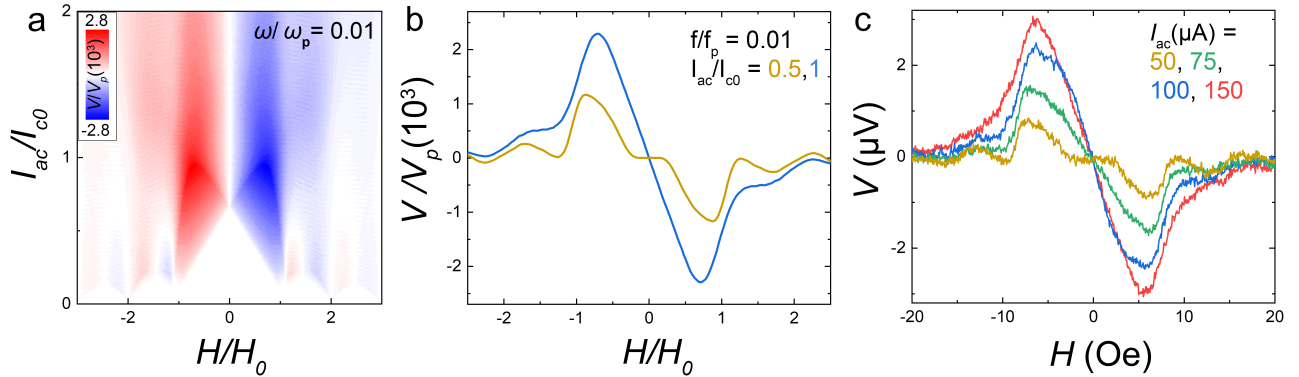
At low applied ac current amplitudes, the rectified voltage changes sign in accordance with the oscillations of the critical current as a function of the applied magnetic field. In contrast, for higher amplitudes $I_{ac} \approx I_c$, the sign no



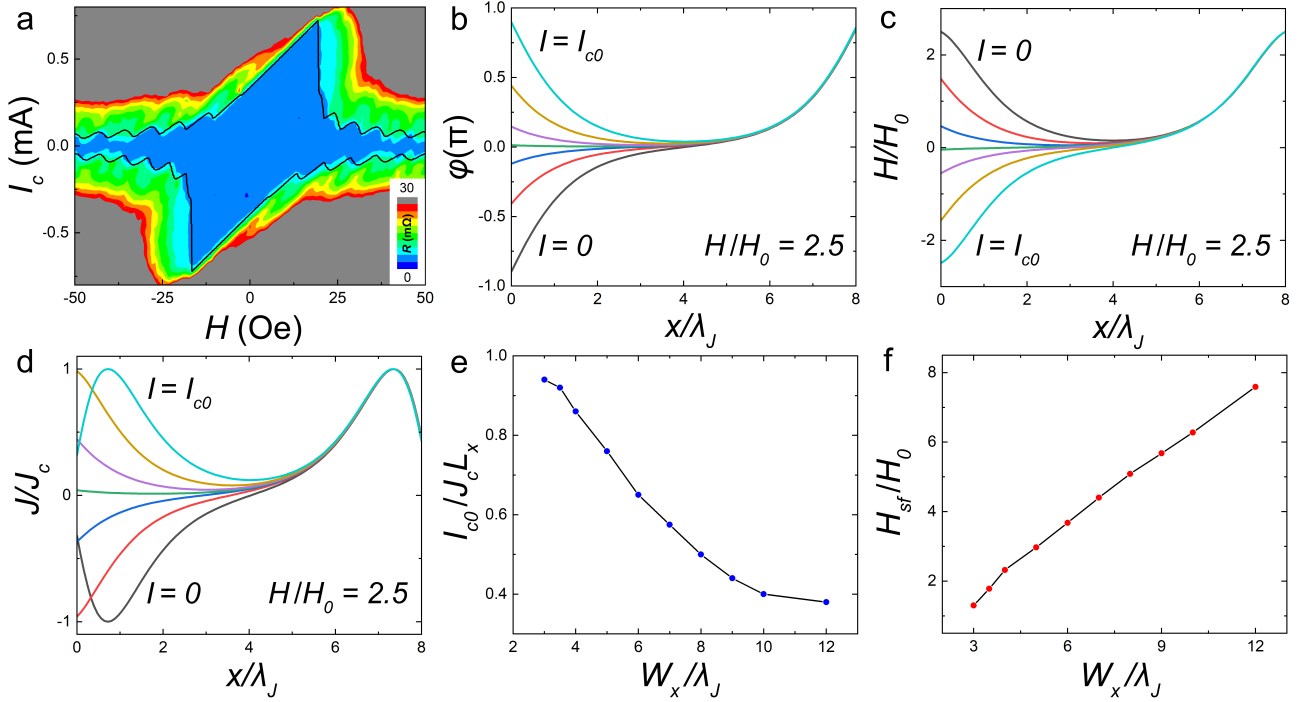
Supplementary Figure S5. **Temperature-dependent characterization of D2.** (a) Critical current versus applied magnetic field measured at temperatures ranging from 4 to 7 K in increments of 0.1 K. (b–f) Various diode parameters as a function of temperature: (b) maximum critical current I_{c0} , (c) self-field H_{sf} , (d) the lower critical field, (e) Josephson penetration depth λ_J , and (f) the effective self-field inductance L_{sf} .

longer reverses and is determined solely by the polarity of the applied magnetic field (see Fig. S6(b) for representative cuts).

Figure S6(c) shows the experimentally determined magnetic-field dependence of the voltage V measured at different



Supplementary Figure S6. **Rectification of transport current: numerical analysis and additional experimental results.** (a) Color-coded map of the calculated rectified voltage as a function of the magnetic field and the AC current amplitude. Calculations were carried out for the same parameters as in Fig. 2(a) $W_x/\lambda_J = 3$ and frequency $\omega/\omega_p = 0.01$. (b) Calculated $V(H)$ for different values of the applied current amplitude. At low current amplitudes, the rectified voltage exhibits sign-changing behavior, whereas at higher amplitudes no sign reversal is observed. In this regime, the sign of the rectified voltage depends only on the polarity of the applied magnetic field. (c) Magnetic-field dependence of the experimentally measured rectified voltage of D2 for different current amplitudes. The quantitative agreement with calculated curves in (b) is evident.



Supplementary Figure S7. **Long junction limit numerical analysis.** (a) Color-coded map of the measured differential resistance of device D2 as a function of magnetic field and bias current through the junction at $T = 5$ K. The black curve represents the calculated critical current obtained using the sine-Gordon formalism, with the experimentally determined junction parameter $W_x/\lambda_J \simeq 8$. (b–d) Calculated, with same L_{sf} as in (a), distributions along the junction length at various values of the bias current, increasing from bottom to top in (b), with a step of approximately $0.2 I/I_{c0}$, and at a constant applied magnetic field of $H/H_0 = 2.5$: (b) phase distribution; (c) normalized magnetic field distribution H/H_0 ; (d) normalized supercurrent density distribution J/J_c . (e) Normalized critical current $I_{c0}/J_c W_x$ as a function of L/λ_J , calculated assuming a constant L_{sf} . A significant decrease is observed with increasing W_x/λ_J . (f) Normalized self-field H_{sf}/H_0 as a function of W_x/λ_J .

ac current amplitudes. The qualitative behaviour closely resembles that obtained in the numerical simulations shown in Fig. S6(a,b). At low driving amplitudes, the rectified voltage exhibits sign reversals, whereas at higher amplitudes its sign is determined solely by the polarity of the applied magnetic field.

SUPPLEMENTARY NOTE 6. NUMERICAL SIMULATIONS

Fig. S7(a) shows the differential resistance of D2 measured at $T = 5$ K in the left-edge bias configuration, along with a fit obtained using the sine-Gordon formalism introduced in the main text, with $W_x/\lambda_J \simeq 8$ (black line). One can see that the fit agrees well with the experimental dependence. Moreover, the value $\lambda_J = 625$ nm is in good agreement with the one determined experimentally; see Fig. S5(e).

Using the same self-field parameter as in Fig. S7(a), we calculated the spatial distributions of the phase [Fig. S7(b)], magnetic field [Fig. S7(c)], and current density $J(x)$ [Fig. S7(d)]. The calculations were performed at a constant external magnetic field of $H/H_0 = 2.5$, corresponding to the field at which the critical current reaches its maximum. The bias current was varied from 0 to I_{c0} in steps of $0.2 I_{c0}$.

As can be observed, without bias current, $I = 0$ (black lines), the phase in (b) acquires equal gradients at both junction edges, and local fields in at the edges in (c) are similar and equal to the external field. The current distribution in (d) exhibits the expected behavior, appearing as an odd function of coordinate x along the junction.

The finite bias current introduces additional Josephson phase gradient at the bias edge (left side in this case), breaking the above mentioned symmetry in the behavior of all three quantities, highlighting the origin of the discussed self-field-induced nonreciprocity.

When the critical current reaches its maximum value and fully compensates the external magnetic flux, both the phase and the current density become even functions, whereas the magnetic field becomes an odd function, effectively

counterbalancing the applied field.

It is important to note that both the bias current and the external magnetic field penetrate over a characteristic length scale of λ_J , thereby validating the approximations used in this work.

As explained in the manuscript, the self-field-like effect is caused by the current-induced Josephson phase gradient in the junction, which is clearly seen in Fig. S7(b). For asymmetric bias from one edge, the phase gradient appears only on one side of the JJ. In the case of symmetric bias, two self-fields of opposite sign are generated at the two edges. However, since each edge carries only half of the total current, the corresponding self-fields are twice weaker than in the asymmetric-bias configuration. As a result, there is no significant difference in the maximum critical current, as can be seen in Fig. 1(d). Note, however, that the presence of a strong self-field-like effect (i.e., a large Josephson phase gradient in the junction) significantly reduces the critical current relative to the unperturbed value, I_{c0} , as demonstrated by the simulations in Fig. 1(d).

Fig. S7(e) and (f) show the calculated values of the normalized critical current, $I_{c0}/J_c W_x$, and the self-field, H_{sf}/H_0 , respectively, using a constant L_{sf} as in Fig. S7(a), plotted as a function of the ratio W_x/λ_J . As can be seen, $I_{c0}/J_c W_x$ decreases significantly—from approximately 1 for $W_x/\lambda_J \leq 3$ to nearly 0.3 at $W_x/\lambda_J = 12$. This result indicates that for long junctions, due to strong asymmetry and concentration of magnetic flux at one edge of the junction, only a small fraction of the critical current is sufficient to drive the system into the flux-flow resistive regime.

This phenomenon demonstrates that the concave-like behavior of I_{c0} observed in Fig. S5(b) arises from the superposition of two effects: the concave increase of the critical current with temperature and the concurrent decrease of its maximum value due to the increasing normalized junction length (i.e., decreasing λ_J).

In contrast to the critical current, the diode self-field exhibits a nearly linear increase. This behavior arises from the penetration of the magnetic field into the junction over a characteristic distance of λ_J .

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