nature portfolio

Peer Review File

Algorithmic Design of Origami Mechanisms and Tessellations

Web links to the author's journal account have been redacted from the decision letters as indicated to maintain confidentiality.

8th Oct 21

Dear Dr Stankovic,

Thank you for submitting your manuscript, "Algorithmic Design of Origami Mechanisms and Tessellations", to Communications Materials. It has now been seen by 3 referees, whose comments are appended below. You will see that while they all find your work of interest, some important points are raised. In particular, the reviewers are asking to provide additional details on the algorithm and optimization procedure, the kinematic constraints, and the advantages/limitations of this algorithm in relation to previous research. Furthermore, both Reviewers #2 and #3 are requesting to provide an easy-to-follow graphic flowchart of the algorithm.

We are interested in the possibility of publishing your study in Communications Materials, but would like to consider your response to these concerns in the form of a revised manuscript before we make a decision on publication. We therefore invite you to revise and resubmit your manuscript, taking into account the points raised.

We are committed to providing a fair and constructive peer-review process. Please don't hesitate to contact us if you wish to discuss the revision in more detail.

When submitting your revised manuscript, please include the following:

-A response letter with a point-by-point reply to each of the referee comments and a description of changes made. Please include the complete referee report in the response letter. Please note that the response letter must be separate to the cover letter to the editors.

-A marked-up version of the manuscript with all changes to the text in a different colored font. Please do not include tracked changes or comments. Please select the file type ‘Revised Manuscript - Marked Up’ when uploading the manuscript file to our online system.

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- If a dataset has a Digital Object Identifier (DOI) as its unique identifier, we strongly encourage including this in the Reference list and citing the dataset in the Data Availability Statement.

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We hope to receive your revised paper within three months; please let us know if you aren’t able to submit it within this time so that we can discuss how best to proceed. If we don’t hear from you, and the revision process takes significantly longer, we will close your file. In this event, we will still be happy to reconsider your paper at a later date, as long as nothing similar has been accepted for publication at Communications Materials or published elsewhere in the meantime.

We understand that due to the current global situation, the time required for revision may be longer than usual. We would appreciate it if you could keep us informed about an estimated timescale for resubmission, to facilitate our planning. Of course, if you are unable to estimate, we are happy to accommodate necessary extensions nevertheless.

Please do not hesitate to contact me if you have any questions or would like to discuss these revisions further. We look forward to seeing the revised manuscript and thank you for the opportunity to review your work.

Best regards,

Dr Aldo Isidori
Associate Editor
Communications Materials

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

In this paper, Walker and Stankovic proposed an algorithm for designing rigidly foldable origami structures. Based on a recent work by one of the authors (Ref. 26) on the Principle of Three Units (PTU) for the rigid foldability conditions of degree-n vertices, the authors introduced four algorithmic operations including expansion, quadrilateral-closing, translating, and mirroring and gluing in this work, which together lead to a new approach for designing complex origami mechanisms and tessellations. Overall, the algorithm is theoretically sound and the examples presented are convincing.

I have a few questions and comments:

1. The proposed method relies heavily on the PTU, but only very few technical details of PTU have been given in the main text or the supplementary information, making the paper difficult to follow. I suggest that the authors include more details of PTU to make the paper more self-contained.

2. What are the limitations of the proposed algorithm? Are the four mentioned algorithmic operations sufficient for generating all kinds of rigidly foldable origami structures?

3. The layer-adding procedure in Fig. 3c looks related to the additive origami design algorithm in Ref. 24. I suggest that the authors include a more detailed discussion on their connections and differences in the section about the quadrilateral-closing operation.

4. What do the colors of the facets represent in Supplementary Movie 1?

Reviewer #2 (Remarks to the Author):

This article presented a generalized approach for the algorithmic design of rigidly-foldable origami structures exhibiting a single kinematic degree of freedom. The Principle of Three Units was extended to quadrilateral facets, generic three-dimensional panel-hinge assemblages and their tessellations. Regular origami patterns were optimized using the interior-point optimizer of the optimization tool box in Matlab and prototypes were fabricated using 3D printing technique to show feasibility.

This article could be considered for publication in communications materials with revision:
1. A flowchart is required for the proposed algorithm, which would also be helpful to present an optimization procedure.
2. Objective function, optimization variables, constraints should be clearly defined in all optimization processes in 16 algorithms.
3. There are several optimizers in the MatLab optimization tool box; the reason of using the interior-point optimizer needs to be discussed.

Reviewer #3 (Remarks to the Author):

The paper "Algorithmic Design of Origami Mechanisms and Tessellations" by Walker and Stankovic is concerned with the design of origami pattern for on-demand shape changes. The work is based on an iterative algorithm that progressively builds origami structures to map a target geometry when folded. The power of the method resides in an iterative approach, which limits the complexity of the design: when growing the structure by one origami vertex, finding a specific target folding pattern only involves a small number of kinematic relations. In addition, they introduce a specific way to handle kinematic constraints that emerge when origami vertices are arranged into plaquettes made of 4 vertices --- a necessary step to construct origami sheets. The authors then demonstrate the applicability of their methods by designing a few examples, from strips and sheets to cylinder and 3d lattices, a few of which are also demonstrated experimentally. While I have a few questions and requests for clarifications (see below), the paper is overall well written and well structured and will be of interest to scientists that work on origami, kirigami and metamaterials. For these reasons, I think that the paper is well suited for Communications Materials.

1. I find the motivation a bit vague at places:
a. what are "non-engineering applications"?;
b. Can the authors give some concrete examples of "novel applications in the design of metamaterials."
c. When the authors say " to explore the potential of origami for the design of metamaterials even more", could they specify what specific advantage does the proposed design approach offer over existing approaches and what new avenues it offers?

2. I miss a generic statement about the number of kinematic paths and the number of degrees of freedom in the current design approach. Most of the structures that are designed have a huge number of kinematic paths, is there a way to know how many they have in advance?

3. On the number of degrees of freedom, I assume that origami that consist of vertices with 4 creases will only have one degree of freedom, am I correct? Is it possible to predict or even design for the number of degrees of freedom in the more complex origami that are introduced?

4. I found the explanation of the "quadrilateral-closing operation" not very clear. There typically is a kinematic constraint when four vertices are connected into a plaquette. How is the optimization run under that constraint is unclear to me.

5. The SI provides very complete information about the design process. I would still find the algorithms a bit easier to parse if there were shown graphically instead of a sequence of "while", "for" , "if" ….

Dear Reviewers,

Thank you very much for the helpful comments to further improve the quality of our manuscript. We carefully considered every comment and address them here, in addition to the changes made in the revised manuscript that are marked-up using a red colored font. Following your comments, we also updated Supplementary Information and Supplementary Movie 1.

We further made minor changes to the manuscript to improve the readability.

The Authors

Reviewer #1 (Remarks to the Author):

In this paper, Walker and Stankovic proposed an algorithm for designing rigidly foldable origami structures. Based on a recent work by one of the authors (Ref. 26) on the Principle of Three Units (PTU) for the rigid foldability conditions of degree-n vertices, the authors introduced four algorithmic operations including expansion, quadrilateral-closing, translating, and mirroring and gluing in this work, which together lead to a new approach for designing complex origami mechanisms and tessellations. Overall, the algorithm is theoretically sound and the examples presented are convincing.

I have a few questions and comments:

1. The proposed method relies heavily on the PTU, but only very few technical details of PTU have been given in the main text or the supplementary information, making the paper difficult to follow. I suggest that the authors include more details of PTU to make the paper more self-contained.

Thank you for your comment. We revised the manuscript as well as the Supplementary Information to include more details regarding PTU. More specifically, in the opening paragraph of the Results and Discussion section in the manuscript we have included a more descriptive explanation of PTU, while the first section of Supplementary Information is revised to contain an in-depth explanation of PTU (please refer to Pages 1-3 in Supplementary Information).

2. What are the limitations of the proposed algorithm? Are the four mentioned algorithmic operations sufficient for generating all kinds of rigidly foldable origami structures?

Currently, we consider one main limitation to be that to apply the quadrilateral-closing operation, Kawasaki's rule must be satisfied at all involved vertices. This excludes all rigidly but not flat-foldable quadrilateral patterns. Furthermore, analogous operations for other faces would enable patterns with faces with more than four edges. Also, expanding the approach to more than one kinematic degree of freedom is possible, but requires small modifications to the current implementation.

Following your comment, we revised the Conclusion section in the main manuscript accordingly. Please refer to the following text there: "Future developments could aim at eliminating the remaining restrictions by introducing a quadrilateral-closing operation that does not require that Kawasaki's rule is satisfied. Also, analogous operations for faces with more than four edges could be introduced. The approach currently can only generate structures with a single kinematic degree of freedom, but could be extended to allow for several kinematic degrees of freedom."

3. The layer-adding procedure in Fig. 3c looks related to the additive origami design algorithm in Ref. 24. I suggest that the authors include a more detailed discussion on their connections and differences in the section about the quadrilateral-closing operation.

Thank you for your comment. We revised the manuscript to contain an explanation of the similarities and differences between the presented procedures and methods found in literature. Ref. 24, specifically, deals with the approximation of a target surface and uses an operation to expand a vertex that does not guarantee flat foldability, whereas our approach generates degree 4 vertex structures that are rigidly- and flat-foldable.

Following your comment, we revised the Results and Discussion section in the main manuscript right after the introduction of the layer adding procedure. Please refer to the following: " Although the layer-adding procedure is similar to existing methods[24,25,30], it is not restricted to discrete sector angles and guarantees flat foldability. Furthermore, the procedure is not restricted by an underlying grid but is versatile as vertices can be added or removed between adding the layers. The layers can also vary in length.".

4. What do the colors of the facets represent in Supplementary Movie 1?

Initially, similar colors were (randomly) assigned to faces in regions that were obtained with the same mirroring or translation operation. However, since the colors carry no information relevant to the reader, we revised Supplementary Movie 1 to use the same color for all faces.

Reviewer #2 (Remarks to the Author):

This article presented a generalized approach for the algorithmic design of rigidly-foldable origami structures exhibiting a single kinematic degree of freedom. The Principle of Three Units was extended to quadrilateral facets, generic three-dimensional panel-hinge assemblages and their tessellations. Regular origami patterns were optimized using the interior-point optimizer of the optimization tool box in Matlab and prototypes were fabricated using 3D printing technique to show feasibility.

This article could be considered for publication in communications materials with revision:

1. A flowchart is required for the proposed algorithm, which would also be helpful to present an optimization procedure.

> Thank you for your comment. To improve the clarity of our manuscript following your comment we have revised it by adding a generic procedure reflecting all aspects of our approach in Fig 1f. The procedure is explained in detail in the 3rd paragraph of the Results and Discussion section in the manuscript. Following your comment, the optimization is now described in-depth in Supplementary Information Sec. 3 using both a flow chart (Figs. 12 and 17 for optimizing a pattern in general and the "Elephant curve" example, respectively), as well as text based explanations (Section 3.1, Page 14). Furthermore, Supplementary Information now contains flowcharts in Figs. 2, 5, 12, 13, 17, 19, 29, 32, 34-37, 46, 49, and 54, to better illustrate our approach.

2. Objective function, optimization variables, constraints should be clearly defined in all optimization processes in 16 algorithms.

> We agree that such a statement is mandatory in the context of optimization problems. In our work, optimization is only used in the context of Fig. 2 of the main manuscript and therefore only affects Algs. 2-5 in Supplementary Information. Alg. 2 is called by Algs. 3-5, such that only Alg. 2 solves optimization problems directly. We revised Supplementary Information (Section 3.1, Page 14) to explicitly state the detailed optimization problem that is solved in Alg. 2.

3. There are several optimizers in the MatLab optimization tool box; the reason of using the interior-point optimizer needs to be discussed.

> The motivation to use the interior-point optimizer for the optimization stems from the fact that the interior-point method-based optimizer in the MATLAB optimization tool box is a general purpose non-linear local optimizer for constrained problems that adheres well to our optimization model formulation. We updated Supplementary Information (Section 3.1) to state this information.

Reviewer #3 (Remarks to the Author):

The paper "Algorithmic Design of Origami Mechanisms and Tessellations" by Walker and Stankovic is concerned with the design of origami pattern for on-demand shape changes. The work is based on an iterative algorithm that progressively builds origami structures to map a target geometry when folded. The power of the method resides in an iterative approach, which limits the complexity of the design: when growing the structure by one origami vertex, finding a specific target folding pattern only involves a small number of kinematic relations. In addition, they introduce a specific way to handle kinematic constraints that emerge when origami vertices are arranged into plaquettes made of 4 vertices --- a necessary step to construct origami sheets. The authors then demonstrate the applicability of their methods by designing a few examples, from strips and sheets to cylinder and 3d lattices, a few of which are also demonstrated experimentally. While I have a few questions and requests for clarifications (see below), the paper is overall well written and well structured and will be of interest to scientists that work on origami, kirigami and metamaterials. For these reasons, I think that the paper is well suited for Communications Materials.

1. I find the motivation a bit vague at places:

Thank you for your comment. We have clarified the motivation for our work by addressing the comments 1.a, 1.b, and 1.c.

a. what are "non-engineering applications"?;

We revised the main text which now refers to approaches that "(…) either address shape matching without guaranteeing rigid foldability applications[18] or target artistic origami[19], (…)". The revised text is now in context of Refs. 18 and 19.

b. Can the authors give some concrete examples of "novel applications in the design of metamaterials."

Thank you for your comment. Compared with existing methods, our approach offers significantly greater versatility in constructing origami structures, thus moving the discovery and analysis of rigidly foldable structures beyond the ubiquitous case-to-case basis, and not limited to a vertex degree or facet type. Thus, we cannot claim that other approaches cannot be used for novel applications in the design of metamaterials regardless of the applications themselves, but what we can say is that our approach offers a much wider range of origami patterns to inspire design of metamaterials for various applications. These various applications of origami in the design of metamaterials we already listed in the first paragraph of the original manuscript. Following your comment, we revised the text in the manuscript such that "novel applications" are rephrased based on our rationale here.

c. When the authors say " to explore the potential of origami for the design of metamaterials even more", could they specify what specific advantage does the proposed design approach offer over existing approaches and what new avenues it offers?

We revised the text in the manuscript accordingly by stating "To enable design practitioners and researchers alike to explore the potential of origami for the design of metamaterials without restrictions to facet or vertex types, (…)". We agree that "even more" is not appropriate and that focusing on the advantages of our method would benefit the discussion in the introductory sections of the manuscript.

Furthermore, we clarified the advantages of our approach as compared to existing methods by adding the statement "Compared with existing methods[20–25], the approach offers significantly greater versatility in constructing origami structures, thus providing a universal tool for the discovery and analysis of rigidly foldable structures."

We also revised the Results and Discussion section in the main manuscript right after the introduction of the layer adding procedure to delineate our approach from other existing methods.

2. I miss a generic statement about the number of kinematic paths and the number of degrees of freedom in the current design approach. Most of the structures that are designed have a huge number of kinematic paths, is there a way to know how many they have in advance?

The number of kinematic paths can be very high and, as exemplified in Fig. 1e, reach 2^n where n is the number of interior vertices, but the exact number can be much lower. We revised the manuscript to state that information and added paragraphs in Supplementary Information Secs. 2.5 and 5.4 to elaborate on the calculation of the exact number and aspects to consider when it is calculated.

3. On the number of degrees of freedom, I assume that origami that consist of vertices with 4 creases will only have one degree of freedom, am I correct? Is it possible to predict or even design for the number of degrees of freedom in the more complex origami that are introduced?

An origami with four creases would correspond to a single degree-4 vertex which has a single kinematic degree of freedom – you are correct. When using our method, the resulting structures will always have a single kinematic degree of freedom. This can be visualized based on the graph-based representation of crease patterns: There is always a single input edge and all dihedral angles are calculated in relation to the dihedral angle at this edge – which therefore represents the only degree of freedom. If more degrees of freedom were desired, several input edges (or source vertices, resp.) would have to be introduced.

We added a statement to the manuscript to clarify this and also to distinguish between the terms “kinematic degree of freedom” and “kinematic path” – which can be hard to tell apart. As explained in our response to your previous comment, we also included a statement regarding the number of kinematic paths.

4. I found the explanation of the "quadrilateral-closing operation" not very clear. There typically is a kinematic constraint when four vertices are connected into a plaquette. How is the optimization run under that constraint is unclear to me.

The kinematic compatibility constraint used here is that the two dihedral angles of the two collinear edges in Fig. 3a are identical for all input dihedral angles. If this is the case, the resulting origami is rigidly foldable. This condition is automatically fulfilled if the sector angles satisfy Kawasaki’s condition and the sector angles and the mode of the quadrilateral-closing vertex are calculated as described in Supplementary Information Sec. 5.1 – and therefore whenever the quadrilateral-closing operation is applied. (No optimization is used or necessary in this context.)

We revised the introduction of the quadrilateral-closing operation in the main manuscript. Please refer to the following changes “Considering, for example, the general case shown in Fig. 3a, the two collinear edges marked with ρ_1 and ρ_2 do not necessarily have an identical dihedral angle for arbitrary dihedral angles ρ_{in}.”, and “(…) the face to the right of the edge marked with ρ_1 always coincides with the face to the left of the edge marked with ρ_2 allowing to (…)”.

5. The SI provides very complete information about the design process. I would still find the algorithms a bit easier to parse if there were shown graphically instead of a sequence of "while", "for" , "if"

Following your comment, we have revised Supplementary Information to include 15 flow charts to better explain the main aspects of our work and the involved algorithms. We have also added a flow chart to the main manuscript in Fig. 1f which is applicable to all algorithmic design processes that are considered in our work. The procedure in the flow chart in in Fig. 1f is explained in detail in the 3rd paragraph of the Results and Discussion section in the manuscript. Supplementary Information now contains flowcharts in Figs. 2, 5, 12, 13, 17, 19, 29, 32, 34-37, 46, 49, and 54, to better illustrate our approach.

10th Dec 21

Dear Dr Stankovic,

Your manuscript titled "Algorithmic Design of Origami Mechanisms and Tessellations" has now been seen again by our referees, whose comments appear below. In light of their advice I am delighted to say that we are happy, in principle, to publish a suitably revised version in Communications Materials under the open access CC BY license (Creative Commons Attribution v4.0 International License).

We therefore invite you to revise your paper one last time to comply with our journal policies and formatting style in order to maximise the accessibility and therefore the impact of your work.

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We hope to hear from you within two weeks; please let us know if the process may take longer.

Best regards,

Dr Aldo Isidori
Associate Editor
Communications Materials

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

The authors have addressed all my concerns in the revised manuscript. I recommend acceptance of the article for publication.

Reviewer #2 (Remarks to the Author):

Authors have addressed my comments. The manuscript has been significantly improved and could be considered for publication.

Reviewer #3 (Remarks to the Author):

The authors have successfully addressed the comments of the reviewers. I therefore recommend publication of the article in its current form.

Dear Reviewers,

Again, thank you very much for the helpful comments that improved the quality of our manuscript.

The Authors

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The authors have addressed all my concerns in the revised manuscript. I recommend acceptance of the article for publication.

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