Supplementary Information for "Fatigue fracture mechanism of amorphous materials from a density-based coarse-grained model"

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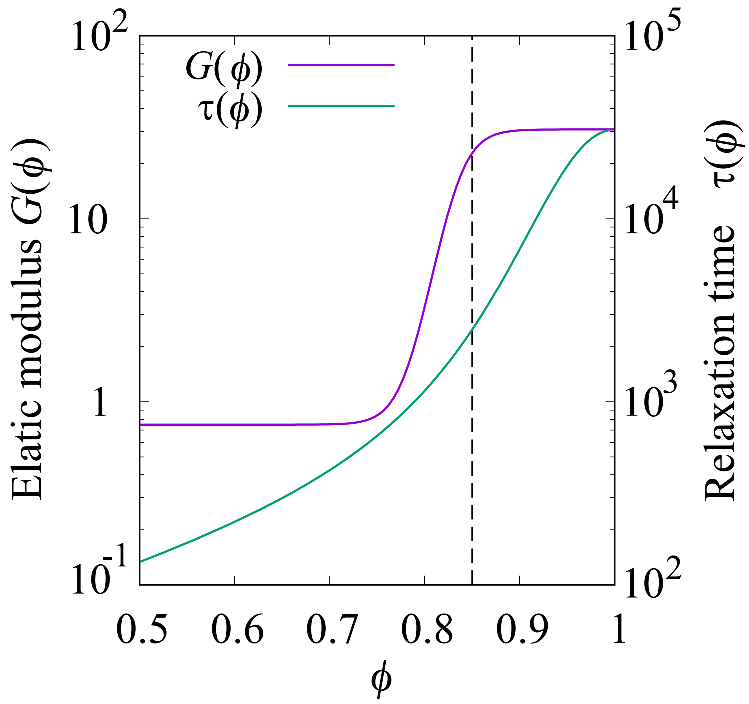
SUPPLEMENTARY NOTE 1: DENSITY DEPENDENCES OF SHEAR ELASTIC MODULUS AND STRUCTURAL RELAXATION TIME

We used density dependences of the shear elastic modulus $G(\rho)$ and structural relaxation time $\tau(\rho)$. By using normalized density $\phi = \rho/\rho_{\mathrm{c}}$, these functional forms can be written as $G(\phi) = G_0\{\tanh[\Lambda_{\mathrm{G}}(\phi \cdot \rho_{\mathrm{c}}/\rho_{\mathrm{G}} - 1)] + 1\} + 0.05G_0$ and $\tau(\phi) = \tau_{\mathrm{B}}\{\exp[\mu(\rho)\phi/(1-\phi)] - 1\} + 0.05\tau_{\mathrm{B}}$, respectively (see Method). Here, $\mu(\phi)$ is given by $\mu(\phi) = \Gamma \tanh[\Lambda_\tau(1/\phi - 1)]$.

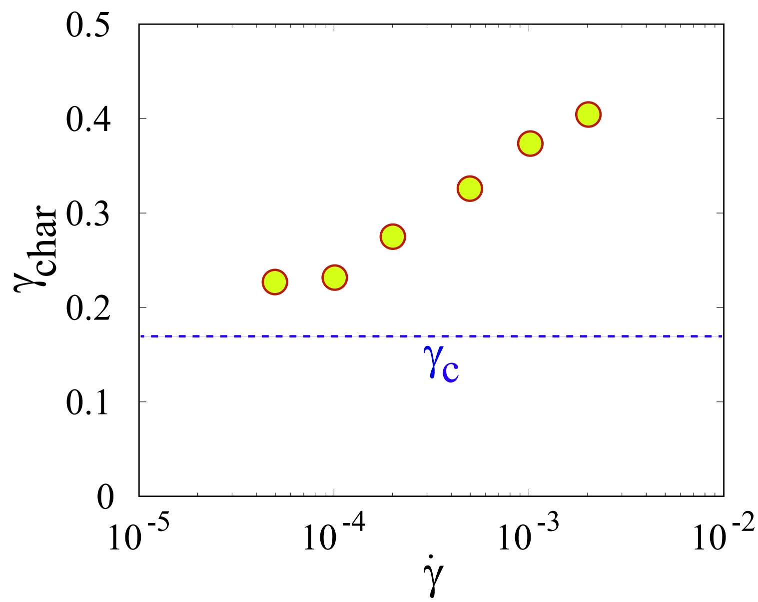
In our numerical simulations, we set the above parameters as $G_0 = 15$, $\rho_{\mathrm{G}}/\rho_{\mathrm{c}} = 0.835$, $\Lambda_{\mathrm{G}} = 28.4$, $\tau_{\mathrm{B}} = 200$, $\Lambda_\tau = 10.5$ and $\Gamma = 0.48$, respectively. The density dependences of $G(\phi)$ and $\tau(\phi)$ are plotted in Supplementary Fig. 1.



Supplementary Figure 1. **Normalized density dependences of shear elastic modulus and structural relaxation time.** Vertical dashed line shows the mean normalized density $\phi = 0.85$

SUPPLEMENTARY NOTE 2: SHEAR RATE DEPENDENCE OF FRACTURE STRAIN UNDER SIMPLE SHEAR

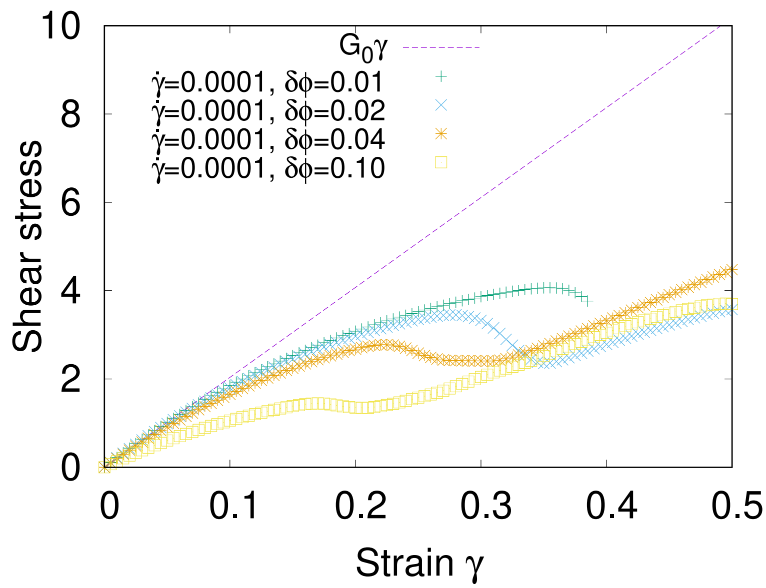
In Fig.1, we saw the upper yield strain, $\gamma_{\rm yield}$, and the fracture strain, $\gamma_{\rm frac}$, are larger than the critical strain $\gamma_{\rm c}$. Now, we define the characteristic strain $\gamma_{\rm char}$ as the strain which $\langle(\delta\phi)^2\rangle$ tends to 0.06. We plot the characteristic strain $\gamma_{\rm char}$ as a function of the shear rate $\dot{\gamma}$ in Supplementary Fig. 2.



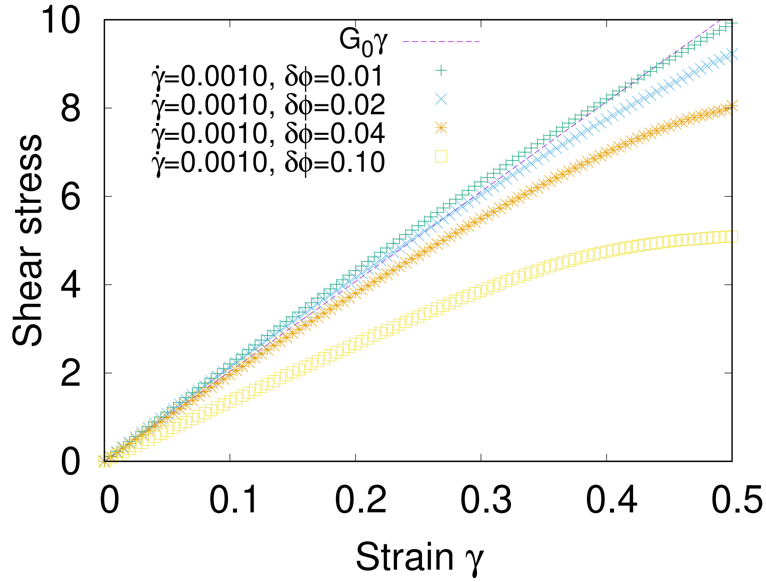
Supplementary Figure 2. **Shear rate dependence of the characteristic strain under simple shear.** The characteristic strain $\gamma_{\rm char}$ is always larger than the critical strain $\gamma_{\rm c}$ indicated by the horizontal dashed blue line, i.e., the onset strain of nonlinear density growth, especially for high $\dot{\gamma}$ (the solid-like fracture regime).

SUPPLEMENTARY NOTE 3: STRESS-STRAIN CURVES DEPENDENCES OF INITIAL DENSITY FLUCTUATION

In our numerical simulations, we set initial normalized density as $\phi_0 = \langle\phi\rangle = 0.85$ and the average variance $\delta\phi = 0.04$. We plotted the stress-strain curves at $\dot{\gamma} = 0.0001$ and 0.0010 for various initial density fluctuation $\delta\phi$ under simple shear in Supplementary Figs. 3 and 4, respectively. We can confirm that the difference in the stress-strain curves in the low-strain region, $0 \leq \gamma \lesssim 0.2$, is small between the density fluctuations $\delta\phi = 0.01, 0.02$ and 0.04. However, we can confirm that the stress-strain curves are significantly different in the high strain region, $\gamma \gtrsim 0.2$, and particularly when the density fluctuation is $\delta\phi = 0.10$. These results show that the preexisting density inhomogeneities (e.g., voids) in materials should significantly affect fracture behaviour and lower the material strength.



Supplementary Figure 3. **Stress-strain curves at** $\dot{\gamma} = 0.0001$ **for various initial density fluctuation** $\delta\phi$ **under simple shear.** With increasing density fluctuation $\delta\phi$, yielding occurs at a lower strain.



Supplementary Figure 4. **Stress-strain curves at $\dot{\gamma} = 0.0010$ for various initial density fluctuation $\delta\phi$ under simple shear.** With increasing density fluctuation $\delta\phi$, the material strength monotonically decreases.