

Supplementary Information:

Superconducting flux qubit with ferromagnetic Josephson π -junction operating at zero magnetic field

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Supplementary Note 1. Lagrangian of π -junction qubit

Let us take the qubit loop shown in the inset of Fig. 1(d). Since the Lagrangian of each junction is defined as the difference between the kinetic energy ($T = \frac{C_i \Phi_0^2}{2} \dot{\delta}_i^2$) and potential energy terms ($U = E_J(1 - \cos \delta)$), the Lagrangian of the whole circuit can be written as

$$\mathcal{L} = T - U = \sum_{i=1}^3 \left(\frac{C_i \Phi_0^2}{2} \dot{\delta}_i^2 - E_{J,i}(1 - \cos \delta_i) \right) + \frac{C_\pi \Phi_0^2}{2} \dot{\delta}_\pi^2 - E_{J,\pi}(1 - \cos(\delta_\pi + \pi)) \quad (\text{S1})$$

where C_i is the capacitance, $\Phi_0 = \Phi_0/2\pi$, Φ_0 is the superconducting flux quantum, $E_{J,i}$ is the Josephson energy given by $E_{J,i} = I_{c,i} \Phi_0 / (2\pi)$, $I_{c,i}$ is the critical current, and δ_i is the dimensionless phase variable. Here, the index i labels the three conventional Josephson junctions ($i = 1, 2, 3$), as well as π for the π -junction. Importantly, the terms that correspond to the π -junction look almost identical to those of the other junctions, except for the π -phase shift in the potential energy term. Flux quantization leads to the constraint:

$$\sum_{i=1}^3 \delta_i + \delta_\pi = \frac{2\pi \Phi_{\text{ext}}}{\Phi_0} + 2\pi n \quad (\text{S2})$$

where n is an integer. In principle, to analyze the quantum dynamics of the different variables, we need to substitute the constraint into the Lagrangian, calculate the Hamiltonian via a Legendre transformation, introduce the quantum commutation relations and then analyze the properties and dynamics of different variables. However, we can already say something about the behavior of the variables based on the Lagrangian in Eq. (S1).

The classical ground state of the system is obtained by minimizing the potential energy. Since the π -junction's critical current is four orders of magnitude larger than those of the other junctions, its potential energy term will dominate in comparison to the other junctions. The classical ground state must therefore have $\delta_\pi \approx \pi$. Furthermore, since the capacitance C_π is comparable to those of the conventional Josephson junctions¹, the strong confinement of the variable δ_π implies that, to a very good approximation, quantum fluctuations in δ_π are negligible compared to the other phase variables δ_i . Regarding the C_π of our SFS junction, we assume that it is of the same order of magnitude as that in Ref. 1. In the Supplementary Information of Ref. 1, Feofanov *et al.* estimated the capacitance C_π of their SFS junction to be less than 10 fF. Considering the capacitance of the larger Josephson junction (JJ) ($C_J = 36.0$ fF) and the smaller JJ ($\alpha C \approx 16.5$ fF) in our flux qubit, we estimate that C_π of our SFS junction is comparable to those of the other junctions.

As a result, we can set $\delta_\pi = \pi$ and $\dot{\delta}_\pi = 0$ to obtain the approximate Lagrangian

$$\mathcal{L} = \sum_{i=1}^3 \left(\frac{C_i \Phi_0^2}{2} \dot{\delta}_i^2 - E_{J,i}(1 - \cos \delta_i) \right) \quad (\text{S3})$$

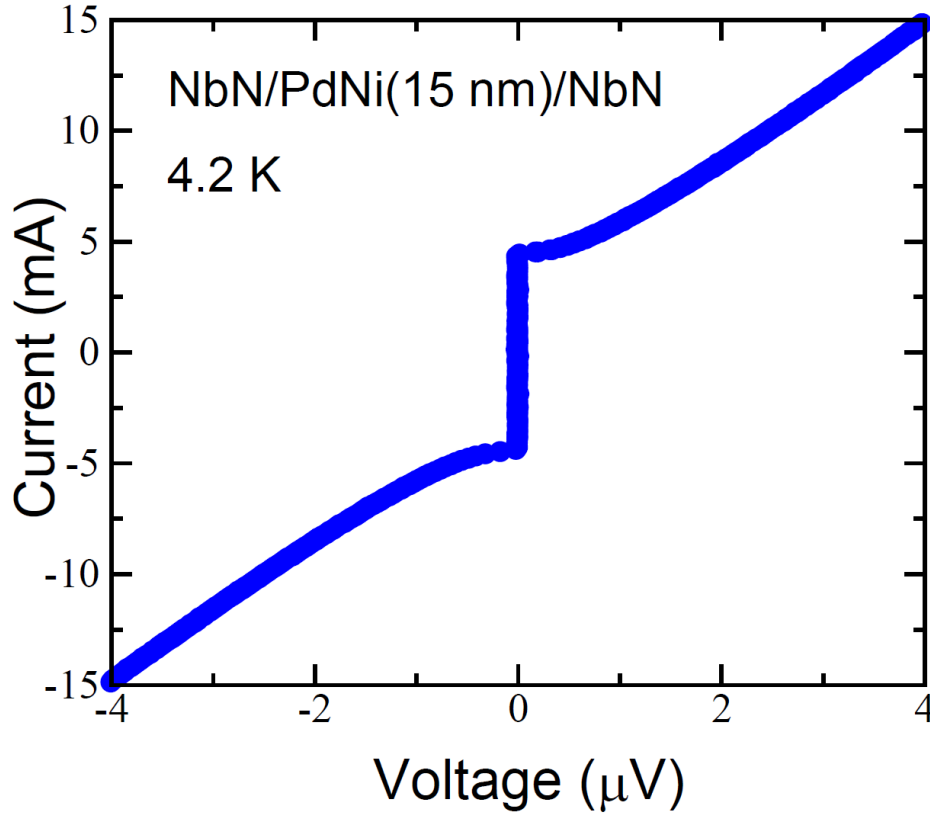
with the flux quantization condition

$$\sum_{i=1}^3 \delta_i = \frac{2\pi \Phi_{\text{ext}}}{\Phi_0} + 2\pi n - \pi \quad (\text{S5})$$

In other words, the Lagrangian of the π -junction flux qubit reduces to that of an equivalent circuit without the π -junction, but with half a flux quantum added to the applied flux bias.

Supplementary Note 2. Estimation of I_c and R_n of π -junction

For the NbN/PdNi/NbN junction with a $10\ \mu\text{m}$ -square-sized junction, we measured I-V characteristics at 4.2 K as shown in the following figure. Their critical current (I_c) and the normal resistance (R_n) are about 4.4 mA and $270.3\ \mu\Omega$. The current density (J_c) of the π -junction is estimated to be about $4.4\ \text{kA cm}^{-2}$. Considering a circular shape with a diameter of $10\ \mu\text{m}$ used for a π -junction in qubit, the extracted I_c and R_n are 3.5 mA and $344.3\ \mu\Omega$.



Supplementary Figure 1. Current-voltage (I-V) characteristics of a $10 \times 10\ \mu\text{m}^2$ π -junction with a 15 nm-thick PdNi interlayer measured at 4.2 K.

Supplementary References

1. Feofanov, A. K. *et al.*, Implementation of superconductor/ferromagnet/superconductor π -shifters in superconducting digital and quantum circuits, *Nat. Phys.* **6**, 593-597 (2010).