

Universal negative magnetoresistance in antiferromagnetic metals from symmetry breaking of electron wave functions

Corresponding Author: Dr Pavel Grigoriev

This file contains all editorial decision letters in order by version, followed by all author rebuttals in order by version.

Version 0:

Decision Letter:

**** Please ensure you delete the link to your author home page in this e-mail if you wish to forward it to your coauthors ****

Dear Dr Grigoriev,

Thank you for submitting your manuscript, "Universal negative magnetoresistance in antiferromagnetic metals from symmetry breaking of electron wave function", to Communications Materials. It has now been seen by 2 referees, whose comments are appended below. You will see that while they find your work of potential interest, they have raised substantial concerns that must be addressed. In light of these comments, we cannot accept the manuscript for publication, but are interested in considering a revised version that addresses these serious concerns, especially the major issues raised by Reviewer #1.

Please note that, if arXiv:2411.03971 has more complete and thorough results, these results should be incorporated in the current paper (assuming that arXiv:2411.03971 is not published first).

We hope you will find the referees' comments useful as you decide how to proceed. Should further experimental data or analysis allow you to address these criticisms, we would be happy to look at a substantially revised manuscript. However, please bear in mind that we will be reluctant to approach the referees again in the absence of major revisions. If the revision process takes significantly longer than three months, we will be happy to reconsider your paper at a later date, as long as nothing similar has been accepted for publication at Communications Materials or published elsewhere in the meantime.

We are committed to providing a fair and constructive peer-review process. Please don't hesitate to contact us if you wish to discuss the revision in more detail.

When submitting your revised manuscript, please include the following:

-A response letter with a point-by-point reply to each of the referee comments and a description of changes made. Please include the complete referee report in the response letter. Please note that the response letter must be separate to the cover letter to the editors.

-A marked-up version of the manuscript with all changes to the text in a different colored font. Please do not include tracked changes or comments. Please select the file type 'Revised Manuscript - Marked Up' when uploading the manuscript file to our online system.

-A clean version of the manuscript. Please select the file type 'Article File'.

-An updated <https://www.nature.com/documents/nr-editorial-policy-checklist.zip> Editorial Policy checklist, uploaded as a 'Related Manuscript File' type. This checklist is to ensure your paper complies with all relevant editorial policies. If needed, please revise your manuscript in response to these points. Please note that this form is a dynamic 'smart pdf' and must therefore be downloaded and completed in Adobe Reader. Clicking this link will download a zip file containing the pdf.

Please use the following link to submit your revised manuscript files:

Link Redacted

** This url links to your confidential home page and associated information about manuscripts you may have submitted or be reviewing for us. If you wish to forward this email to co-authors, please delete the link to your homepage first **

Please do not hesitate to contact me if you have any questions or would like to discuss the required revisions further. Thank you for the opportunity to review your work.

Best regards,

Oleksandr Pylypovskyi
Editorial Board Member
Communications Materials
orcid.org/0000-0002-5947-9760

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

In this manuscript, Grigoriev et al use a four-band tight-binding model and Born approximation trying to explain the experimentally observed negative magnetoresistance (MR) in antiferromagnetic (AFM) EuSn_2As_2 , and claim this is a universal behavior for AFM metals, or the so-called negative isotropic MR (NIMR). Here are some major comments.

1. The key result of the inverse parabolic dependence of MR on magnetic field H in Eq.(10) and Fig.3 directly comes from Eqn.(2), where the change of exchange-splitting as a function of H is simply written down by citing Ref.27, Kittel's Quantum Theory of Solids textbook. However, in Ref.27, there is no such formula of field dependence for exchange-splitting.

After reading the other arXiv paper from the same authors Ref.25 (Pervakov et al arXiv:2411.03971) Appendix A, Eq.(5-7), it turns out this formula is derived from the classical spin moment reorientation with an applied H . However, if only spin is reoriented, the size of the total spin moment, thus exchange-splitting, should stay the same, which is from the electron exchange interaction, usually calculated in density functional theory (DFT) in the collinear form, especially for Eu with half-filled 4f orbitals carrying a local moment. Thus, it is not clear that the main result in Eq.(2) of field dependence for exchange-splitting in the current manuscript is well founded.

2. Also given the newer Ref.25, arXiv:2411.03971 also covers the same theoretical analysis and experimental MR data of EuSn_2As_2 in more details (as well ARPES data and DFT calculations) than the current manuscript arXiv:2405.18046, it seems the newer paper is more complete and is the main paper on EuSn_2As_2 .

3. For EuSn_2As_2 , two groups have proposed a different magnetic structure around 2K below the A-type AFM with $T_N=21$ K from magnetic susceptibility and MR measurements in Ref.9, Li et al, PRB 104, 054435 (2021) and Ref.29, Pakhira et al, PRB 104, 174427 (2021). How does such a different magnetic structure affect the theoretical analysis?

4. If the proposed NIMR behavior from the classical spin reorientation is universal as claimed for AFM metals, more evidence and analysis from other AFM metals with both local and itinerant magnetism should be presented.

Thus, based on these major comments, the manuscript in the current form does not meet the high standard of Commun. Mater.

Reviewer #2 (Remarks to the Author):

In this work, based on theoretic modeling, the authors clarified the origin of the negative isotropic magnetoresistance observed in layered AFM semimetals. The proposed NIMR was further confirmed in experimental measurement of EuSn_2As_2 . The authors should address the following major points before the manuscript may be considered for publication:

(1) The authors should specify the definition of Z_2 symmetry and why such symmetry was broken in EuSn_2As_2 .

(2) There is a transition from NIMR to classical MR. What is the origin of such transition? Please explain. In addition, is it possible to induce the transition from FM to AFM or vice versa by external field in this work?

(3) The spin-orbit coupling was ignored in this work. The authors should comment on the effect of SOC on the proposed NIMR.

**** Visit Nature Research's author and referees' website at www.nature.com/authors for information about policies, services and author benefits****

Communications Materials is committed to improving transparency in authorship. As part of our efforts in this direction, we are now requesting that all authors identified as 'corresponding author' create and link their Open Researcher and Contributor Identifier (ORCID) with their account on the Manuscript Tracking System prior to acceptance. ORCID helps the scientific community achieve unambiguous attribution of all scholarly contributions. You can create and link your ORCID from the home page of the Manuscript Tracking System by clicking on 'Modify my Springer Nature account' and following the instructions in the link below. Please also inform all co-authors that they can add their ORCIDs to their accounts and that they must do so prior to acceptance.

<https://www.springernature.com/gp/researchers/orcid/orcid-for-nature-research>

For more information please visit <http://www.springernature.com/orcid>

If you experience problems in linking your ORCID, please contact the Platform Support Helpdesk.

Version 1:

Decision Letter:

**** Please ensure you delete the link to your author homepage in this e-mail if you wish to forward it to your coauthors ****

Dear Dr Grigoriev,

Thank you for submitting your revised manuscript, "Universal negative magnetoresistance in antiferromagnetic metals from symmetry breaking of electron wave function", to Communications Materials. It has now been seen again by the 2 referees, whose comments are appended below. You will see that while they find your work improved, some important points are still raised by Reviewer #1.

We remain interested in the possibility of publishing your study in Communications Materials, but we kindly invite you to revise your manuscript one last time before we make a decision on publication.

We are committed to providing a fair and constructive peer-review process. Please don't hesitate to contact us if you wish to discuss the revision in more detail.

When submitting your revised manuscript, please include the following:

-A response letter with a point-by-point reply to each of the referee comments and a description of changes made. Please include the complete referee report in the response letter. Please note that the response letter must be separate to the cover letter to the editors.

-A marked-up version of the manuscript with all changes to the text in a different colored font. Please do not include tracked changes or comments. Please select the file type 'Revised Manuscript - Marked Up' when uploading the manuscript file to our online system.

-A clean version of the manuscript. Please select the file type 'Article File'.

In the event that your manuscript is accepted we will provide detailed guidance on our journal policies and formatting. You may however wish to ensure that the manuscript complies with our house style at this stage. See our style and formatting guide (<https://www.nature.com/documents/commsj-phys-style-formatting-guide-accept.pdf>) and checklist (<https://www.nature.com/documents/commsj-phys-style-formatting-checklist-article.pdf>) for reference.

Data availability statements and data citations policy: All Communications Materials manuscripts must include a section titled "Data Availability" at the end of the Methods section or main text (if no Methods). More information on this policy, and a list of examples, is available at http://www.nature.com/authors/policies/data/data-availability-statements-data-citations.pdf.

In particular, the Data availability statement should include:

- Accession codes for deposited data
- Other unique identifiers (such as DOIs and hyperlinks for any other datasets)

- At a minimum, a statement confirming that all relevant data are available from the authors
- If applicable, a statement regarding data available with restrictions
- If a dataset has a Digital Object Identifier (DOI) as its unique identifier, we strongly encourage including this in the Reference list and citing the dataset in the Data Availability Statement.

DATA SOURCES: We strongly encourage authors to deposit all new data associated with the paper in a persistent repository where they can be freely and enduringly accessed. We recommend submitting the data to discipline-specific, community-recognized repositories, where possible and a list of recommended repositories is provided at <http://www.nature.com/sdata/policies/repositories>.

If a community resource is unavailable, data can be submitted to generalist repositories such as [figshare](https://figshare.com/) or [Dryad Digital Repository](http://datadryad.org/). Please provide a unique identifier for the data (for example a DOI or a permanent URL) in the data availability statement, if possible. If the repository does not provide identifiers, we encourage authors to supply the search terms that will return the data. For data that have been obtained from publicly available sources, please provide a URL and the specific data product name in the data availability statement. Data with a DOI should be further cited in the methods reference section.

Please refer to our data policies at <http://www.nature.com/authors/policies/availability.html>.

Please use the following link to submit your documents:

Link Redacted

**** This url links to your confidential home page and associated information about manuscripts you may have submitted or be reviewing for us. If you wish to forward this email to co-authors, please delete the link to your homepage first ****

We hope to receive your revised paper within six weeks; please let us know if you aren't able to submit it within this time so that we can discuss how best to proceed. If we don't hear from you, and the revision process takes significantly longer, we will close your file. In this event, we will still be happy to reconsider your paper at a later date, as long as nothing similar has been accepted for publication at Communications Materials or published elsewhere in the meantime.

Please do not hesitate to contact me if you have any questions or would like to discuss these revisions further. We look forward to seeing the revised manuscript and thank you for the opportunity to review your work.

Best regards,

Oleksandr Pylypovskyi, PhD
Editorial Board Member
Communications Materials
orcid.org/0000-0002-5947-9760

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

In the revised manuscript, the authors have added Eqn.(2), (4) and the associated new paragraphs to explain the derivation of Eqn.(3). They have added a new paragraph and the associated Supplementary Note to consider spin-orbit coupling (SOC) effect. They have also provided a new section and Fig.6 to apply the NIMR model to six other AFM materials besides EuSn₂As₂.

With these revisions, the manuscript has been improved. However, there are still two major issues.

1. Regarding to the derivation of Eqn.(3), the newly added Eqn.(4) and the right side of Eqn.(3) are for the magnetization and susceptibility of the Eu moments from the half-filled local Eu 4f electrons and its field dependence. But the left side of Eqn. (3), $E_{\text{ex}}(H)$, is defined as the difference of the exchange-splitting for the conduction electrons on two AFM sublattice. As the authors acknowledged in the Reply letter that the size of the AFM moment does not change because it is from the electron-exchange interaction of the local 4f electrons. The induced exchange splitting of the conduction electron is also independent of magnetic field, which is calculated in DFT in the collinear form. Then the equality in Eqn.(3) and the field dependence for $E_{\text{ex}}(H)$ is more of an ad hoc justification than a rigorous proof. The effect of local moment reorientation on conduction electron should have been more systematically treated in RKKY-type analysis.

2. The use of "Z2-symmetry" here for the simple A-type AFM order is very confusing. In modern condensed matter theory, Z2 is a topological classification of band structure for being even or odd from mod(2) of symmetry-protected topological invariants. It provides more information than magnetic space group symmetries. But here the relation of the two A-type AFM sublattices is already adequately described with magnetic space group symmetries. Using "Z2-symmetry" here can cause unnecessary confusion for the readers.

Reviewer #2 (Remarks to the Author):

The authors have addressed my concerns and I recommend the revised manuscript for publication.

**** Visit Nature Research's author and referees' website at <http://www.nature.com/authors> for information about policies, services and author benefits****

Communications Materials is committed to improving transparency in authorship. As part of our efforts in this direction, we are now requesting that all authors identified as 'corresponding author' create and link their Open Researcher and Contributor Identifier (ORCID) with their account on the Manuscript Tracking System prior to acceptance. ORCID helps the scientific community achieve unambiguous attribution of all scholarly contributions. You can create and link your ORCID from the home page of the Manuscript Tracking System by clicking on 'Modify my Springer Nature account' and following the instructions in the link below. Please also inform all co-authors that they can add their ORCIDs to their accounts and that they must do so prior to acceptance.

<https://www.springernature.com/gp/researchers/orcid/orcid-for-nature-research>

For more information please visit <http://www.springernature.com/orcid>

If you experience problems in linking your ORCID, please contact the [Platform Support Helpdesk](http://platformsupport.nature.com/).

Version 2:

Decision Letter:

**** Please ensure you delete the link to your author homepage in this email if you wish to forward it to your coauthors ****

Dear Dr Grigoriev,

Thank you once again for submitting your manuscript, "Universal negative magnetoresistance in antiferromagnetic metals from symmetry breaking of electron wave function," to Communications Materials. It has now been seen again by the referees, whose comments are appended below. The concerns of our reviewers have now been addressed, but there are some amendments needed before we can accept your paper.

We ask that you edit your manuscript according to the attached table. **Please read this document carefully as we will be unable to further assess your revised paper until these important points are addressed.**

Please outline all revisions made in the right-hand column and return the completed table with your updated manuscript files as a Related Manuscript file.

Please use the link below to submit your revised files:

Link Redacted

When resubmitting, please provide a marked-up manuscript with all changes highlighted, as well as a clean version of your paper.

We hope to receive this updated version of your paper within 1 week, but please let us know if you find that you need more time.

Best regards,

Oleksandr Pylypovskyi, PhD
Editorial Board Member
Communications Materials
orcid.org/0000-0002-5947-9760

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The concerns have been addressed by the authors and the manuscript is now publishable.

Version 3:

Decision Letter:

**** Please ensure you delete the link to your author homepage in this email if you wish to forward it to your coauthors ****

Dear Dr Grigoriev,

Thank you once again for submitting your manuscript, "Universal negative magnetoresistance in antiferromagnetic metals from symmetry breaking of electron wave function," to Communications Materials. The concerns of our reviewers have been addressed, but there are some amendments needed before we can accept your paper.

We ask that you edit your manuscript according to the attached table. **Please read this document carefully as we will be unable to further assess your revised paper until these important points are addressed.**

Please outline all revisions made in the right-hand column and return the completed table with your updated manuscript files as a Related Manuscript file.

Please use the link below to submit your revised files:

Link Redacted

When resubmitting, please provide a marked-up manuscript with all changes highlighted, as well as a clean version of your paper.

We hope to receive this updated version of your paper within 1 week, but please let us know if you find that you need more time.

Best regards,

Oleksandr Pylypovskyi, PhD
Editorial Board Member
Communications Materials
orcid.org/0000-0002-5947-9760

Open Access This Peer Review File is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were

made.

In cases where reviewers are anonymous, credit should be given to 'Anonymous Referee' and the source.

The images or other third party material in this Peer Review File are included in the article's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

To view a copy of this license, visit <https://creativecommons.org/licenses/by/4.0/>

We thank the referees for their careful reading our manuscript and for the comments. Below is our detailed reply to these comments with the corresponding list of changes in our manuscript. All changes and new additions are also highlighted in blue text in the revised manuscript.

Response to Reviewer #1:

Referee comment: In this manuscript, Grigoriev et al use a four-band tight-binding model and Born approximation trying to explain the experimentally observed negative magnetoresistance (MR) in antiferromagnetic (AFM) EuSn₂As₂, and claim this is a universal behavior for AFM metals, or the so-called negative isotropic MR (NIMR). Here are some major comments.

Our response. We appreciate the reviewer's important suggestions for our manuscript. We have replied to the comments, revised our manuscript accordingly, and added supplemental materials.

Referee comment 1. The key result of the inverse parabolic dependence of MR on magnetic field H in Eq.(10) and Fig.3 directly comes from Eqn.(2), where the change of exchange-splitting as a function of H is simply written down by citing Ref.27, Kittel's Quantum Theory of Solids textbook. However, in Ref.27, there is no such formula of field dependence for exchange-splitting.

After reading the other arXiv paper from the same authors Ref.25 (Pervakov et al arXiv:2411.03971) Appendix A, Eq.(5-7), it turns out this formula is derived from the classical spin moment reorientation with an applied H . However, if only spin is reoriented, the size of the total spin moment, thus exchange-splitting, should stay the same, which is from the electron exchange interaction, usually calculated in density functional theory (DFT) in the collinear form, especially for Eu with half-filled 4f orbitals carrying a local moment. Thus, it is not clear that the main result in Eq.(2) of field dependence for exchange-splitting in the current manuscript is well founded.

Our response. We agree that in Ref.[27] (Ref. [29] in the revised version) there is no Eqn.(2) (i.e. Eq. (3) in the revised version) for the field dependence of exchange-splitting. This formula is well-known (see, e.g., new Refs. [27,28]) and follows directly from (i) the condition of the fixed total spin of magnetic atoms, $S_{\parallel}^2 + S_{\perp}^2 = S^2$, valid both in classical and quantum mechanics (in quantum mechanics we only have $S^2 = (S+1)S$), and (ii) an approximately constant magnetic susceptibility $\chi = M/H$ in the AFM state, as derived under simple approximations in the Kittel's textbook Ref.[29] and is clearly seen from experimental data on magnetization, e.g. in Fig. 4b. Indeed, from summing $S_{\parallel}^2 + S_{\perp}^2 = S^2$ over each AFM sublattice we obtain for the magnetization of each AFM sublattice $i=1,2$: $M_{i\parallel}^2 + M_{i\perp}^2 = M_i^2$. The total magnetization $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$ also comes from the magnetic atoms (Eu in our case), and the AFM order parameter $\mathbf{L}_{\text{AFM}} = \mathbf{M}_1 - \mathbf{M}_2$. In a magnetic field $\mathbf{M} = \mathbf{M}_{1\parallel} + \mathbf{M}_{2\parallel} = 2\mathbf{M}_{1\parallel} = \chi\mathbf{H}$, and $\mathbf{L}_{\text{AFM}} = \mathbf{M}_{1\perp} - \mathbf{M}_{2\perp} = 2|\mathbf{M}_{1\perp}|$. Hence, $L_{\text{AFM}}^2(H) = L_{\text{AFM}}^2(0) - M^2 = L^2 - (\chi H)^2$, which immediately gives the right part of Eq. (3).

As the referee correctly writes, the spin of magnetic atoms is only reoriented by magnetic field. Hence, the size of the total spin moment S^2 (thus exchange-splitting, as follows from the Heisenberg exchange coupling $-J\mathbf{S}\mathbf{\sigma}$), should stay the same. This splitting originating from the electron exchange interaction is usually calculated in density functional theory (DFT) in the collinear form, especially for Eu with half-filled 4f orbitals carrying a local moment. Indeed, it is this splitting which we call $E_{\text{ex}}(H=0)$ in Eq.(3) of the revised version and which we estimate from our DFT calculations to be $\sim 30\text{meV}$ in EuSn₂As₂. And this splitting is indeed independent of magnetic field. However, the exchange splitting $E_{\text{ex}} = E_{\text{ex}}(H)$, entering our NIMR mechanism

and Eqs. (2) and (3), is the difference of exchange energy on two different AFM sublattices for each spin component of conducting electrons, as follows e.g. from the Hamiltonian in Eq. (5). Hence, $E_{\text{ex}}=E_{\text{ex}}(H)$ is proportional to $L_{\text{AFM}} = \mathbf{M}_1 - \mathbf{M}_2$.

We wish to emphasize once again that the AFM order parameter $L_{\text{AFM}} = \mathbf{M}_1 - \mathbf{M}_2$, defined as the difference of magnetization of two AFM sublattices $L_{\text{AFM}} = \mathbf{M}_1 - \mathbf{M}_2 \propto \langle \mathbf{S}_1 \rangle - \langle \mathbf{S}_2 \rangle$, also gives the difference between the averaged spin orientation $\langle \mathbf{S}_i \rangle$ of magnetic atoms on these two AFM sublattices and, hence, determines the exchange energy splitting E_{ex} of conducting electrons between two AFM sublattices. Therefore, $E_{\text{ex}} \propto L_{\text{AFM}}(H)$, as we write in the left part of Eq. (3).

Changes to the manuscript:

To facilitate the reading of our paper, we added this explanation, namely two new paragraphs and Eq.(4), to the main text after Eq. (3) and changed the notation $M_{\text{AFM}} \rightarrow L_{\text{AFM}}$. For clarity, two other notations are also changed: ΔE_{ex} is replaced with E_{ex} , and $E_{\pm}$ is replaced with $\varepsilon_{\pm}$. We have also added two more references [27, 28] to the papers where Eq. (3) is given or can be easily derived from. We also show in **Supplementary Note 1** more detailed data to substantiate the linear $M(H)$ dependence in the AFM state of EuSn₂As₂ (ESA) at low temperatures $T \ll T_N$.

Referee comment 2. Also given the newer Ref.25, arXiv:2411.03971 also covers the same theoretical analysis and experimental MR data of EuSn₂As₂ in more details (as well ARPES data and DFT calculations) than the current manuscript arXiv:2405.18046, it seems the newer paper is more complete and is the main paper on EuSn₂As₂.

Our response. Our Ref. [25], arXiv:2411.03971 is the experimental paper (unpublished and even not yet submitted anywhere). There are too many minor experimental details and auxiliary measurements, which are not directly relevant to the proposed mechanism of negative magnetoresistance. Therefore, the inclusion of all these details would smear out the main simple and clear idea of the present paper. However, in the revised version of our manuscript, we have added the most relevant material from Ref. [25], arXiv:2411.03971, and some new information. In particular, in the revised version of our manuscript we added (i) the derivation of Eq. (3); (ii) Supplementary Materials, namely, Notes 1 – 5, where we presented additional experimental data and our results of DFT band-structure calculations, as well as extended discussions on relevant issues, and (iii) a completely new section devoted to universality of our model and to its comparison with experiments on other materials. This added material is important for substantiating our main conclusion and to show the universality of proposed mechanism.

The main goal of our paper is to present the explanation and the theoretical model, which describes the observed isotropic negative magnetoresistance, and to show its universality. Our performed and presented measurement results are aimed to show that this new mechanism of negative magnetoresistance captures correctly major features of experimental data in real AFM compounds.

Changes to the manuscript:

According to this referee comments we have made a substantial revision of our manuscript. In the revised version we have **added** the following new material:

1) To emphasize the universality of the proposed mechanism we have added the new subsection “On the universality of the proposed magnetoresistance model” to the discussion with a new Figure 6, where we perform a qualitative comparison of the proposed theory with experimental data from six (!) other measurements with ESA and five (!) other compounds.

2) The relevant theoretical analysis from Ref. [25], arXiv:2411.03971, e.g., after Eq. (3). We have also added two more references [27, 28] to the papers where Eq. (3) can be derived from or was given.

3) Five new Supplementary Notes 1-5 with new figures S1-S3. In particular, Notes 2 and 5 discuss and substantiate with experimental data our conclusion on the isotropy of NIMR and on the absence of topological effects in transport. Note 3 discusses the alignment of the Eu magnetic moments, relevant to comment 3 of the first referee. Note 4 shows our DFT band structure calculation, estimating the spin-orbit coupling, relevant to Comment 3 of the second referee. Note 1 shows our experimental data on the field and temperature dependence of magnetization, relevant to Comment 2 of the second referee and to Comment 1 of first referee.

Referee comment 3. For EuSn_2As_2 , two groups have proposed a different magnetic structure around 2K below the A-type AFM with $T_N=21$ K from magnetic susceptibility and MR measurements in Ref.9, Li et al, PRB 104, 054435 (2021) and Ref.29, Pakhira et al, PRB 104, 174427 (2021). How does such a different magnetic structure affect the theoretical analysis?

Our response. Indeed, in earlier papers regarding ESA, the two possible orientations of Eu magnetic moments (in-plane, IP, or out-of-plane, OOP) were suggested. In these papers the authors confirm that “magnetic ground states at low temperatures are mostly A-type AFM states, i.e., the magnetic moments are ferromagnetically aligned in the layer but antiferromagnetically coupled interlayer ” (quoted verbatim!). This completely agrees with our model of NIMR that is applicable for both types of magnetic moments orientation (in-plane, or out-of-plane), and is isotropic.

Changes to the manuscript:

We have added **Supplementary Note 3** where this somewhat controversial issue of the magnetic moments orientation in ESA is critically reviewed. We have provided simple empirical arguments in favor of the in-plane orientation. Finally, we noted there that the debatable issue of the Eu moments direction in ESA at zero magnetic field, along the c-axis, or along the (ab)-plane is insignificant for the proposed model of NIMR that is applicable for both cases, and is isotropic. In our argumentation in the **Supplementary Note 3** we refer to **Fig. S1** of the **Note 1**.

Referee comment 4. If the proposed NIMR behavior from the classical spin reorientation is universal as claimed for AFM metals, more evidence and analysis from other AFM metals with both local and itinerant magnetism should be presented.

Our response and changes made. To emphasize the universality of the proposed mechanism we have added the **new subsection** “On the universality of the proposed magnetoresistance model” to the discussion with a **new Figure 6**, where we perform a qualitative comparison of the proposed theory with experimental data from six (!) other measurements on ESA and five (!) other compounds.

Response to Reviewer #2:

Referee comment: In this work, based on theoretic modeling, the authors clarified the origin of the negative isotropic magnetoresistance observed in layered AFM semimetals. The proposed NIMR was further confirmed in experimental measurement of EuSn_2As_2 . The authors should address the following major points before the manuscript may be considered for publication:

Our response: We appreciate the reviewer's careful review of our paper. We have responded to all the comments and revised the manuscript accordingly.

Referee comment 1. The authors should specify the definition of Z_2 symmetry and why such symmetry was broken in EuSn_2As_2 .

Our response. In mathematics any binary symmetry is called Z_2 symmetry. In our case this Z_2 symmetry means that two AFM sublattices are equivalent, which holds only in the absence of AFM order. In order to avoid confusion with topological invariant we do not use the symbol Z_2 for the topological invariant related to topological insulators or semimetals and emphasize in the manuscript that our Z_2 symmetry to the permutation of AFM sublattices *is irrelevant to topology*. We have already written the definition of Z_2 symmetry on page 2, in the beginning of Section "Results": "The AFM order violates the binary Z_2 symmetry $1 \leftrightarrow 2$ between two magnetic sublattices (adjacent layers in EuSn_2As_2 , see Fig. 1). Its violation leads to spatial redistribution of wave functions for various spin projections and their concentration near the corresponding AFM sublattices... This Z_2 symmetry to the permutation of AFM sublattices is irrelevant to topology and means equal energies and weights of electron eigenfunctions on each AFM sublattice."

Changes made. In the revised version, we clarified this point more in the paragraph before Eq. (2) by referring to equations in our paper.

Referee comment 2. There is a transition from NIMR to classical MR. What is the origin of such transition? Please explain. In addition, is it possible to induce the transition from FM to AFM or vice versa by external field in this work?

Our response. Our NIMR mechanism holds only in the AFM state, i.e. when (i) the temperature is below the Neel temperature and (ii) when the magnetic field is below the field of complete spin polarization H_{sf} . At fields $H > H_{sf}$ the AFM order is destroyed, NIMR vanishes and our mechanism of NIMR is not applicable. In contrast, the classical positive MR exists (only for current perpendicular to the magnetic field) in a much wider interval of external parameters, but in the AFM state it is weaker than the proposed NIMR effect in EuSn_2As_2 and in many other layered AFM compounds. One observes a sum of these two opposite contributions. Hence, a transition from NIMR to the classical MR happens when the AFM order is destroyed (by magnetic field or by temperature). In higher fields, the magnetoresistance is described by the textbook parabolic dependence for two-band semimetals. Thus, the "AFM $\rightarrow$ FM transition" (in referee's terms) corresponds to the complete polarization of Eu magnetic moments when the applied field H exceeds H_{sf} and vice versa. This transition also reveals in magnetization curves, shown in Fig. 4b of the main text or in the Supplementary Note 1).

Changes to the manuscript:

We have added a new sentence explaining this issue after Eq. (13): "The sharp transition from negative to positive magnetoresistance at $H = H_{sf}$ happens because our NIMR effect, which is stronger than the classical positive magnetoresistance in A-type AFM layered metals, exists only in the AFM state at $H < H_{sf}$ —and vanishes at $H > H_{sf}$ ". We have also added **Supplementary Note 1**, to show temperature evolution of the magnetization in external field ("AFM to FM transition" in the referee's terms).

Referee comment 3. The spin-orbit coupling was ignored in this work. The authors should comment on the effect of SOC on the proposed NIMR.

Our response. Usually, the spin-orbit coupling E_{SO} is much weaker than the exchange coupling $J \sim E_{ex}$. Therefore, we ignore it. A weak spin-orbit coupling E_{SO} gives a correction to the predicted NIMR effect, which is small by the parameter E_{SO}/E_{ex} . At $E_{SO} \ll J$, the spin-orbit coupling does not affect our NIMR mechanism considerably. Even at $E_{SO} = E_{ex}$ our NIMR mechanism survives, because it gives the effect averaged over all electron quasi-momenta and contains only the exchange splitting, because the spin-orbit term in the Hamiltonian is averaged (over electron momenta) to almost zero and affects only in the next orders of the perturbation theory. Of course, at $E_{SO} > J$ our NIMR mechanism quantitatively modifies, which requires further investigation. However, in the studied compound the exchange splitting $E_{ex} \cong 30\text{meV}$, while the spin-orbit coupling $E_{SO} \sim 5\text{meV} \ll E_{ex}$, as follows from our DFT calculations (see Supplementary Note 4). Hence, we can safely neglect the spin-orbit coupling.

Changes to the manuscript:

We have added our estimate of the spin-orbit coupling as compared to the exchange splitting relevant to NIMR mechanism and the corresponding discussion as a new paragraph to the Discussion section of our manuscript. We have also added one new relevant sentence to the section Results before Eq. (5) of our revised manuscript. We have added the **Supplementary Note 4** with DFT calculations of the SO effect on the band structure.

List of main changes to the manuscript according to reviewer's comments:

1. According to comment 1 of reviewer 1, we added a brief explanation and derivation of Eq. (3), namely, two new paragraphs and new Eq. (4), to the main text after Eq. (3). We have also added two more references [27, 28] to the papers where Eq. (3) is given or can be easily derived from.
2. According to the comments 2 and 4 of reviewer 1, we have added the **new subsection** "On the universality of the proposed magnetoresistance model" to the discussion with a new Figure 6, where we perform a qualitative comparison of the proposed theory with experimental data from six other measurements from ESA and five other compounds. In the revised version, we also have added the relevant theoretical analysis from Ref. [25], arXiv:2411.03971 to our manuscript, e.g., two paragraphs after Eq. (3) and new Eqs. (2) and (4). We have also added **Supplementary Materials, Notes 1-5**. Hence, we have made a substantial revision of our manuscript according to these referee comments.
3. According to the comment 3 of reviewer 1, we have added **Supplementary Note 3** where the issue of the magnetic moments orientation in ESA is critically reviewed. We have provided simple empirical arguments in favor of the in-plane orientation. Finally, we added in **Supplementary Note 3** that the direction of the Eu moments in ESA at zero magnetic field, along the c-axis, or along the (ab)-plane **is insignificant** for the proposed model of NIMR that is applicable for both cases, and is isotropic.
4. According to the comment 1 of reviewer 2, although the definition of the Z_2 symmetry to the permutation of AFM sublattices is already explained in the first paragraph of section "Results. Model and a qualitative consideration.", we clarified somewhat more our definition of Z_2 symmetry in the paragraph before Eq. (2) by referring to the particular equations in our paper.
5. According to the comment 2 of reviewer 2, we have added a new clarifying sentence after Eq. (13): "The sharp transition from negative to positive magnetoresistance at $H=H_{sf}$ happens

because our NIMR effect, which is stronger than the classical positive magnetoresistance in A-type AFM layered metals vanishes when the AFM order is destroyed at $H > H_{sf}$.”

6. According to the comment 3 of reviewer 2, we have added our estimate of spin-orbit coupling as compared to the exchange splitting relevant to NIMR mechanism and the corresponding discussion as a new paragraph to the Discussion section of our manuscript and Supplementary Note 4. We have also added one new relevant sentence to the section Results before Eq. 5 of our revised manuscript.

7. Four notations are changed: $M_{AFM} \rightarrow L_{AFM}$, $\Delta E_{ex} \rightarrow E_{ex}$, $E_{\pm} \rightarrow \varepsilon_{\pm}$, and $\Delta E_{\zeta} \rightarrow \Delta \varepsilon_{\zeta}$.

8. Five **Supplementary Notes** are added in order to substantiate our model, approximations used, the viewpoint, and to clarify the debatable issues in the field.

9. Many minor stylistic corrections are made.

We thank the referees for their careful reading of our manuscript and for positive evaluation of our work. Below is our detailed reply to the comments of the second report of Reviewer #1 and the corresponding list of changes in our manuscript. All changes and new additions are also highlighted in blue in the revised manuscript.

Second report of Reviewer #1: In the revised manuscript, the authors have added Eqn.(2), (4) and the associated new paragraphs to explain the derivation of Eqn.(3). They have added a new paragraph and the associated Supplementary Note to consider spin-orbit coupling (SOC) effect. They have also provided a new section and Fig.6 to apply the NIMR model to six other AFM materials besides EuSn₂As₂.

With these revisions, the manuscript has been improved. However, there are still two major issues.

1. Regarding to the derivation of Eqn.(3), the newly added Eqn.(4) and the right side of Eqn.(3) are for the magnetization and susceptibility of the Eu moments from the half-filled local Eu 4f electrons and its field dependence. But the left side of Eqn.(3), $E_{\text{ex}}(H)$, is defined as the difference of the exchange-splitting for the conduction electrons on two AFM sublattice. As the authors acknowledged in the Reply letter that the size of the AFM moment does not change because it is from the electron-exchange interaction of the local 4f electrons. The induced exchange splitting of the conduction electron is also independent of magnetic field, which is calculated in DFT in the collinear form. Then the equality in Eqn.(3) and the field dependence for $E_{\text{ex}}(H)$ is more of an ad hoc justification than a rigorous proof. The effect of local moment reorientation on conduction electron should have been more systematically treated in RKKY-type analysis.

2. The use of “Z₂-symmetry” here for the simple A-type AFM order is very confusing. In modern condensed matter theory, Z₂ is a topological classification of band structure for being even or odd from mod(2) of symmetry-protected topological invariants. It provides more information than magnetic space group symmetries. But here the relation of the two A-type AFM sublattices is already adequately described with magnetic space group symmetries. Using “Z₂-symmetry” here can cause unnecessary confusion for the readers.

Our response to the second report of Reviewer #1:

1. Response to comment 2.

According to the second referee’s comment, in order to avoid confusion, we changed the symbol Z_2 to T_2 in all our discussions about the binary symmetry between AFM sublattices throughout the manuscript.

2. Response to comment 1, regarding the derivation of Eqn.(3).

We disagree that the proposed explanation “is more of an ad hoc justification than a rigorous proof.” We think it is rather general, as argued below.

2.1. First, by definition, the RKKY interaction is the **interaction between local spins** (of magnetic ions) via the conduction electrons. The RKKY interaction together with the essentially larger direct exchange and superexchange coupling between the magnetic Eu ions is already included in their magnetic coupling, leading to the A-type AFM ordering. **Hence, we do not need to include again the RKKY interaction explicitly to our model that deals with conduction electrons solely and presumes that the magnetic order in the host lattice is already fixed.**

To substantiate quantitatively the argument that RKKY interaction is negligible for the AFM ordering in EuSn₂As₂ (ESA), we present below estimates for the actual exchange interlayer coupling between the Eu spins and compare it with the estimate of the RKKY interaction.

(i) The interlayer exchange, $J \sim 0.2 \text{ meV}$, can be found from the observable quantities, such as the AFM transition temperature $T_N = 24 \text{ K}$ or the field $H_{sf}^{ab} = 3.6 \text{ T}$ of complete spin polarization (see Fig. 4b of our manuscript). Indeed, in the mean-field theory the transition temperature $k_B T_N \sim J z S(S+1)/3$, where for EuSn_2As_2 $T_N = 24 \text{ K}$, $z=2$, $S=7/2$. This gives $J \sim 0.2 \text{ meV}$. Also, the field $H_{sf}^{ab} = 3.6 \text{ T}$ of complete spin polarization gives the exchange effective field $H_{sf}^{ab} \approx H_{ex} \approx J/(M_s S^2)$, where $M_s = 7 \mu_B$ - the saturation magnetization (see Refs. [8,24]). The latter also gives $J \sim 0.1 \text{ meV}$.

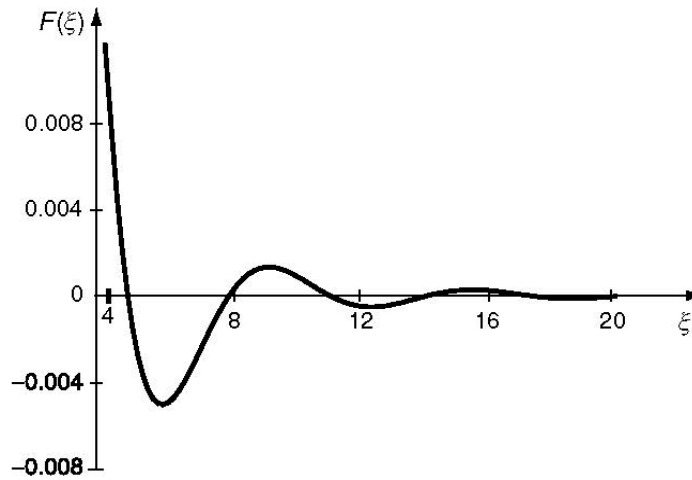
(ii) In order to estimate RKKY interaction in EuSn_2As_2 , one can use the analytical formula (5.30) and Fig. 5.8 from the book by J.M.D. Coey, Magnetism and Magnetic Materials, CAMBRIDGE UNIVERSITY PRESS, ISBN-13 978-0-511-67743-4:

$$\mathcal{J}_{eff} \approx \frac{9\pi \mathcal{J}_{sf}^2 v^2 F(\xi)}{64 \mathcal{E}_F}, \quad (5.30)$$

$$F(\xi) = (\sin \xi - \xi \cos \xi)/\xi^4, \quad \xi = 2k_F r,$$

Figure 5.8

The RKKY function $F(\xi)$.
Note that $F(\xi)$ becomes very large when $\xi < 4$.



In Eq. (5.30) v denotes the number of conduction electrons per atom, the on-site interaction between a core spin $\mathbf{S}$ and a conduction electron spin $\mathbf{s}$ is $-J_{sf} \mathbf{S} \cdot \mathbf{s}$, where $J_{sf} \approx 0.2 \text{ eV}$ [see p.141 Coey book, cited above]. The Fermi energy for ESA is of the same order, $E_F = 0.07 \text{ eV}$, estimated from the measured carrier density $10^{20}/\text{cm}^3$. However, $v \approx 0.006 \ll 1$ for such semimetal as EuSn_2As_2 , thus providing the smallness, $v^2 = 3.6 \times 10^{-5}$ of J_{eff} in eq.(5.30) above. The measured carrier density $n \approx 10^{20}/\text{cm}^3$ also gives $k_F = (3\pi^2 n)^{1/3} = 1.43 \times 10^7 \text{ cm}^{-1}$. For anisotropic case k_F across the layers is even larger. The function $F(\xi)$ additionally decreases J_{RKKY} by 1 order of magnitude, because the estimated $\xi = 2k_F r > 3.72$ and $F(\xi = 3.72) \approx 0.013$ (for the Eu-Eu interlayer distance of 1.3 nm). Hence, the estimated (using Eq. 5.30) RKKY coupling between Eu spins in EuSn_2As_2 is by 2-3 orders of magnitude smaller than the actual Eu-Eu interlayer spin coupling found from the AFM transition temperature or from the field H_{sf} of complete polarization. Moreover, according to the Figure above, the RKKY coupling is of the opposite sign, giving ferromagnetic Eu-Eu coupling rather than the observed A-type AFM. This shows that the RKKY coupling is not the main source of the Eu-Eu interaction leading to AFM order.

(iii) Another estimate of the RKKY coupling in A-type AFM EuSn_2As_2 can be found using the exchange coupling between FM layers through a paramagnetic interlayer. In the case of a thick conducting interlayer, the analytical formula for the effective exchange coupling per unit area (Eq. (3.14) from the classical work [P. Bruno, Phys. Rev. B **52**, 411 (1995)]; and also [Phys. Rev. B **49**, 13231 (1994)] takes the form

$$J \approx \frac{\hbar^2 k_F^2 a^2}{4\pi^2 m d^2} \cdot \text{Im}[\Delta r^2 \exp(2ik_F d)] , \quad (1)$$

where $k_F \approx 1.43 \times 10^9 \text{ m}^{-1}$ is the Fermi wave number in semimetal EuSn_2As_2 , $a=b=0.42 \text{ nm}$ is the in-plane lattice constant, m is the effective mass of charge carriers; $d=1.3 \text{ nm}$ is the interlayer distance. This gives the

prefactor $\frac{\hbar^2 k_F^2 a^2}{4\pi^2 m d^2} \approx 0.4 \text{ meV}$ in EuSn_2As_2 . The dimensionless constant Δr^2 is related to the reflection coefficient at the boundary between the interlayer and the FM layer $|\Delta r^2| < 1$. In the case of conducting interlayer, the exponential factor in this formula (1) gives the well-known RKKY-type oscillations, while in the case of insulating interlayer (potential barrier) it gives an exponential decrease of exchange coupling because $k_F \rightarrow i\kappa$. The interlayer coupling in strongly anisotropic metals is closer to the second case, because the interlayer electron transfer is reduced by the tunneling through a potential barrier, giving the strong anisotropy of conducting properties. In EuSn_2As_2 the resistivity anisotropy is rather strong: $\rho_{zz}/\rho_{xx} \approx 130$, suggesting the additional smallness of coupling constant J in formula (1) by ~ 2 orders of magnitude coming from the factor $\text{Im}[\Delta r^2 \exp(-2\kappa d)] < 0.01$. Hence, the estimate of $J < 0.004 \text{ meV}$ using Eq. (1) also gives a too small contribution of the RKKY interaction to the interlayer AFM coupling in EuSn_2As_2 and is also of the opposite sign. Unfortunately, Eq. (1) derived for thick layers gives only a rough estimate of J in our case.

(iv) And at last, there are two additional experiment-based arguments supporting that the AFM coupling in EuSn_2As_2 is determined NOT by RKKI interaction:

1) In Ref. [Jian Yan et al., Phys. Rev. **110**, 115111 (2024)] an effect of Eu substitution by non-magnetic Ca_x ($x=0.0 - 0.25$) was studied in the sister compound, EuIn_2As_2 . It was found that as the Ca concentration x increases from 0 to 20%, the carrier density (determined from Hall resistance) decreases by a factor of 4, whereas T_N decreases only by 30%. As the authors conclude: “These results show that the Ca substitution tunes the Fermi energy and Fermi wave vector while keeping the AFM magnetic ground state”. This contradicts both above formulas for RKKI interaction. In other words, the free carriers (i.e. the RKKY interaction) is not the major driving force in AFM ordering of the Eu layers.

2) In Ref. [Hualei Sun et al., Sci. China **64** (11), 118211 (2021)], the EuSn_2As_2 crystal was studied under pressure. It was found that as pressure increases from 0 to 10 GPa, the carrier density increases by a factor of 7, while T_N increases only from 24 to 77 K, i.e. only by a factor of 3.2. This means that the $T_N \sim J$ dependence on the carrier density (and on k_F) for ESA is rather weak (sublinear) and monotonic, contrary to Eq. (5.30) above where $J_{\text{eff}} \propto n^{4/3}$.

2.2. For our model of magnetoresistance, we only need the **exchange interaction between the magnetic ions (or even their sublattices) and conduction electrons**. It is nearly a point-like Heisenberg interaction $-\mathbf{J} \mathbf{S} \boldsymbol{\sigma}$ between the spin $\mathbf{S}$ of magnetic ion and the spin $\boldsymbol{\sigma}$ of conduction electrons, because the exchange coupling is strong only in those regions where the wave functions of conduction electrons overlap with the localized electrons of magnetic ions. The wave function $\psi_{\sigma}(\mathbf{r})$ of conduction electron is delocalized and extended to many magnetic ions. In particular, $\psi_{\sigma}(\mathbf{r})$ is delocalized on each AFM sublattice (conducting layer in EuSn_2As_2). Therefore, it feels an average spin $\langle \mathbf{S} \rangle_i$ of magnetic ions on each AFM sublattice i , which is proportional to its total magnetization $\mathbf{M}_i$. Hence, we do not need to consider the RKKY-type interaction between individual Eu atoms. Instead, we only need their mean-field average spin $\langle \mathbf{S} \rangle_i \propto \mathbf{M}_i$, which creates a spin-dependent potential for conduction electrons.

Possibly, here is a confusion in terminology. Usually (and apparently the referee thinks so), the exchange term in the Hamiltonian $J \mathbf{\tilde{e}}_1 \mathbf{\tilde{e}}_2$ is the isotropic exchange interaction with the exchange energy J **between Eu atoms or layers**. In our previous reply J denoted the exchange coupling between the localized Eu spins $\mathbf{S}$ and the spins $\boldsymbol{\sigma}$ of delocalized conduction electrons. To avoid this confusion, below and in the revised manuscript we replaced notation and use $\mathbf{J}$ for the exchange integral of delocalized conduction electrons with localized electrons of Eu-atoms.

For our NIMR effect, coming from the redistribution of $\psi_{\sigma}(\mathbf{r})$ between two AFM sublattices as given by Eq. (6) of our manuscript, we need the difference of weights of $\psi_{\sigma}(\mathbf{r})$ on two different AFM sublattices. And this difference of weights is proportional to the difference E_{ex} , entering the Hamiltonian in Eq. (5) of the averaged exchange energy $\langle -\mathbf{J} \mathbf{S} \boldsymbol{\sigma} \rangle$ of conducting electrons on two different AFM sublattices. Since $\langle \mathbf{S} \rangle_i \propto \mathbf{M}_i$, we get $E_{\text{ex}} \propto -\mathbf{J} \boldsymbol{\sigma} (\langle \mathbf{S} \rangle_1 - \langle \mathbf{S} \rangle_2) \propto -\mathbf{J} \boldsymbol{\sigma} (\mathbf{M}_1 - \mathbf{M}_2) = -\mathbf{J} \boldsymbol{\sigma} \mathbf{L}$. **This gives the left-side part of Eq. (3).** The

low index σ in the wave function $\psi_\sigma(\mathbf{r})$ is the projection of conducting-electron spin on the AFM order parameter $\mathbf{L}$.

These arguments are quite general, not ad hoc. They are applicable to all AFM metals. The NIMR effect is small if the electron hopping rate (transfer integral) between AFM sublattices is much larger than the exchange coupling E_{ex} . However, our NIMR effect is strong in most layered metals with A-type AFM order.

2.3. Possibly, by the comment about RKKY-type analysis the referee means that we need to consider the feedback influence of the redistribution of the spin-density of conduction electrons by AFM on the exchange interaction between the Eu atoms and on AFM coupling. Again, (i) this is already included in the magnetic coupling between the magnetic Eu ions, leading to the A-type AFM ordering. (ii) This redistribution may only slightly affect the RKKY interaction between the Eu atoms, which we already have shown to be small, because the main contribution to the exchange interaction between the Eu atoms comes from the direct exchange and superexchange coupling via intermediate atoms rather than from the RKKY mechanism. (iii) The redistribution of spin density of conduction electrons by AFM order is small by parameter $\gamma \ll 1$, while their charge density remains practically unchanged, making a small effect even on RKKY coupling.

2.4. Regretfully, the referee apparently missed reading the papers [27] and [28] (noted on page 2 of our paper, right column, just above Eq. (2)), where the same Eq.(3) was derived for the simplest case $\mathbf{H} \parallel (\text{ab})$.

For the referee convenience, below we attach to our response a copy of the page of referred paper [27] where we highlighted the excerpt which verbatim reproduces our Eq.(3) for $\mathbf{H} \parallel (\text{ab})$.

Since the problem considered has the axial symmetry, we can assume (without any loss of generality) that $\mathbf{H}_0$ lies in the zy plane making an angle ψ with the easy plane, which coincides with the xy plane. Then, we find that in the ground state

$$\begin{aligned} M_{0x} &= 0, & L_{0x} &= L_0 \approx 2M_s \sqrt{1 - \left(\frac{H_0}{2H_E}\right)^2}, & \chi_{\perp} &= \frac{1}{B}, \\ M_{0y} &= \frac{H_y}{B} = \frac{H_0 \cos \psi}{B}, & L_{0y} &= 0, & \chi_{\parallel} &= \frac{1}{B + b}, \\ M_{0z} &= \frac{H_z}{B + b} = \frac{H_0 \sin \psi}{B + b}, & L_{0z} &= 0, & \frac{\Delta\chi}{\chi} &= \frac{H_A^b}{2H_E} \end{aligned} \quad (3)$$

List of the main changes to the manuscript according to reviewer's comments:

1. According to comment 1 of reviewer 1, in the section "Model and a qualitative consideration" on page 3 we added two new paragraphs substantiating Eq. (3) in more detail.
2. According to comment 1 of reviewer 1, in the Discussion on page 7 we added a new paragraph explaining why we ignore the influence of the redistribution of spin-density of conduction electrons on the exchange interaction between the Eu atoms. We also added new Ref. [37] and replaced the notation J by $\mathbf{J}$.
3. According to comment 2 of reviewer 1, we removed the symbol Z_2 from all our discussions about the binary symmetry between AFM sublattices and replaced it with T_2 .