

Scalable Sondheimer oscillations driven by commensurability between two quantizations

Corresponding Author: Professor Zengwei Zhu

This file contains all editorial decision letters in order by version, followed by all author rebuttals in order by version.

Version 0:

Decision Letter:

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Dear Professor Zhu,

Thank you again for submitting your manuscript "Scalable Sondheimer oscillations driven by commensurability between two quantizations" to Communications Materials. We have now received reports from 3 reviewers and, based on their comments, we have decided to invite a revision of your work. You will find the reviewers' reports below. While they find your work of interest, they have raised important points which must be addressed in a revised manuscript.

To allow us to move forward with your work, we also ask that you edit your manuscript according to the attached table.

Please read this document carefully as we will be unable to further assess your revised paper until these important points are addressed.

Please outline all revisions made in the right-hand column and return the completed table with your updated manuscript files as a Related Manuscript file.

When resubmitting, please also include:

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- A clean version of your revised manuscript with no mark-ups
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We hope to receive your revised paper within six weeks, but we understand that the revisions may take longer. Please let us know if you find that the revision process will take substantially more time.

We are committed to providing a fair and constructive peer-review process. Please do not hesitate to contact me if you have any questions or would like to discuss these revisions further. We look forward to seeing the revised manuscript and thank you for the opportunity to review your work.

Best regards,

Aldo

Dr Aldo Isidori
Senior Editor
Communications Materials

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

X. Guo and colleagues report on the discovery of Sondheimer oscillations in cadmium single crystals. This phenomenon - Sondheimer oscillations - is generally not widely known or extensively studied in the condensed matter physics community, but it has attracted renewed interest following recent experimental developments. The present manuscript contributes valuable novel insights by demonstrating that the observed oscillations originate from a commensurability between two distinct quantizations.

The manuscript is clearly and carefully written, with an excellent presentation of the experimental data and corresponding analysis. The figures are of high quality, and the discussion is logical and coherent. All major claims are well supported by the data, and the authors' arguments are both careful and convincing.

Overall, this is a high-quality and timely contribution to the research on Sondheimer oscillations. I recommend publication in Communications Materials.

Reviewer #2 (Remarks to the Author):

The manuscript COMMSMAT-25-0957 "Scalable Sondheimer oscillations driven by commensurability between two quantizations" presents interesting new idea and experimental results, worthing publication in Communication Materials. However, the manuscript does not provide a theory of the observed effect, only the idea about the direction in which the theory should be developed to explain (if lucky) this phenomenon. The proposed theory in Sec. IIIC on page 11 is not quite clear. Maybe this can be partially improved by stylistic revision. In particular, (1) why the right panel of Fig. 8c describes the odd number? Maybe more depicted Landau levels and dimensional-quantization levels will make this clearer. (2) Why Eq. (12) describes the magnetic flux times the elementary charge e ? This statement is repeated after Eq. 13. (3) The comparison in Fig. 9 shows a considerable frequency mismatch, especially upper panels. Usually, the frequency of magnetic oscillations is described by theory quite accurately. what is the possible origin of this mismatch?

There are other stylistic drawbacks. For example,

(i) in the caption to Fig. 2a,b, the authors write amplitude of oscillations while the oscillations themselves are plotted.

(ii) There are misprints. For example, before Eq. (5) the power of B should be $-5/2$ instead of -5 .

(iii) I don't like the last phrase of Sec. IV Conclusions "We then showed this allows us to understand the empirical scaling as a simple expression in which the key parameter is a dimensionless parameter set by the Fermi surface geometry." Maybe better start it as "This may help to understand ..". As I understand, this dimensionless parameter is Ω given by Eq. 9, but it is set not only by the Fermi-surface geometry.

To summarize, I think the paper may be published in Communication Materials after a stylistic revision.

Reviewer #3 (Remarks to the Author):

I enjoyed reading the manuscript by Guo et al. The manuscript looks at a classical effect called Sondheimer oscillations (first proposed in the 1950s) and examines the interplay with Landau oscillations. I think one point not sufficiently emphasized by the authors is that in previous studies such as Ref. 17 (van Delft et al.) on WP2 there was a big separation in scale between Landau level spacing and the confinement spacing, and as a result these could be taken as independent and additive. By contrast, in the current work these energy scales are comparable, and the authors make a convincing case that when the Landau level spacing becomes comparable to the k_z sub band spacing the two quantizations become commensurate, leading to periodic reinforcement in conductivity. This provides a quantum origin to the Sondheimer oscillations.

In my view, this is an interesting new twist on some old physics, and could potentially be used as a complementary probe of Fermi surface topology.

While I am supportive that this work should be published in Communication Materials, there are some concerns that the authors should address before publication. Once these issues are addressed, the manuscript would make a valuable addition to the literature.

1. The style of presentation was a bit unusual with most of the figures showing experimental data scaled a certain way and then judging goodness by eye e.g. Fig 3 shows that scaling by B^4 does not stabilize oscillation amplitude except in the 4th sample beyond a critical field. I would prefer more quantitative fits with error bars and confidence intervals.

2. I think the temperature data is very limited (only between 2K and 8K). The paper would benefit from more data below 2K, or if the authors are unable to do this, they should explain the reason why in the main text.

3. While the experimental data is solid (e.g. the scaling of the data, and the interplay between Landau levels and Sondheimer oscillations seen through exponential damping having the same scale as the oscillation period, the oscillations disappearing when the cyclotron frequency approaches $k_B T$, etc.) the theoretical picture is internally consistent but phenomenological. The authors should clarify that their scaling forms are empirical fits supported by heuristic arguments, rather than give the impression that it is derived from a microscopic model.

4. The relation that $B/\Delta B \approx 3$ (Fig. 5b) is intriguing. From the figure, it seems to vary from 2.5 to 3.3. Do the authors think this is a universal value of 3 (with experimental error) or just that it scales by a factor close to 3? Some further discussion of this factor would benefit future studies.

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Version 1:

Decision Letter:

Dear Professor Zhu,

Thank you again for submitting your revised manuscript "Scalable Sondheimer oscillations driven by commensurability between two quantizations" to Communications Materials. We have now received reports from the 2 reviewers, which are copied below. Based on their comments, we are delighted to accept your paper for publication.

Please note that in advance of your paper being published we will host an early access version, known as an 'Article in Press,' on our journal website. For more information on this initiative please see our [author guidelines](https://support.springernature.com/en/support/solutions/articles/6000281821-what-is-an-article-in-press-).

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We look forward to publishing your paper.

Best regards,

Dr Aldo Isidori
Senior Editor
Communications Materials

Reviewers' comments:

Reviewer #2 (Remarks to the Author):

The manuscript COMMSMAT-25-0957 "Scalable Sondheimer oscillations driven by commensurability between two quantizations" has been improved after the revision. A convincing theory of the observed dependences is still absent, but now the authors write that "A rigorous account of our experimental finding is missing and remains a challenge to theory." and indicate which points remain unexplained. The paper contains interesting and detailed experimental investigation of the title effect. I think the manuscript is suitable for publication in Communication Materials.

Reviewer #3 (Remarks to the Author):

I have carefully reviewed the revised manuscript and the accompanying rebuttal letter. The authors have made a reasonable effort to engage with the questions raised during the review process and, in my judgment, have addressed them at a level that is sufficient for publication. There are aspects I would have preferred to see handled differently -- for example, presenting the quantitative fits with error bars directly in the main text rather than relegating them to the supplemental material, and obtaining additional data below 2 K. But these limitations do not materially weaken the central results.

The discussion surrounding the estimate $B/\Delta \approx \pi$ is somewhat assertive; however, the authors appropriately flag this point as speculative. Taken as a whole, the manuscript is sound and makes a meaningful contribution. I am satisfied with the revisions and am prepared to support publication in Communications Materials.

***As a new journal, we would greatly appreciate any comments you have about your experience at Communications Materials. I hope that we have been able to meet your expectations and look forward to working with you again in the future.

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Reply to the reviewers

We thank three reviewers for their review and comments on our manuscript.

Reviewer #1 (Remarks to the Author):

X. Guo and colleagues report on the discovery of Sondheimer oscillations in cadmium single crystals. This phenomenon - Sondheimer oscillations - is generally not widely known or extensively studied in the condensed matter physics community, but it has attracted renewed interest following recent experimental developments. The present manuscript contributes valuable novel insights by demonstrating that the observed oscillations originate from a commensurability between two distinct quantizations.

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Overall, this is a high-quality and timely contribution to the research on Sondheimer oscillations. I recommend publication in Communications Materials.

We are very grateful for this appreciation of our work.

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However, the manuscript does not provide a theory of the observed effect, only the idea about the direction in which the theory should be developed to explain (if lucky) this phenomenon. The proposed theory in Sec. IIIC on page 11 is not quite clear. Maybe this can be partially improved by stylistic revision.

We thank the referee for this positive evaluation. We agree that our paper does not include a rigorous theoretical account of the experimental data.

In particular, (1) why the right panel of Fig. 8c describes the odd number? Maybe more depicted Landau levels and dimensional-quantization levels will make this clearer.

Fig. 8c has been reproduced below. It sketches allowed states in a (E, k_z) plane. Spatial confinement imposes $k_z = nk_d$. Each vertical line represents such a discrete k_z value, which is allowed and no restriction in energy. Each oblique line represents a Landau level, with discrete energy (and a dispersion along k_z). Now, k_z can be either positive or negative. On the other hand, n can be only positive. This is the origin of odd/even dichotomy. When the number of states sharing the same Landau level is an even integer times the number of states sharing the same k_z , both these constraints on quantization can be satisfied. On the other hand, when the ratio of the two degeneracies is an odd integer, the incommensurability between the two is extreme. In the even case (left panel), the chemical potential (which is set by the electron density) is located at the intersection of the oblique and vertical lines, satisfying both constraints. In the odd case (right panel), chemical potential would be at the farthest possible distance between two neighboring intersections. We have

tried to clarify this in the revised version. We have added a clarification in the caption of Figure 8.

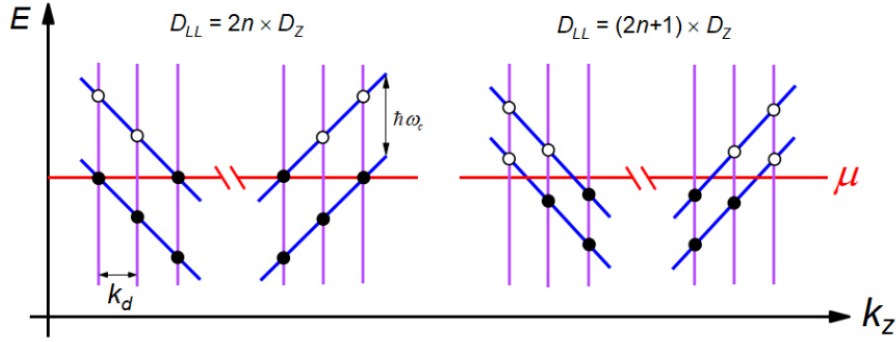


Fig. 8c: Occupied states in even (left panel) and odd (right) number.

(2) Why Eq. (12) describes the magnetic flux times the elementary charge e ? This statement is repeated after Eq. 13.

The general expression for quantum tunneling is $\propto e^{-s/\hbar}$. As a consequence, it disappears when $\hbar \rightarrow 0$. In this expression has the dimensions of a classical action (See for example Physica D **21**, 341 (1986)). Eq.12 gives the expression of action in our case: $s = eBk_d^{-1}(\partial A/\partial k_z)^{-1}$, where e is the elementary charge. According to equation 8, $k_d(\partial A/\partial k_z)$ represents the area of a flat ring carved on the Fermi surface in the momentum space. Therefore, $k_d^{-1}(\partial A/\partial k_z)^{-1}$ is an area in direct space. Multiplied by B , it represents magnetic flux, the conjugate of the electric charge. To enhance clarity, we have incorporated explanatory notes in Supplementary Materials Note 7.

(3) The comparison in Fig. 9 shows a considerable frequency mismatch, especially upper panels. Usually, the frequency of magnetic oscillations is described by theory quite accurately. what is the possible origin of this mismatch?

This is a correct, sharp and relevant observation. Figure 9 compares experimental data with what is expected according to equation 14. The dimensionless Ω , which sets the frequency depends on the thickness, assumed to be the nominal thickness of each sample. The frequency mismatch seen in the 12.6-micron sample may be due to a difference between the effective and the nominal thickness. However, there is more than this. In all samples, in the first period, there is a mismatch between the data and the expectation. This feature is not about frequency mismatch. It may contain interesting physics beyond the scope of the present work. We have added a short discussion of these points.

There are other stylistic drawbacks. For example,

(i) in the caption to Fig. 2a, b, the authors write amplitude of oscillations while the oscillations themselves are plotted.

Thank you for this careful reading of our manuscript. This has been corrected to "The oscillatory component of Sondheimer oscillations..." in the revised manuscript.

(ii) There are misprints. For example, before Eq. (5) the power of B should be $-5/2$ instead of -5 .

Thanks, it has been corrected in the revised manuscript.

(iii) I don't like the last phrase of Sec. IV Conclusions "We then showed this allows us to understand

the empirical scaling as a simple expression in which the key parameter is a dimensionless parameter set by the Fermi surface geometry.” Maybe better start it as “This may help to understand ..”. As I understand, this dimensionless parameter is Ω given by Eq. 9, but it is set not only by the Fermi-surface geometry.

Indeed! Our statement was ambiguous and misleading. To enhance clarity, it has been revised accordingly: “This led us to an empirical scaling in which the key parameter is a dimensionless parameter set by a combination of length scales which decided by the Fermi surface geometry, the thickness and the magnetic length. A rigorous theory of our experimental finding remains a challenge to theory.”

To summarize, I think the paper may be published in Communication Materials after a stylistic revision.

Thank you.

Reviewer #3 (Remarks to the Author):

I enjoyed reading the manuscript by Guo et al. The manuscript looks at a classical effect called Sondheimer oscillations (first proposed in the 1950s) and examines the interplay with Landau oscillations. I think one point not sufficiently emphasized by the authors is that in previous studies such as Ref. 17 (van Delft et al.) on WP2 there was a big separation in scale between Landau level spacing and the confinement spacing, and as a result these could be taken as independent and additive. By contrast, in the current work these energy scales are comparable, and the authors make a convincing case that when the Landau level spacing becomes comparable to the k_z sub band spacing the two quantizations become commensurate, leading to periodic reinforcement in conductivity. This provides a quantum origin to the Sondheimer oscillations.

In my view, this is an interesting new twist on some old physics, and could potentially be used as a complementary probe of Fermi surface topology.

Many thanks for these positive remarks.

While I am supportive that this work should be published in Communication Materials, there are some concerns that the authors should address before publication. Once these issues are addressed, the manuscript would make a valuable addition to the literature.

1. The style of presentation was a bit unusual with most of the figures showing experimental data scaled a certain way and then judging goodness by eye, e.g. Fig 3 shows that scaling by B^4 does not stabilize oscillation amplitude except in the 4th sample beyond a critical field. I would prefer more quantitative fits with error bars and confidence intervals.

Thank you for this insightful suggestion. In Fig. 3, where the oscillation conductivity is scaled by B^4 , the amplitude becomes essentially constant once the field exceeds threshold field, B_1 . Following referee’s suggestion, we have added a quantitative sine-function fitting to describe these periodic oscillations in the supplementary Fig.S3, shown below. We have also added a yellow rectangular bar to show the margin of error for B_1 . Furthermore, sine fits have been added to Fig. 4 e–h and Fig. 7c as visual guides.

Yes! The referee is right. We first show that according to our experimental data $B_0/\Delta B = 2.9 \pm 0.4$. Then, in the discussion section, we show that by assuming $B_0 \cong \pi\Delta B$ (an assumption compatible with our margin of error) one can fit the whole data set by a concise empirical expression (equation 14), in which the period and the amplitude of the oscillations scale with an identical parameter Ω . We make this explicit in the revised version.

List of changes:

All changes in the main text are highlighted in red. The specific revisions are detailed below:

1. Before Eq.5 (the main text on page 8), the power of B^{-5} has been corrected to $B^{-2.5}$.
2. On page 11 (the end of left panel), to improve the reader's understanding of the quantum tunneling action (s), the sentence of "Our empirical equation 4 includes a $\exp(-B/B_0)$ term..." has been replaced by "Equation 4 includes a $\exp(-B/B_0)$ term, which indicates quantum..."
3. At the end of the main text on page 11, we have replaced the sentence "One can see that our data fit a sinusoidal function whose amplitude is accurately given by Equation..." with: "For each sample, Ω has been extracted from Equation 9 and the thickness of the sample..."
4. In the conclusion, we have revised "We then showed this allows us to understand the empirical scaling as..." to "This led us to an empirical scaling in which the key parameter is..."
5. In the caption of Fig. 2, we have replaced "The amplitude of oscillations" with "The oscillatory component of Sondheimer oscillations..."
6. In Fig. 4 e-h and Fig. 7c, we have added sine-fitting curves as visual guides. Accordingly, the phrase "The dashed line indicates the sine fitting" has been added to the respective figure captions.
7. To enhance clarity, the caption of Fig.8c has revised.
8. In Supplementary Materials Note 7, we have added the following sentence to clarify that the tunneling action (s) represents a magnetic flux multiplied by the elementary charge e : "In Eq. 12, the tunneling action is: $s = eBk_d^{-1}(\partial A/\partial k_z)^{-1}...$ "
9. In Supplementary Fig.S3, we have added a quantitative sine-function fit to describe these periodic oscillations. We have also added a yellow rectangular bar to show the margin of error for B_1 . Also, the corresponding caption now includes the following sentences: "The dashed line indicates the sine fitting. The yellow bar indicates the error margin for B_1 ."