

Oil droplets are severely overlooked in risk assessments

Håvard G. Frøysa, Raymond Nepstad, Elin Sørhus,
Carey E. Donald, Sonnich Meier, Frode B. Vikebø

Supplementary Information

Contents

1	Model details.....	1
1.1	Turbulent mixing rate	1
1.2	Viscosity	1
1.3	Rise velocity.....	1
1.4	Dissipation rate of kinetic energy	3
1.5	Collision rate due to difference in rise velocity	4
1.6	Regime for collision rates due to mixing.....	5
1.7	Collision rate contributions.....	6
2	Statistics from the example simulation	7
2.1	Vertical distribution of eggs.....	7
2.2	Dissipation rate of kinetic energy	8
2.3	Egg stage duration.....	9
2.4	Oil composition.....	10
2.5	Size distribution of collided mass	11
2.6	Exposure time distributions	11
	Supplementary references	12

Supplementary methods

1 Model details

This section contains additional details about functions used in the numerical model.

1.1 Turbulent mixing rate

The expression for the mixing rate in the turbulent regime is

$$\theta_i^{\text{mix}} = \theta_i^{\text{turb}} = C_M \cdot L_i^{\frac{7}{3}} \cdot \epsilon^{\frac{1}{3}}$$

where C_M is a proportionality constant. Matching the expressions for viscous and turbulent regime at $\eta_0 = 2L_i$ to get a continuous function between the two regimes gives

$$C_M = C_V \cdot 2^{-\frac{2}{3}} \cdot \sqrt{15C_K}$$

and by using the Bandara value for C_V this in total gives $C_M = 2^{5/6} \cdot \sqrt{\pi \cdot C_K} \approx 4.6$. MacKenzie et al. [65] uses $C_M = \pi \cdot \sqrt{2} \approx 4.4$ for larvae and prey which fits well with our implementation, while Pecseli et al. [60] talk about empirical values in the range from 6 to 7.

1.2 Viscosity

Dynamic viscosity μ is calculated as a function of temperature and salinity as

$$\mu = 10^{-3} \cdot (1.7915 - 0.0538 \cdot T + 0.0007 \cdot T^2 + 0.0023 \cdot S) \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

where T is the temperature value in Celsius and S is the salinity value in psu. This formula is a fit to values reported by Sverdrud et al. [1, p. 69]. The kinematic viscosity ν as usual is

$$\nu = \frac{\mu}{\rho_w} \quad , \quad [\nu] = \frac{\text{m}^2}{\text{s}}$$

where ρ_w is the density of the water in kg per m³.

1.3 Rise velocity

The rise velocity of oil droplets and eggs are calculated in the same way assuming a perfect sphere. For a given diameter D and density difference $\Delta\rho = \rho_{\text{water}} - \rho_{\text{sphere}}$, the rise velocity assuming a small Reynolds number is given by Stokes' law as

$$w = \frac{1}{18} \frac{g \cdot D^2 \cdot \Delta\rho}{\mu} \quad , \quad [w] = \frac{\text{m}}{\text{s}}$$

where the positive direction is upwards (rising). To check the assumptions, we calculate

$$\text{Re} = \frac{|w| \cdot D \cdot \rho_w}{\mu}$$

where we accept $\text{Re} \leq 0.5$ as a small Reynolds number. This condition holds for the characteristic quantities of this manuscript, but if $\text{Re} > 0.5$ we use

$$w = 0.0875 \cdot \text{sgn}(\Delta\rho) \cdot |\Delta\rho|^{\frac{2}{3}} \cdot \mu^{-\frac{1}{3}} \cdot \left(D - 0.4 \frac{9\mu^2}{\rho_w \cdot g \cdot |\Delta\rho|} \right)$$

which is an empirical formula from Dallavalle [2] with proportionality coefficient to assure match with Stokes' law at $\text{Re} = 0.5$. Resulting rise velocities for the egg and droplet sizes used in the manuscript are shown in Figure S1 for the example environment of 7 °C and 35 psu.

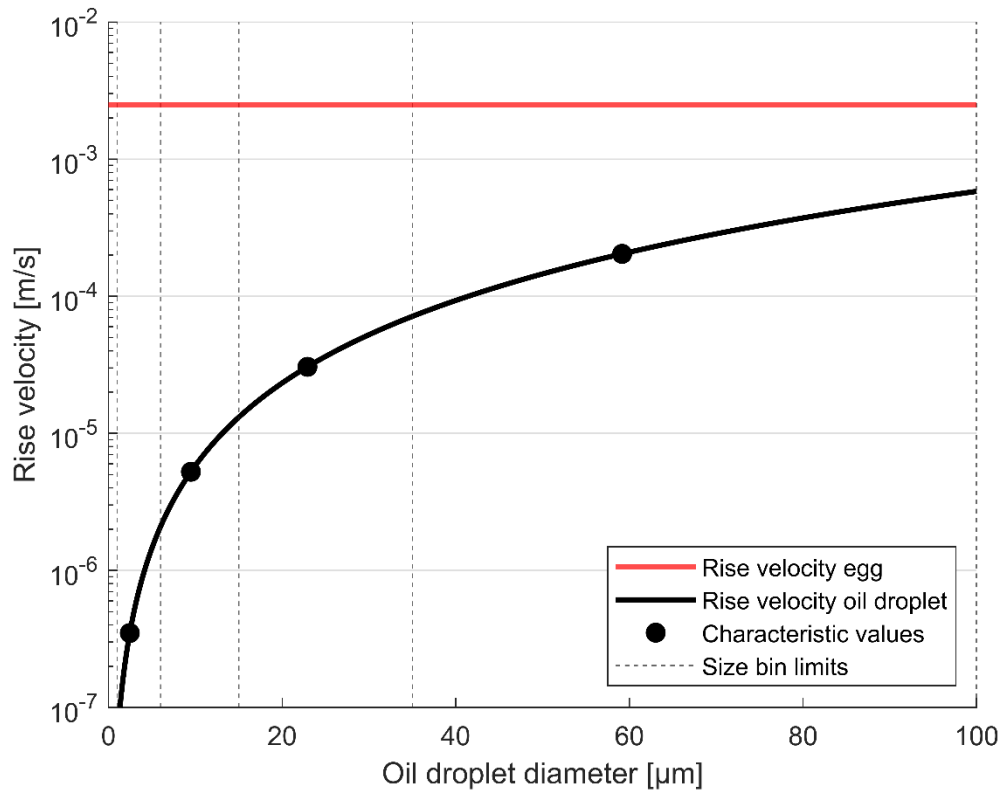


Figure S1: Rise velocity for oil droplets as a function of droplet size. It is assumed a water temperature 7 °C and a salinity of 35 psu giving a dynamic viscosity of $1.53 \cdot 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}$. The characteristic values are for the four droplet sizes used in the simulations. The rise velocity for a haddock egg with a diameter of 1.45 mm and buoyancy of 30.68 psu is included for reference.

1.4 Dissipation rate of kinetic energy

The dissipation rate ϵ is not available directly in the model and must be interfered from other quantities. Let W be the horizontal wind speed at the sea surface as given by the atmospheric model. The dissipation rate of turbulent kinetic energy at depth z is then calculated as

$$\epsilon = \frac{5.82 \cdot 10^{-9} \cdot W^3}{z} \quad , \quad [\epsilon] = \frac{\text{m}^2}{\text{s}^3}$$

using formula (10) in MacKenzie et al. [3]. Resulting profiles as function of depth are shown for three example wind speeds in Figure S2. Several other functions for the dissipation rate are available, see e.g. Esters et al. [4]. However, these are typically more complicated and might require more detailed parameterizations beyond the scope of this manuscript.

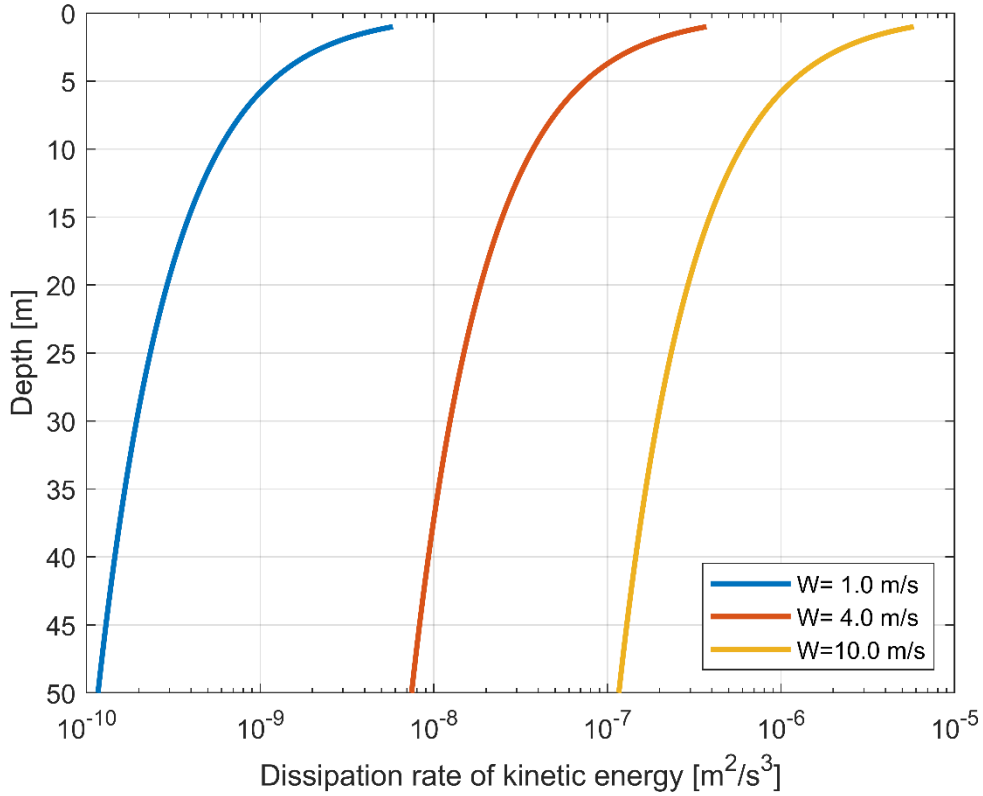


Figure S2: Dissipation rate of kinetic energy (denoted ϵ) as function of depth for different horizontal wind speed W at the sea surface. This is the turbulence quantity used to calculate collision rates. Based on MacKenzie et al. [3].

1.5 Collision rate due to difference in rise velocity

The collision rate due to difference in rise velocity is given by the function

$$\theta_i^{\text{rise}} = \frac{\pi}{4} (D_e + D_i)^2 \cdot |w_e - w_i| = \pi \cdot L_i^2 \cdot |w_e - w_i|$$

where L_i^2 increases and $|w_e - w_i|$ decreases with increasing droplet size. The latter can be seen from Figure S1 where the rise velocity w_e of the egg is constant while the rise velocity of the droplets increases with droplet size and gets close to w_e . In Figure S3 the resulting collision rate as a function of droplet size is shown for the example environment of 7 °C and 35 psu. The collision rate is almost constant for the four characteristic droplet sizes. The interpretation of this is that all droplets are small compared to the egg and have a neglectable rise velocity compared to the egg. Thus, the egg will essentially collide with all droplets within a small cylinder independent of the properties of the individual droplets.

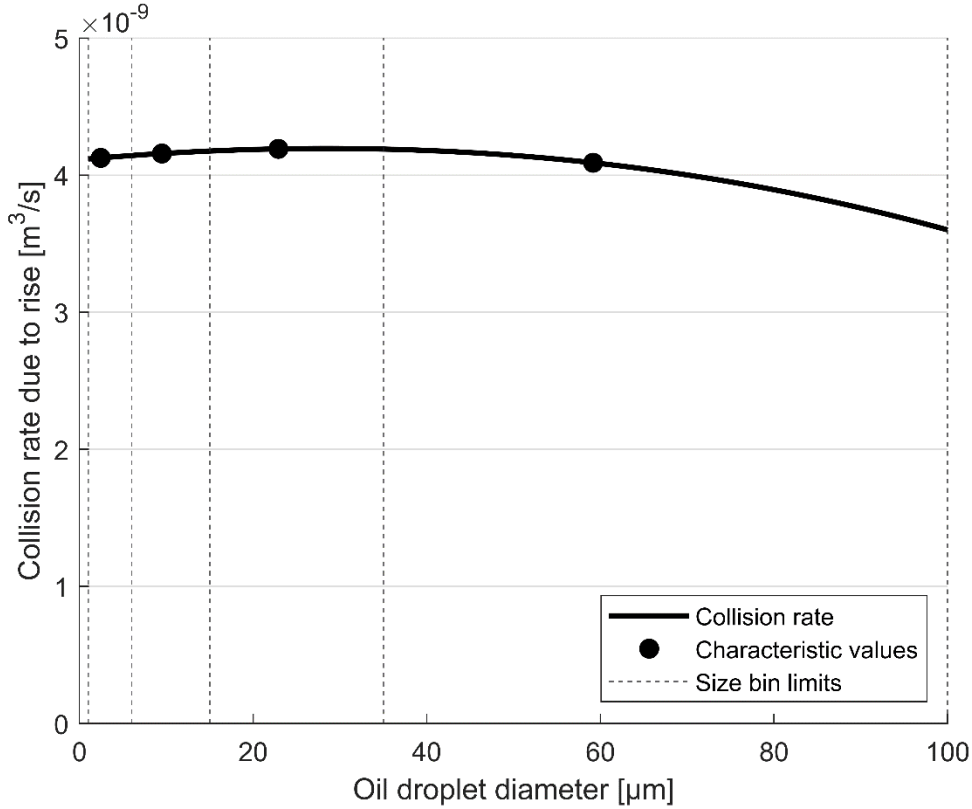


Figure S3: Collision rate between egg and oil droplet due to different rise velocity as a function of droplet size for the mean egg buoyancy of 30.68 psu. It is assumed a water temperature of 7 °C and a salinity of 35 psu.

1.6 Regime for collision rates due to mixing

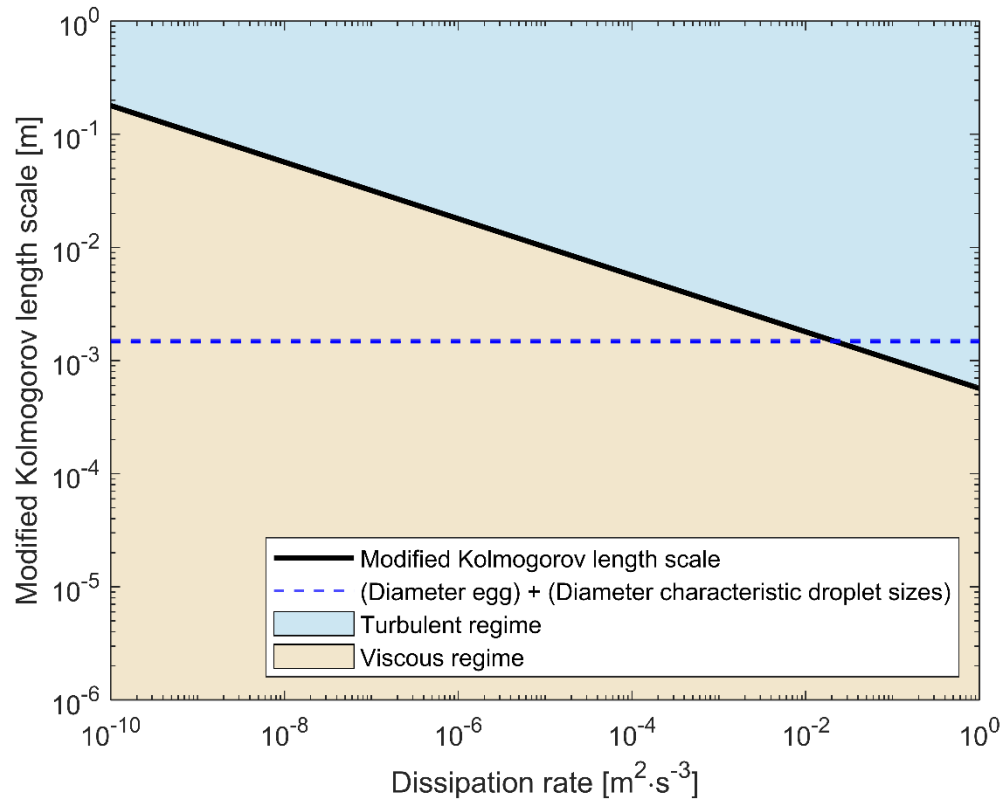


Figure S4: Modified Kolmogorov length scale and the sums of egg and droplet diameter for the four characteristic droplet sizes. The four droplet sizes appear as one line (dashed blue) since the differences in droplet diameters are small compared to the egg diameter. As seen from Figure S7, dissipation rates in the simulation are such that the sum of diameters stay below the length scale and thereby keeping the mixing within the viscous regime.

1.7 Collision rate contributions

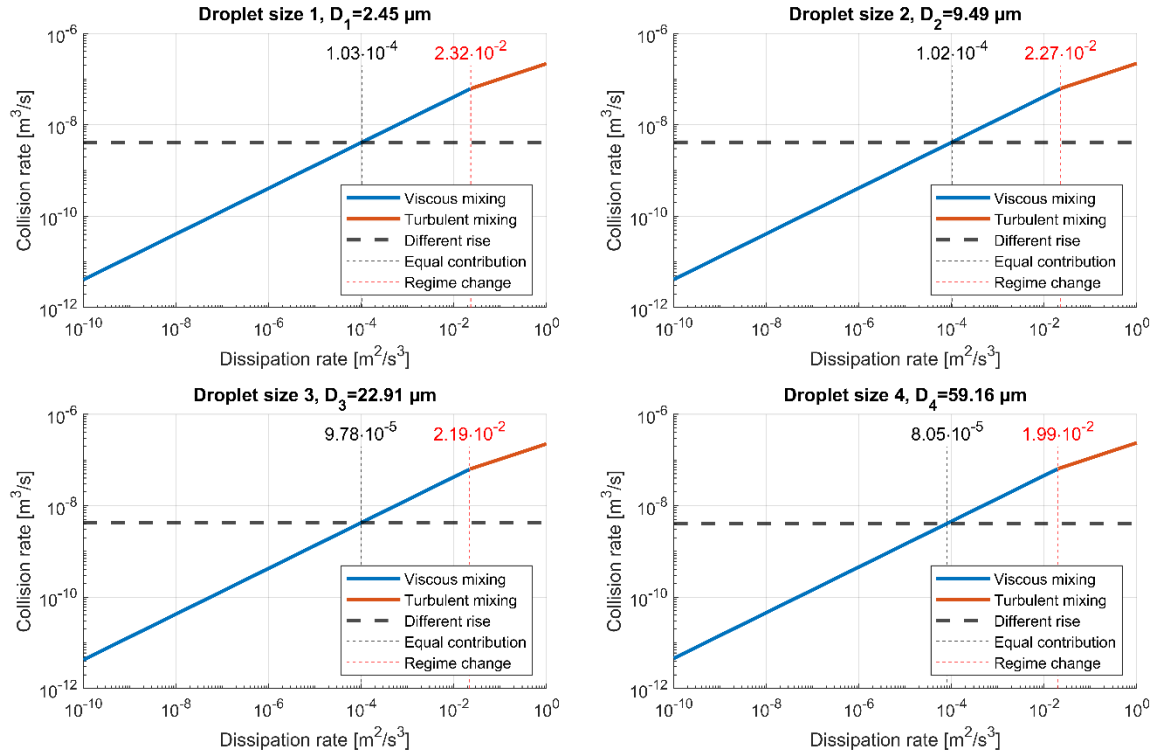


Figure S5: Collision rates as a function of dissipation rate for the four characteristic droplet sizes assuming a temperature of $7^\circ C$, salinity of 35 psu and the mean egg buoyancy of 30.68 psu. The collision rates due to difference in rise velocities are independent of the dissipation rate and thus constant for each droplet size. The collision rates due to mixing are increasing with dissipation rate. At $\sim 10^{-4} m^2 \cdot s^{-3}$ the mixing and rise contribute equally to the collision rates. At $\sim 2 \cdot 10^{-2} m^2 \cdot s^{-3}$ the mixing regime changes from viscous to turbulent (as can also be seen in Figure S4).

2 Statistics from the example simulation

This section contains methods relevant statistics from the example simulation.

2.1 Vertical distribution of eggs

Vertical egg movements are modelled using a binned biased random walk with bins of 2 m. This gives possible egg depths in meter of all odd whole numbers. Resulting profile for egg distribution averaged over a full month is shown in Figure S6.

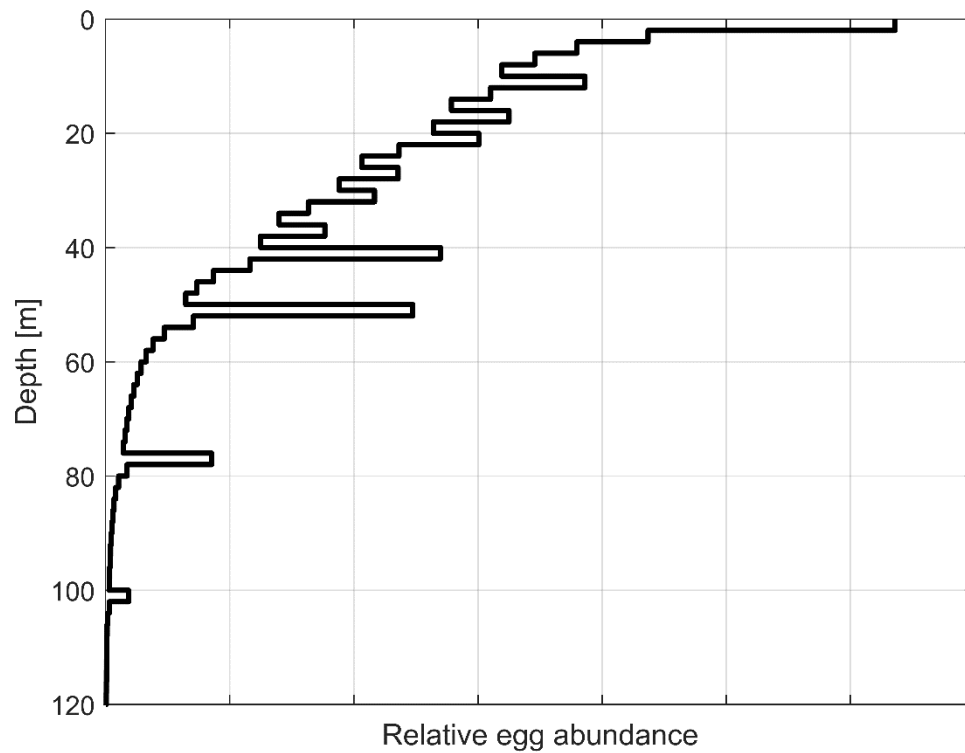


Figure S6: Average depth distribution for eggs in April. The profile is created using all depth values of all eggs in April of the simulation, in total 141,396,078 values.

2.2 Dissipation rate of kinetic energy

As seen in section S1.3, the dissipation rate in the model is a function of surface wind and the depth. The resulting model values are shown in Figure S7.

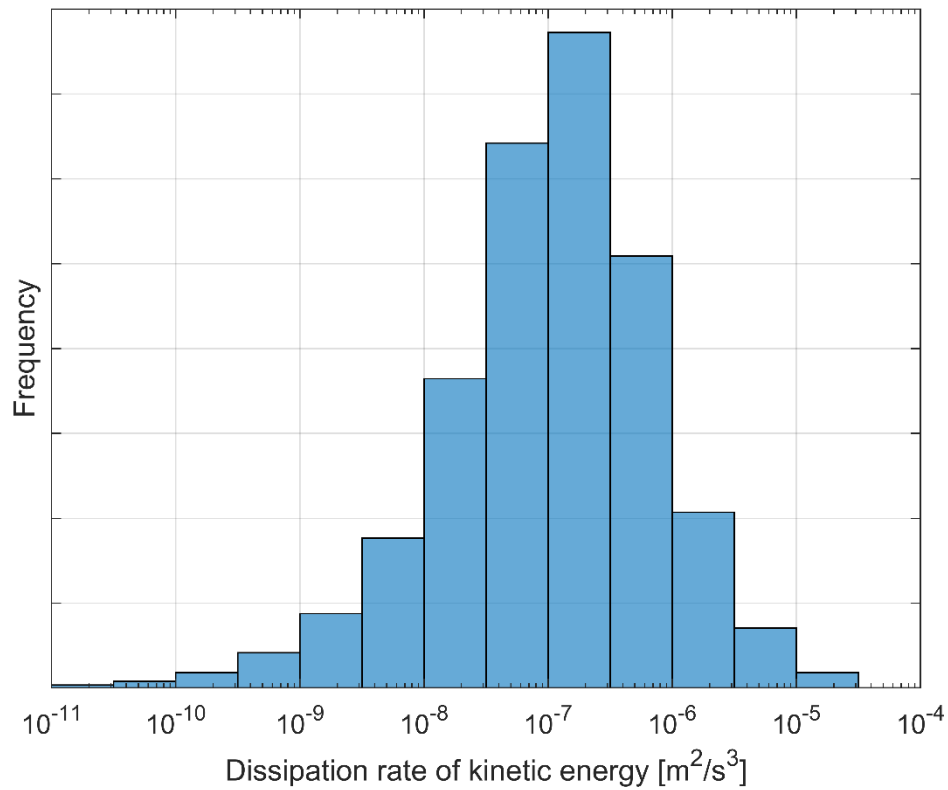


Figure S7: Dissipation rates experienced by the eggs in the model during their egg stage. The histogram considers all values during the egg stage for all eggs. With an average egg stage duration of 1546 time steps (12.9 days) and 189,000 eggs, this results in the histogram being created from a total of $\sim 2.9 \cdot 10^8$ values.

2.3 Egg stage duration

The hatching time of an egg in the model is a function of experienced ambient temperature as described in section 2.1. Resulting hatching times are given in Figure S8. The minimum is 10.2 days, the maximum is 17.8 days and the median is 12.7 days.

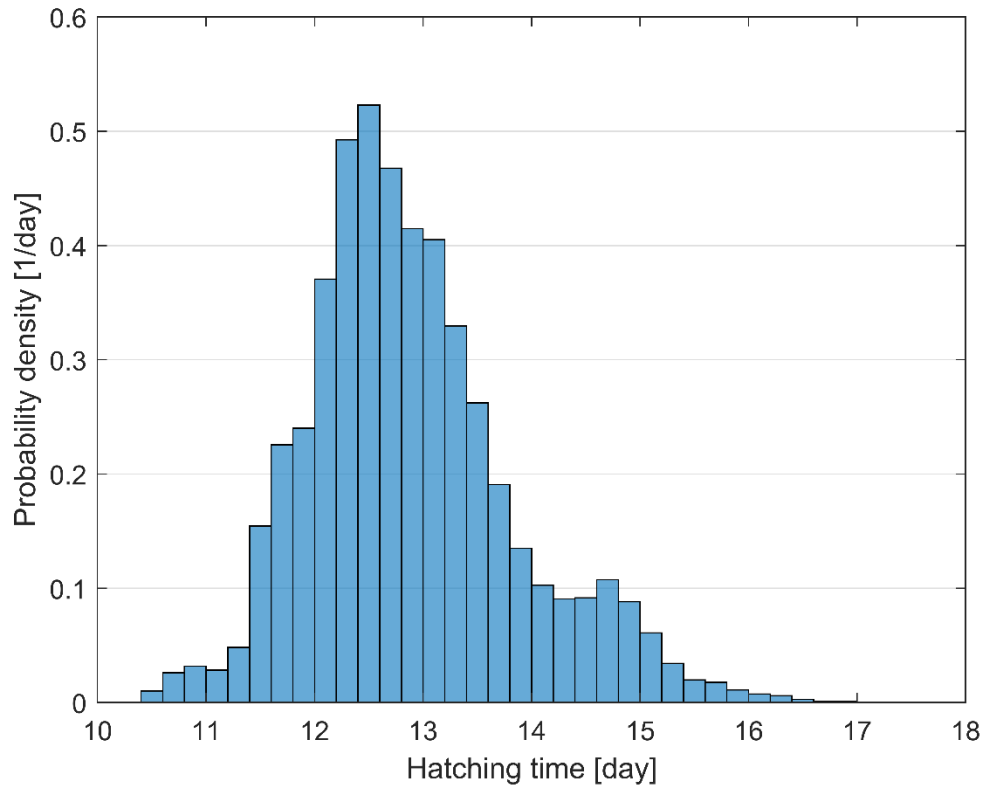


Figure S8: Histogram of hatching times in the model for all eggs.

2.4 Oil composition

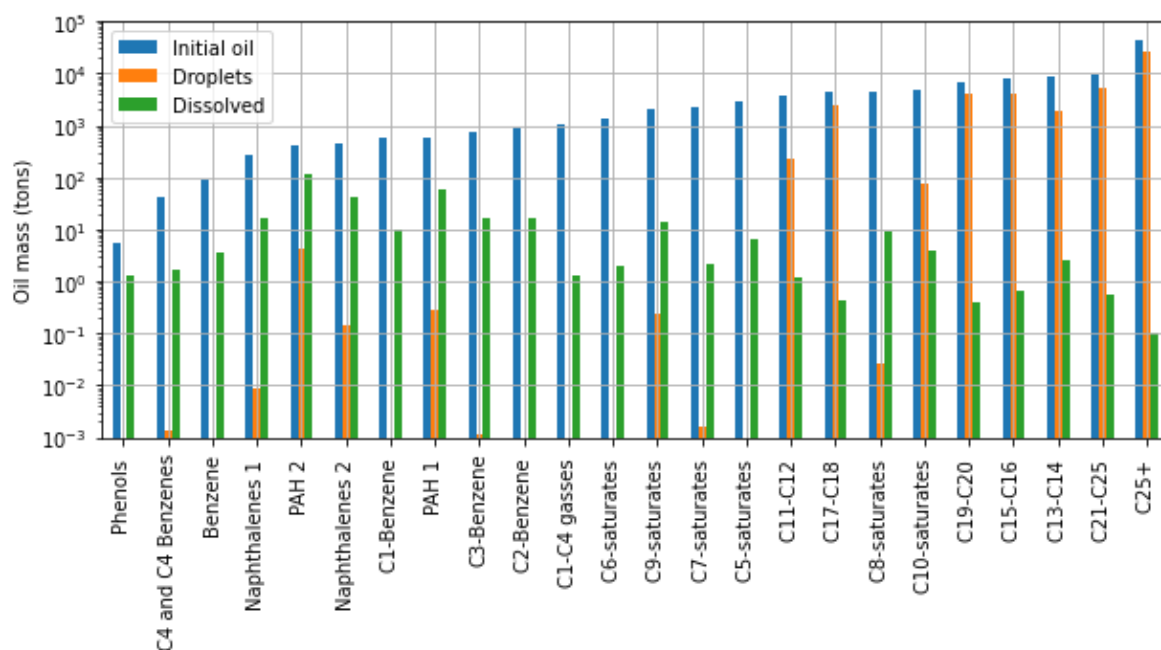


Figure S9: Composition of oil pseudo-components in the simulation, calculated as time average over April for the entire spatial domain.

2.5 Size distribution of collided mass

Having the accumulated droplet mass of each egg, the total contribution from each size bin is shown in Table S1. The distribution is almost identical to the relative mass distribution between the bins used to distribute the dispersed oil into bins. This is an effect of small differences in the collision rates between different droplet sizes as seen in Figure S5, making the collision contribution dominated by the droplet spectrum which has large differences between the bins.

Bin	Accumulated droplet mass contribution	Normalized relative mass concentration
1	0.0275%	0.0265%
2	0.692%	0.662%
3	6.95%	6.62%
4	92.3%	92.7%

Table S1: Contribution to accumulated droplet mass between the bins (middle column) for all accumulated mass, i.e., for all eggs. The normalized relative mass distribution for the droplet spectrum (right column) from Table 1 is included for reference. Note that both columns sum to 100%, but values in the table have been rounded to three significant figures.

2.6 Exposure time distributions

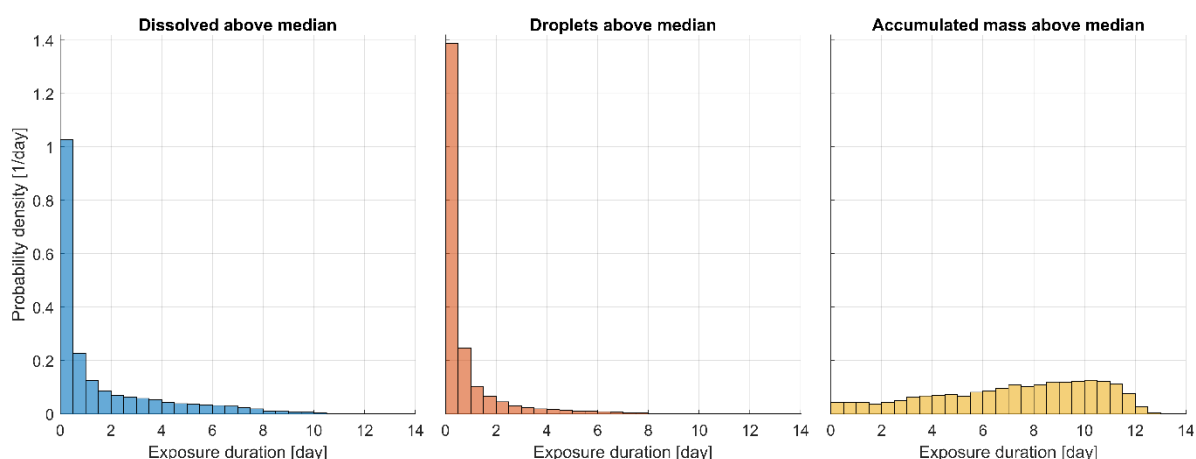


Figure S10: Distributions of exposure durations above the median value for the eggs exposed above the median. The left panel considers all eggs exposed above the median dissolved concentration, the middle all above the median droplet concentration and the right all above the median accumulated mass. The right panel is equal to the right panel in Figure 6 and the middle panel in Figure S11. See Figure 5 for median values.

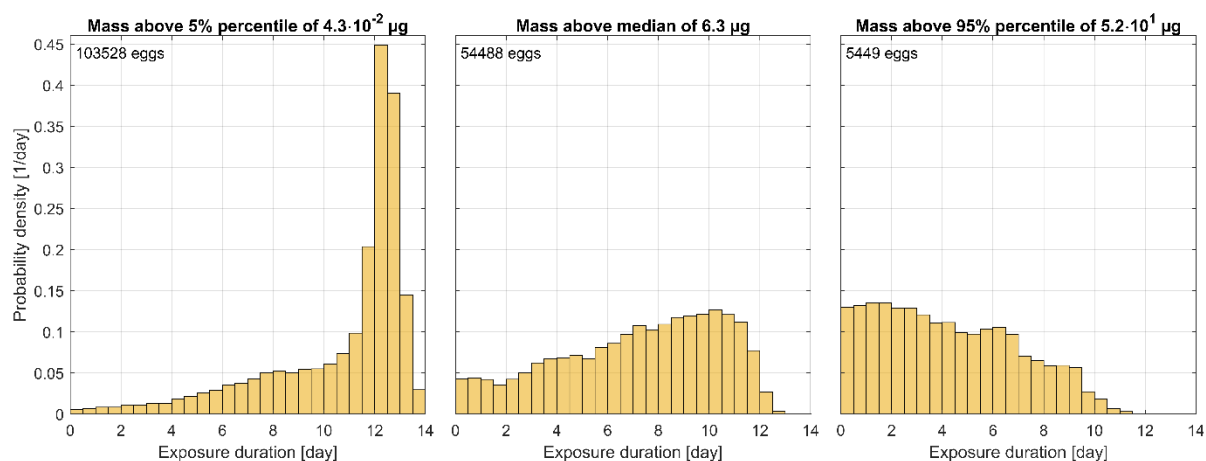


Figure S11: Distributions of exposure durations for the eggs above the different percentiles of accumulated droplet mass considered in the right panel of Figure 5. Each panel considers all eggs exposed above the given limit. The middle panel is equal to the right panel in Figure 6 and the right panel of Figure S10.

Supplementary references

- [1] B. R. MacKenzie, T. J. Miller, S. Cyr and W. C. Leggett, "Evidence for a dome-shaped relationship between turbulence and larval fish ingestion rates," *Limnology and Oceanography*, vol. 39, no. 8, pp. 1790-1799, 1994.
- [2] H. U. Sverdrup, M. W. Johnson and R. H. Fleming, *The Oceans: Their physics, chemistry, and general biology*, vol. 1087, Prentice-Hall New York, 1942.
- [3] J. M. Dallavalle, *Micromeritics: The technology of fine particles*, New York: Pitman, 1948.
- [4] L. Esters, Ø. Breivik, S. Landwehr, A. ten Doeschate, G. Sutherland, K. H. Christensen, J.-R. Bidlot and B. Ward, "Turbulence scaling comparisons in the ocean surface boundary layer," *Journal of Geophysical Research: Oceans*, vol. 123, no. 3, pp. 2172-2191, 2018.