

# **Supplementary Information for “Automatic Beam Optimization Method for Scanning Electron Microscopy based on Electron Beam Kernel Estimation”**

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for  $i$  in range (number of sweep images):

$$f_i = \text{standard\_normalization}(f_i)$$

$u = \text{standard\_normalization}(u)$

for  $n$  in range(number of iterations):

update  $u$ :

$$\frac{\partial F_u}{\partial u} = \frac{1}{n} \sum_{i=0}^n [\overline{k_{\Sigma_i}} \otimes (k_{\Sigma_i} \odot u(x + v_i) - f_i)] - \lambda \nabla \cdot \frac{\nabla u}{|\nabla u|}$$

$$u \leftarrow u - \epsilon_u \frac{\partial F_u}{\partial u}$$

update kernels:

for  $i$  in range (number of sweep images):

$$\frac{\partial F_{\Sigma_i}}{\partial k_{\Sigma_i}} = (k_{\Sigma_i} \odot u(x + v_i) - f_i) \otimes \overline{u(x + v_i)}$$

$$\frac{\partial k_{\Sigma_i}}{\partial \Sigma_i} = \frac{1}{K} \int_{\mathbf{x}} -\frac{1}{2} k_{\Sigma_i}(\mathbf{x}) (\Sigma_i^{-1} - \Sigma_i^{-1} \mathbf{x} \mathbf{x}^T \Sigma_i^{-1}) d\mathbf{x}$$

$$\frac{\partial F_{\Sigma_i}}{\partial \Sigma_i} \leftarrow \text{clipping} \left( \frac{\partial F_{\Sigma_i}}{\partial k_{\Sigma_i}} \cdot \frac{\partial k_{\Sigma_i}}{\partial \Sigma_i} \right)$$

$$\Sigma_i \leftarrow \text{clipping} \left( \Sigma_i - \epsilon_{\Sigma} \frac{\partial F_{\Sigma_i}}{\partial \Sigma_i} \right)$$

$$k_{\Sigma_i} = \text{get\_kernel\_image\_from\_Gaussian\_covariance}(\Sigma_i)$$

update  $v$ :

for  $i$  in range (number of sweep images):

$$v_{i1} \leftarrow -\frac{\sum_{\text{pixels}} [(k_{\Sigma_i} * u_x) \cdot (k_{\Sigma_i} * (v_{i2} u_y + u) - f_i)]}{\|k_{\Sigma_i} * u_x\|_2^2}$$

$$v_{i2} \leftarrow -\frac{\sum_{\text{pixels}} [(k_{\Sigma_i} * u_y) \cdot (k_{\Sigma_i} * (v_{i1} u_x + u) - f_i)]}{\|k_{\Sigma_i} * u_y\|_2^2}$$

$$\epsilon_u \leftarrow \eta_{\text{decay}} \cdot \epsilon_u$$

$$\epsilon_{\Sigma} \leftarrow \eta_{\text{decay}} \cdot \epsilon_{\Sigma}$$


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**Supplementary Algorithm 1** | Algorithm 1 illustrates the beam kernel estimation process, demonstrating the operations required to single update the parameter set of SEM. Please refer to the manuscript for the meanings of each character and operational symbol. The function named *clipping* refers to the process of multiplying a specific scalar with a matrix, preventing the elements of the matrix from exceeding a certain limit in both covariance and its gradient. This method is employed to clip the matrix, mitigating overshooting and enhancing the convergence speed. The function named *get\_kernel\_image\_from\_Gaussian\_covariance* extracts an image of a specified size of Gaussian kernel using covariance matrix.