

Supplementary Information

Results of the comparison between the classical physics-informed neural network and the proposed multi-level physics-informed neural network

The difference in prediction performance between the classical physics-informed neural network (c-PINN) and the proposed multi-level physics-informed neural network (ml-PINN) is compared, as illustrated in Fig. S1. This figure shows the results for Case 1 of the 1D beam structure, as detailed in Table 1 and Fig. 4b of the main text. It should be noted that both c-PINN and ml-PINN employ fully connected neural networks with identical capacities, as illustrated in Fig. S1. It clearly shows that ml-PINN outperforms c-PINN in terms of prediction accuracy and computational efficiency. In addition, Table S1 provides detailed results for all four cases on the 1D beam structure using both c-PINN and ml-PINN (refer to Table 1 of the main text for cases details). As shown in Table S1, even after performing 10^4 iterations, c-PINN still exhibits significantly higher errors compared to ml-PINN in predicted displacements. Additionally, it is worth mentioning that ml-PINN achieves the superior result while requiring only a quarter of computation time compared to c-PINN under the same training parameters.

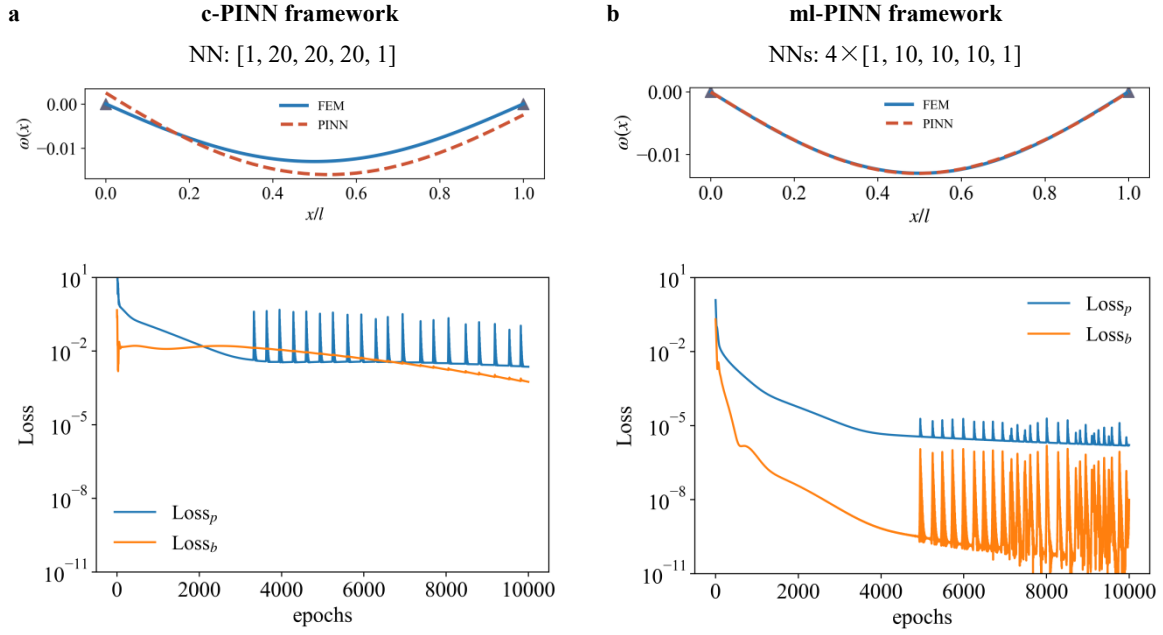


Fig. S1 | Comparison results of computation for 1D beam structure using classical physics-informed neural network (c-PINN) and multi-level physics-informed neural network (ml-PINN). **a** Predicted outcomes and loss decline curve of c-PINN exhibit poor prediction accuracy and convergence difficulties. $Loss_b$ and $Loss_p$ represent the loss functions from boundary constraints and PDEs, respectively. The notation [1, 20, 20, 20, 1] means that the neural network consists of an input layer with 1 node, 3 hidden layers with 20 nodes each, and an output layer with 1 node. **b** Predicted outcomes and loss decline curve of ml-PINN demonstrate rapid convergence of the loss function along with improved prediction accuracy. It should be note that the single network of [1, 20, 20, 20, 1] for c-PINN and four networks of [1, 10, 10, 10, 1] for ml-PINN possess the same capacity of fully connected neural networks.

Table. S1 Comparison results of computation for 1D beam structure obtained by classical physics-informed neural network (c-PINN) and multi-level physics-informed neural network (ml-PINN)

Cases	Methods	Error (%)	Time (s/epoch)
Case 1	c-PINN	1.2~56.9	4.3×10^{-3}
	ml-PINN	0.6 ± 0.5	1.1×10^{-3}
Case 2	c-PINN	2.6~107.4	4.3×10^{-3}
	ml-PINN	2.4 ± 0.8	1.1×10^{-3}
Case 3	c-PINN	11.4~52.0	4.5×10^{-3}
	ml-PINN	1.1 ± 1.0	1.1×10^{-3}
Case 4	c-PINN	15.2~104.3	4.5×10^{-3}
	ml-PINN	0.3 ± 0.1	1.1×10^{-3}

Notes: The c-PINN is an abbreviation for classical PINN. $Loss_p$ and $Loss_b$ denote the loss function of

the PDEs and the loss function of boundary constraints, respectively. $Error = \frac{\|\omega_{PINN} - \omega_{FEM}\|_2^2}{\|\omega_{FEM}\|_2^2}$

where ω_{PINN} and ω_{FEM} represent the value of the vertical displacement computed by ml-PINN and finite

element method respectively.