

Supplementary Materials:

Magnetic Localization and Manipulation of Locking Synchronous Motors

Michiel Richter, Lukas Masjothusmann, Pavlo Makushko,
Venkatasubramanian Kalpathy Venkiteswaran, Denys Makarov,
and Sarthak Misra

Description of Supplementary Files

- Sup. Text 1: Description why the b_{ratio} of a rotating dipole is at most 2.
- Sup. Figure 1: Photographs of inner and outer planar coils with dimensions.
- Sup. Figure 2: Cross-section views of the motor fabrication process.
- Sup. Figure 3: Setup for internal and external motor temperature characterization.
- Sup. Figure 4: Setup for characterization of the rotor field, and validation of localization.
- Sup. Figure 5: 3D representation of a rotating magnetic dipole.
- Sup. Figure 6: Hall effect sensor noise and field comparison of stator and rotor.
- Sup. Figure 7: Flow diagram of the localization process.
- Sup. Figure 8: Partial derivatives of the rotor field isoline parameters.
- Sup. Figure 9: Setup for demonstration of magnetic localization and manipulation.
- Sup. Figure 10: Tracked positions and rotation axes through time.
- Sup. Figure 11: Simulated motor localization accuracy.
- Sup. Figure 12: Electronic circuit diagram of the custom coil driver for motor power.

Description of Additional Supplementary Files

Sup. Movie 1: Synchronous rotor rotation at different frequencies.

Sup. Movie 2: Phase transition of low melting point alloy.

Sup. Movie 3: Validation of the motor localization algorithm.

Sup. Movie 4: Motor localization and manipulation in a tubular network.

Supplementary Text 1 – Field ratio of rotating dipoles

Given the magnetic point dipole approximation, defined as

$$\mathbf{b}(\mathbf{p}) = \frac{\mu_0}{4\pi} \left(\frac{3\mathbf{p}(\boldsymbol{\mu} \cdot \mathbf{p})}{\|\mathbf{p}\|^5} - \frac{\boldsymbol{\mu}}{\|\mathbf{p}\|^3} \right), \quad (1)$$

where $\mathbf{b}(\mathbf{p}) \in \mathbb{R}^3$ is the magnetic field at some position ($\mathbf{p} \in \mathbb{R}$), and $\boldsymbol{\mu} \in \mathbb{R}^3$ is the dipole moment. Consider that the dipole is rotating, and that $\boldsymbol{\mu}$ is perpendicular to the rotation axis ($\hat{\boldsymbol{\omega}}$). We define an angle of rotation ($\phi \in [0, 2\pi)$), let the dipole's rotation axis be its local z-axis ($\boldsymbol{\omega} = \hat{\mathbf{z}}$), and define the rotated dipole moment as $\boldsymbol{\mu}_\phi = \mathbf{R}_z(\phi)(\mu\hat{\mathbf{x}})$.

The field is maximum when $\boldsymbol{\mu}$ and $\mathbf{p}$ are parallel, i.e. for $\boldsymbol{\mu}_0 = \mu\hat{\mathbf{x}}$ and $\mathbf{p} = p\hat{\mathbf{x}}$,

$$\begin{aligned} \mathbf{b}_{\max}(\mathbf{p}) &= \frac{\mu_0}{4\pi} \left(\frac{3p\hat{\mathbf{x}}(\mu\hat{\mathbf{x}} \cdot p\hat{\mathbf{x}})}{p^5} - \frac{\mu\hat{\mathbf{x}}}{p^3} \right) \\ &= \frac{\mu_0}{4\pi} \left(\frac{3p^2\mu\hat{\mathbf{x}}(\hat{\mathbf{x}} \cdot \hat{\mathbf{x}})}{p^5} - \frac{\mu\hat{\mathbf{x}}}{p^3} \right) \\ &= \frac{\mu_0}{4\pi} \left(\frac{2\mu}{p^3} \right) \hat{\mathbf{x}}. \end{aligned}$$

Conversely, the field is minimum when $\boldsymbol{\mu}$ and $\mathbf{p}$ are perpendicular, i.e. for $\boldsymbol{\mu}_{\pi/2} = \mu\hat{\mathbf{y}}$ and $\mathbf{p} = p\hat{\mathbf{x}}$,

$$\mathbf{b}_{\min}(\mathbf{p}) = \frac{\mu_0}{4\pi} \left(-\frac{\mu}{p^3} \right) \hat{\mathbf{y}}.$$

Therefore, the ratio between the maximum and minimum field strength, for a rotating dipole, is at most two,

$$b_{\text{ratio}} = \frac{\|\mathbf{b}_{\max}(\mathbf{p})\|}{\|\mathbf{b}_{\min}(\mathbf{p})\|} = 2.$$

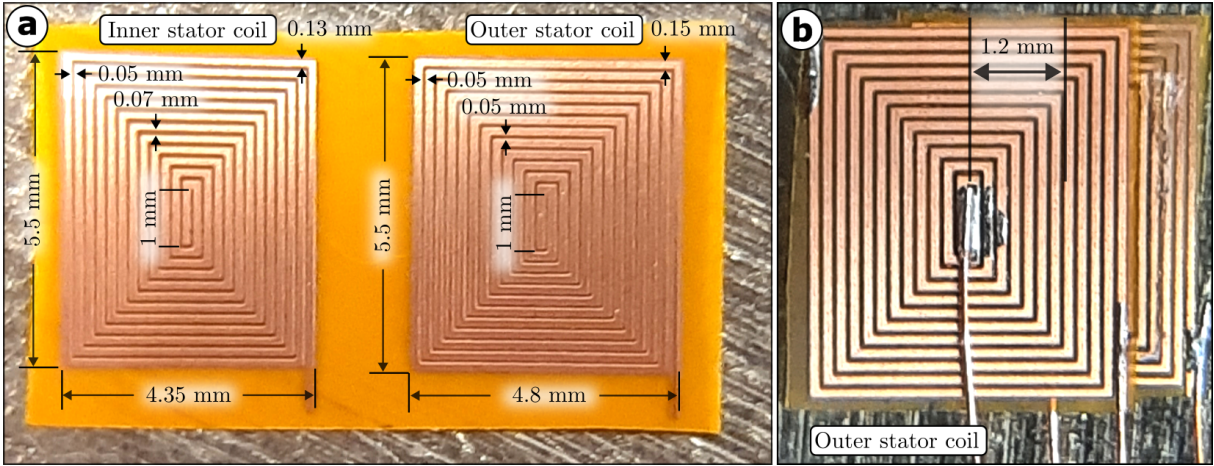


Fig. S1: Planar coils design. **a** Rectangular planar coils with annotated dimensions. **b** Stacked inner and outer stator coils with a center-to-center offset displacement. Soldered copper power leads are visible. Photograph taken prior to cylindrical deformation to form stator coils.

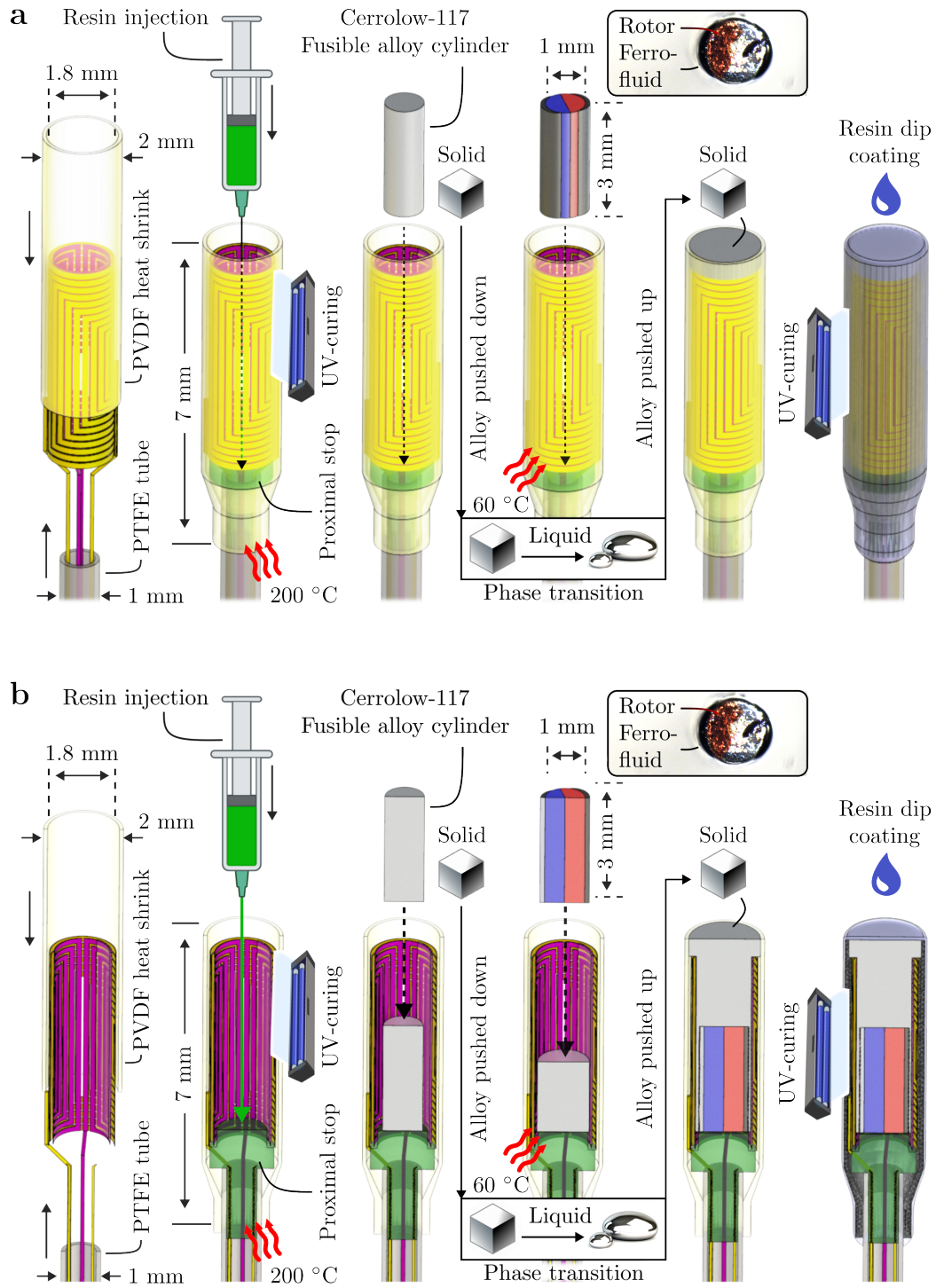


Fig. S2: Fabrication process of the permanent magnet synchronous motor. a Non-cross-sectional view of fabrication steps. **b** Cross-section view showing internal arrangements of components.

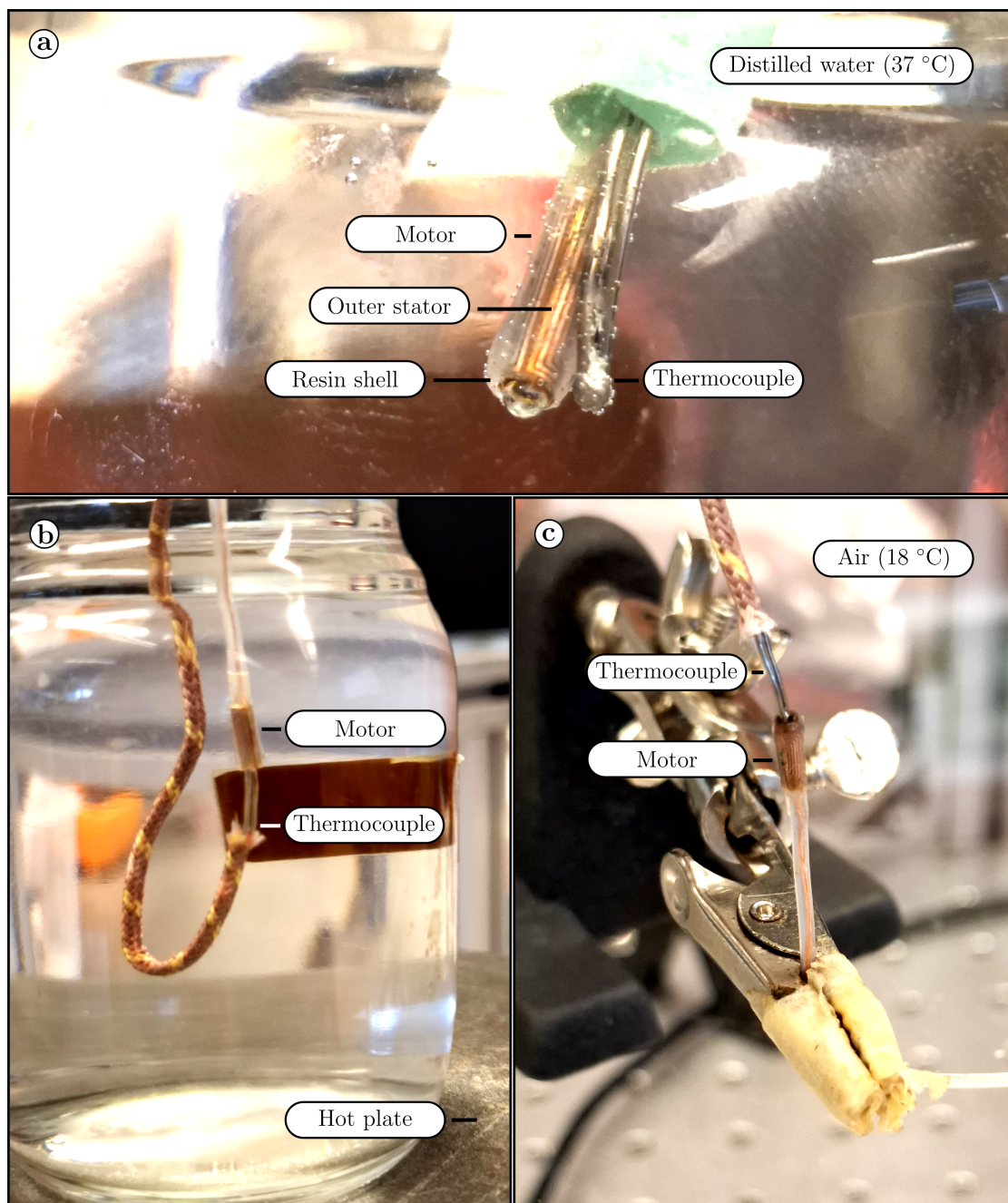


Fig. S3: Temperature measurements. **a** External temperature measurement inside a bath of distilled water at 37 °C. Visible are the motor with outer stator and resin shell, but without the permanent magnet rotor, and a thermocouple placed along its outer surface. **b** Internal temperature measurement inside a bath (glass jar) of distilled water at 37 °C. The thermocouple is placed inside the motor's rotor cavity. The water is heated with a hot plate. **c** Internal temperature measurement inside air at 18 °C.

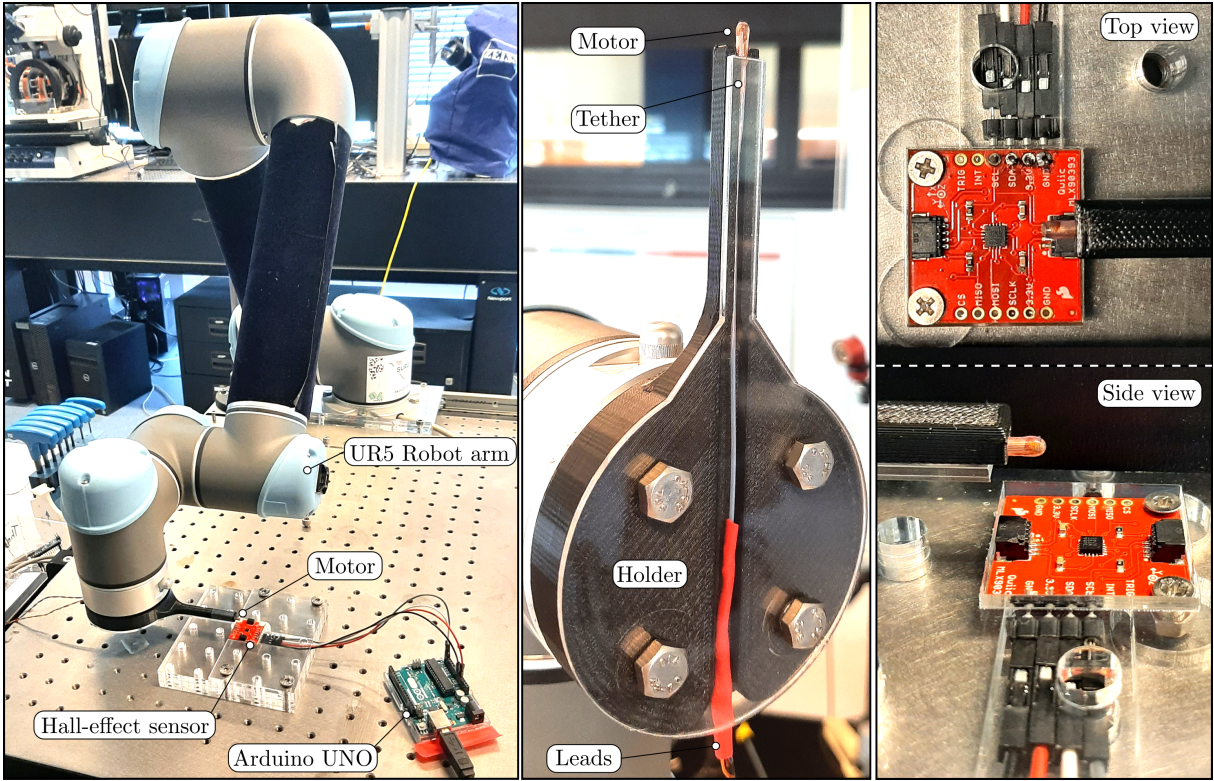


Fig. S4: Experimental setup for rotor field characterization and validation of the localization algorithm. The setup consists of a UR5 serial robotic arm, connected to a 3D-printed holder that positions the motor horizontally above a Hall effect sensor. The sensor is connected to an Arduino UNO, which is itself connected to a PC.

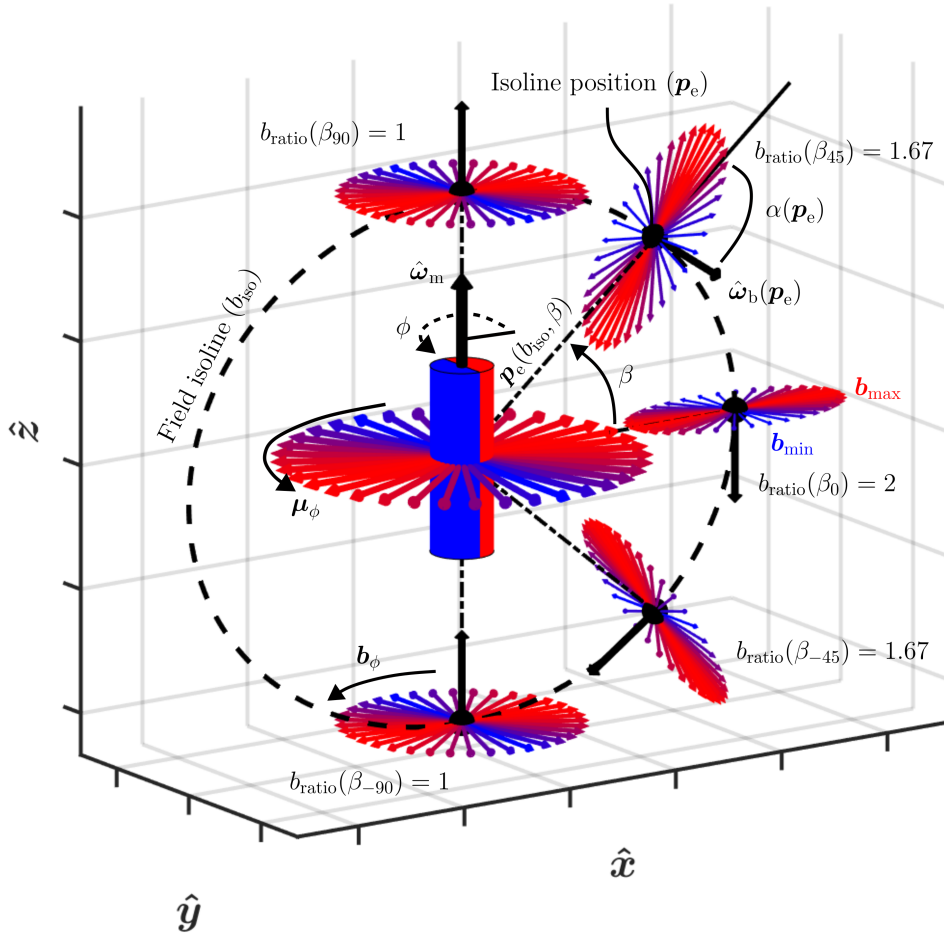


Fig. S5: A 3D representation of a rotating magnetic dipole (rotor magnet) with perpendicular magnetization and rotation axes. The magnet has rotation axis ($\hat{\omega}_m$) and rotation angle (ϕ), that orient the rotating dipole moment (μ_ϕ). During magnet rotation, at any position in space, the field (b_ϕ) rotates in-plane with a field rotation axis ($\hat{\omega}_b$). The field has an elliptically varying field strength between maximum ($b_{\max}$) and minimum ($b_{\min}$) values, with an associated field ratio ($b_{\text{ratio}} = \|b_{\max}\|/\|b_{\min}\|$). Within the magnetization plane of the magnet, spanned by $\hat{\omega}_m$ and $\hat{\mu}_{\phi=0}$, a field isoline ellipse (dashed line) is defined of constant field strength (b_{iso}). Each position on the ellipse, termed the isoline position ($p_e(b_{\text{iso}}, \beta)$), depends on b_{iso} and an isoline position angle ($\beta = \angle p_e, \mu_0$). Then, each $p_e(b_{\text{iso}}, \beta)$ is associated with field vectors of maximum strength ($b_{\max}(p_e)$), minimum strength ($b_{\min}(p_e)$), and rotation ($\hat{\omega}_b(p_e)$), as well as scalars of field ratio ($b_{\text{ratio}}(p_e)$) and field angle ($\alpha(p_e) = \angle p_e, \hat{\omega}_b(p_e)$).

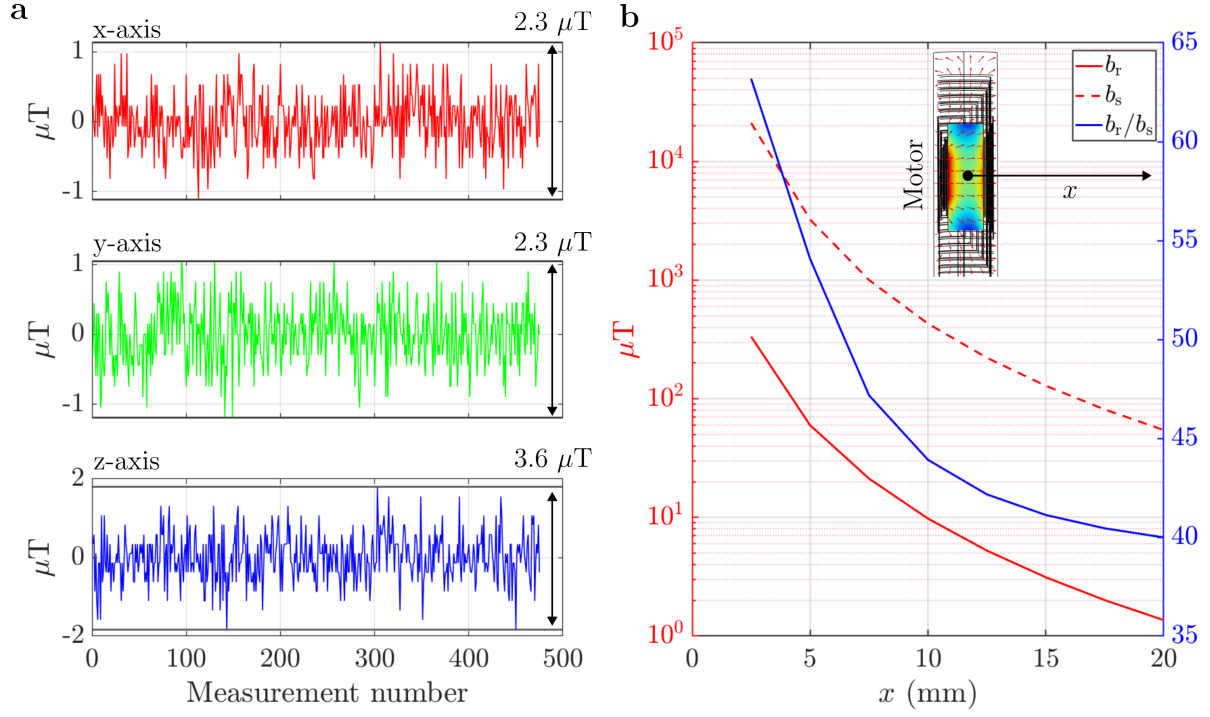


Fig. S6: Hall effect sensor noise and comparison between magnetic fields of the rotor and stator. **a** Noise signal of the Hall effect sensor obtained from a mean-correction of static background field measurements. **b** Finite element simulated magnetic fields of the permanent magnet rotor (b_r) and electromagnet stator (b_s), corrected for driving current (200 mA). In addition, the ratio between rotor and stator fields (b_r/b_s) is shown.

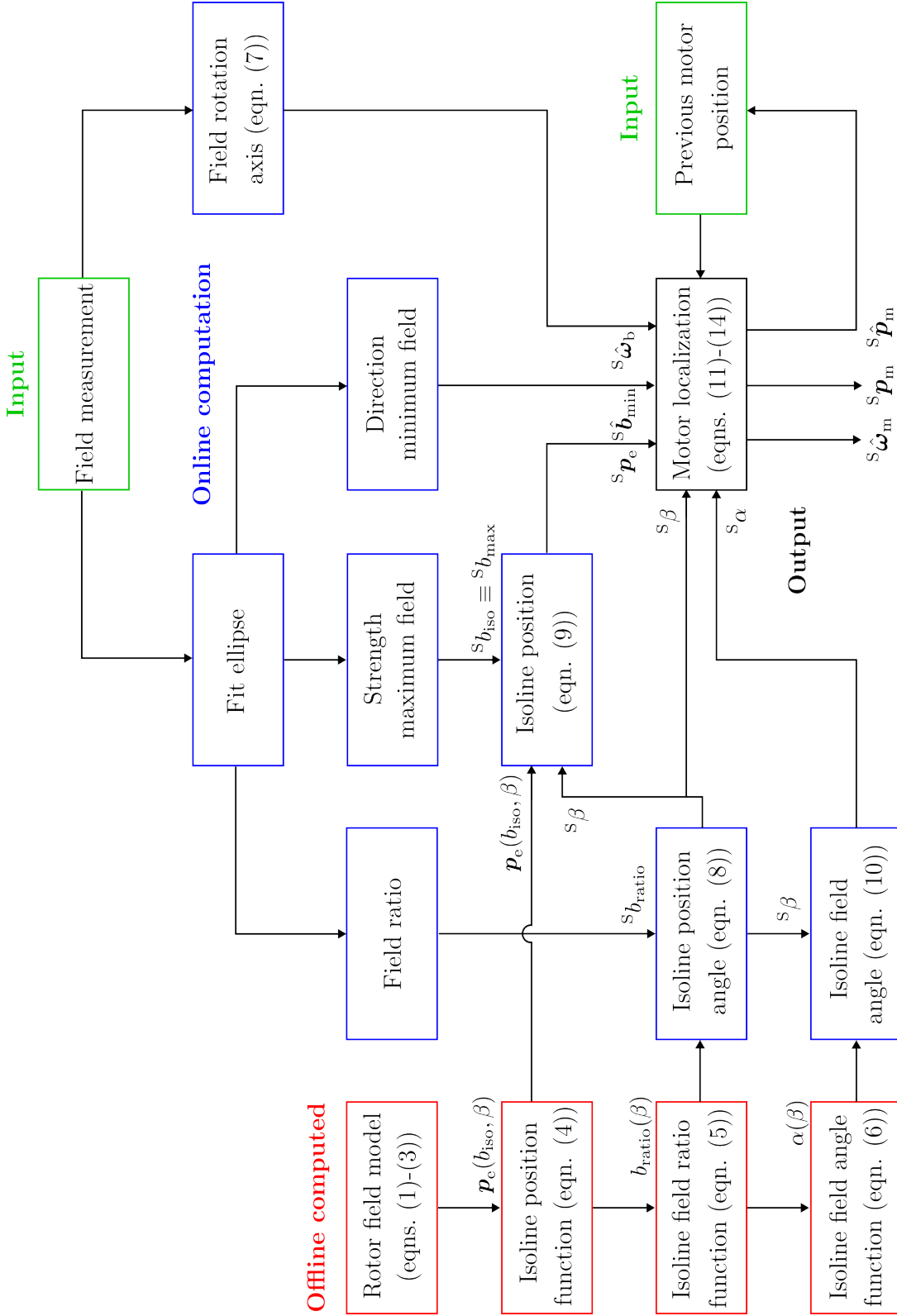


Fig. S7: Flow diagram of the localization algorithm. Offline computed rotor field isoline functions are used in conjunction with field measurements to provide online estimations of the motor's position and rotation axis.

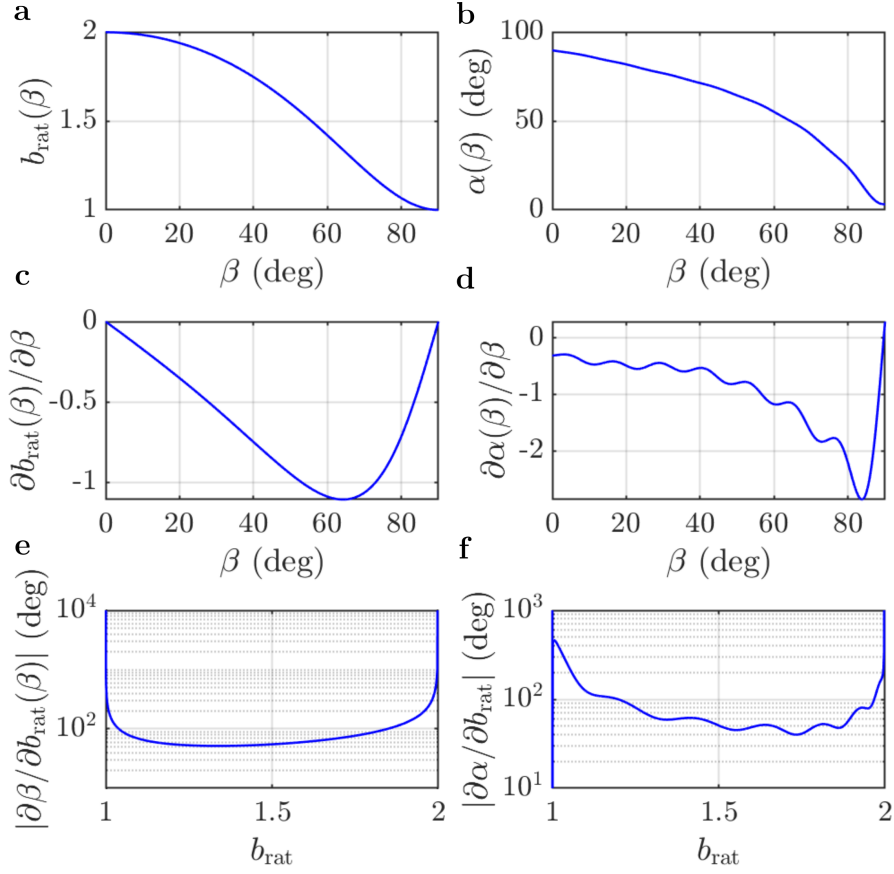


Fig. S8: Error sensitivity of the (angular) rotor isoline functions. **a** Output of the rotor isoline function for field ratio ($b_{\text{rat}}(\beta) := b_{\text{ratio}}(\beta)$) at isoline position angles (β). **b** Output of the rotor isoline function for field angle ($\alpha(\beta)$) at isoline position angles (β). **c** Gradient of $b_{\text{ratio}}(\beta)$ with respect to β . **d** Gradient of $\alpha(\beta)$ with respect to β . The oscillatory profile is due to the sinusoidal function (eqn. (6) of main text) that is fit to $\alpha(\beta)$. **e** Error sensitivity of β with respect to field ratio: $\partial \beta / \partial b_{\text{rat}} = 1 / (\partial b_{\text{rat}} / \partial \beta)$. **f** Error sensitivity of α with respect to field ratio: $\partial \alpha / \partial b_{\text{rat}} = (\partial \alpha / \partial \beta) \cdot (\partial \beta / \partial b_{\text{rat}})$.

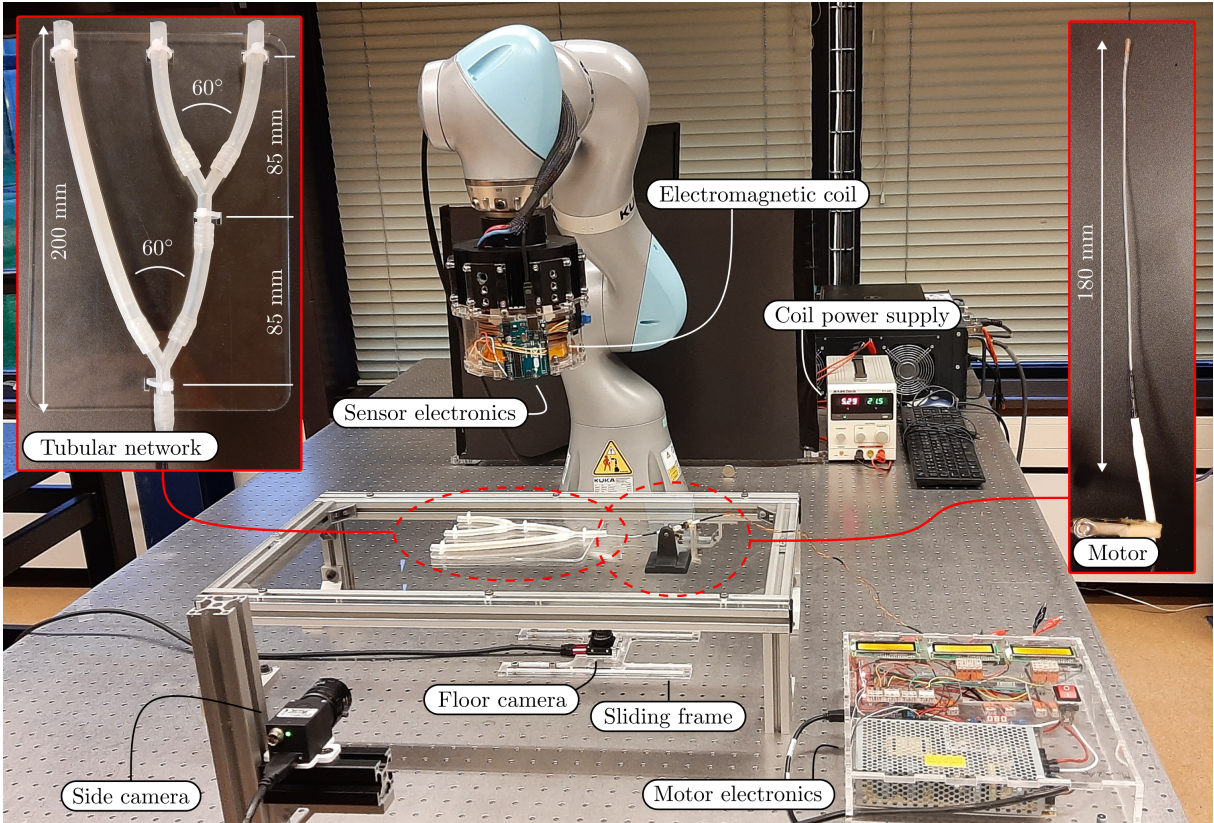


Fig. S9: Overview of the experimental setup for demonstration of magnetic localization and manipulation. A permanent magnet synchronous motor (PMSM) is navigated through a tubular network using an external electromagnetic coil. Localization is performed by a tri-axial eye-in-hand magnetometer attached to the tip of the coil.

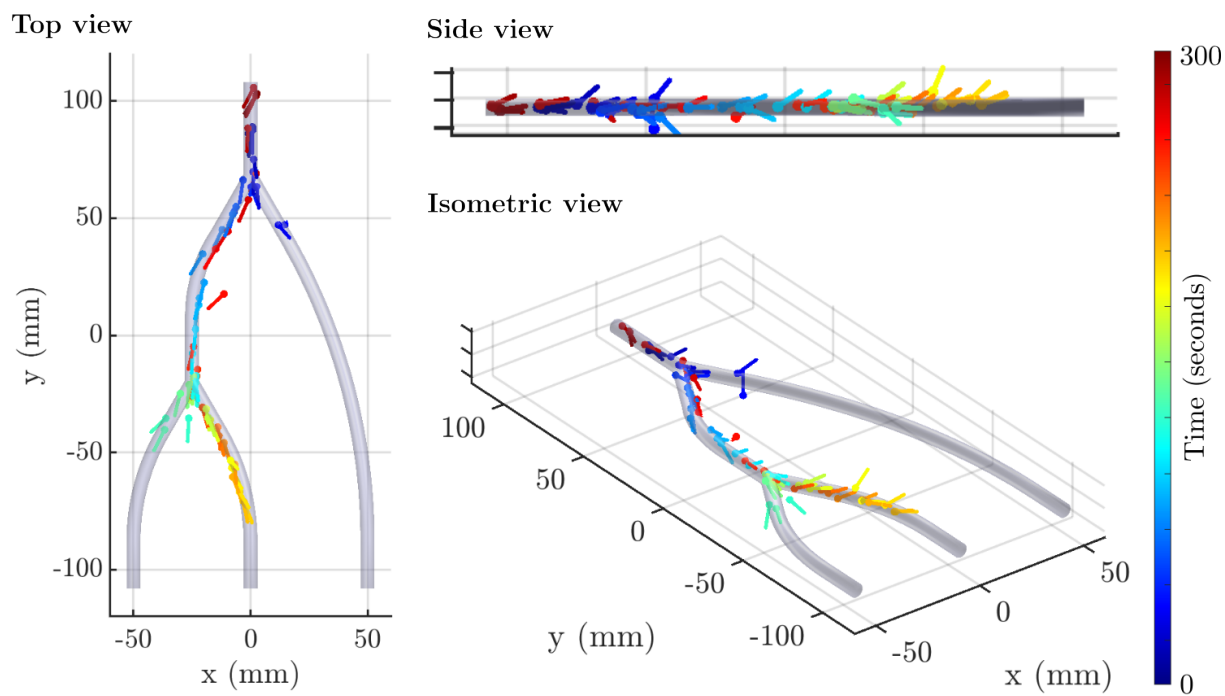


Fig. S10: Experiment results of magnetic localization and manipulation. Tracked positions (dots) and rotation axes (arrows) of the permanent magnet synchronous motor (PMSM) in the tubular network over time, for different viewpoints.

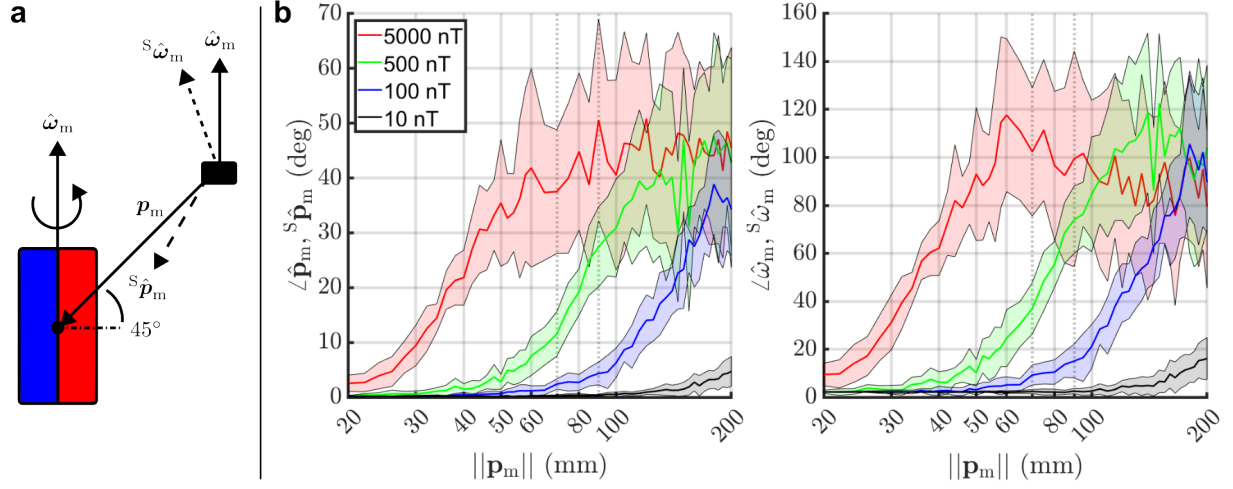


Fig. S11: Simulated angular errors of reconstructed position and rotation axis vectors for different sensor noise levels. **a** Configuration of the rotor magnet and magnetometer considered in simulation. The rotor magnet is associated with a position vector ($\mathbf{p}_m$) and rotation axis ($\hat{\omega}_m$), as well as a reconstructed position vector ($\hat{\mathbf{p}}_m$) and rotation axis ($\hat{\omega}_m$) from simulated sensor field measurements. **b** Simulated angular discrepancies between known and reconstructed position vectors and rotation axes over a range of motor-sensor distances and for different magnitudes of sensor noise. Mean values (solid line) and standard deviations (shaded area) are obtained from twenty simulations using uniformly distributed random noise values of shown maximum magnitudes ($\{10, 100, 500, 5000\}$ nT). For reference, our Hall effect magnetometer showed noise magnitudes of 4000 nT.

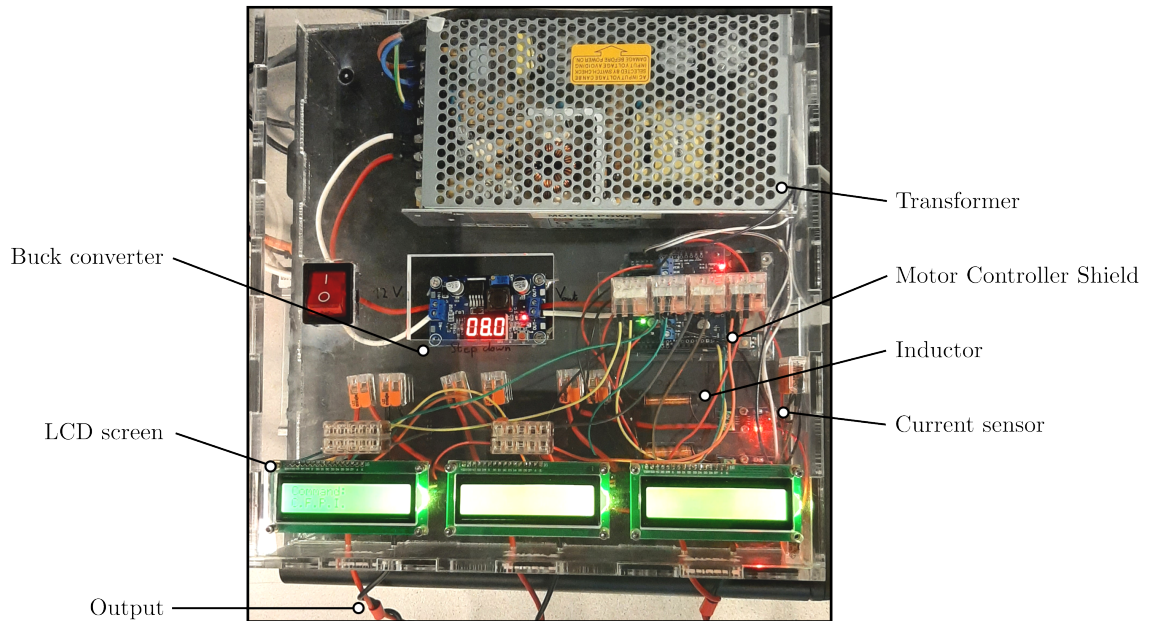
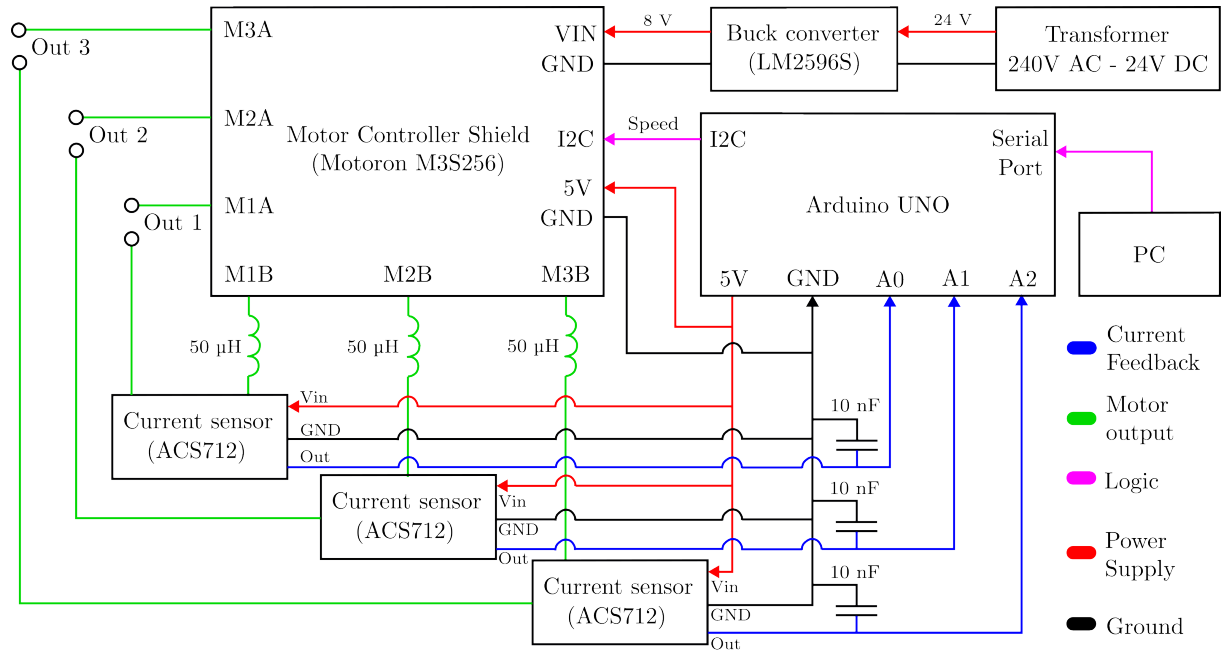


Fig. S12: Electronic circuit diagram of the custom coil driver. Desired stator current, frequency, and phase commands are sent from a PC to an Arduino UNO. These commands are converted to a motor speed (integer in range of -800 to +800), with the sign denoting the direction of current flow and the magnitude representing the PWM duty cycle. Closed-loop current regulation via current sensors is used to iteratively update the motor speed value to match the commanded quadrature current magnitudes and direction through the motor's stator coils (out 1-2).