

# Supplementary Information For Ultra-High Extinction Ratio Optical Modulation via Interference-Engineered Mach–Zehnder Modulator

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## 1 Math Model for ER-Boost MZM

The overall schematic of the ER-Boost MZM and its electronic bias control architecture is illustrated in Fig. 1. An optical carrier from a continuous-wave laser is injected into the integrated dual-branch MZM, which comprises a modulation branch for high-speed data encoding and an ER enhancement branch designed to suppress residual optical leakage through destructive interference. Each branch is driven by independent electrical paths with programmable DC bias voltages ( $V_{\text{bias,M}}$ ,  $V_{\text{bias,E}}$ , and  $V_{\text{bias,P}}$ ) and one shared high-speed RF input ( $V_{\text{RF}}$ ). An FPGA-based board generates the high-speed modulation signal through its GTH interface, which is subsequently amplified and delivered to the M-branch for data encoding. In parallel, the control board dynamically adjusts the DC bias voltages via DAC–amplifier stages, while a photodiode continuously monitors the average output power to provide feedback for extinction optimization. The entire bias control system, occupying an area of  $80 \text{ mm} \times 40 \text{ mm}$ , operates in coordination with the optical frontend to realize real-time ER stabilization.

To rigorously analyze the extinction enhancement behavior of the ER-Boost MZM, we derive the mathematical expressions for the modulated optical field and its corresponding output power. The proposed dual-branch structure comprises an E-path and a M-path, each implemented with asymmetrical Y-junctions characterized by power splitting ratios  $\gamma_{\text{E}}$  and  $\gamma_{\text{M}}$ , respectively. A global power allocation parameter  $\gamma$  determines the relative weighting between the two branches. Furthermore, an adjustable

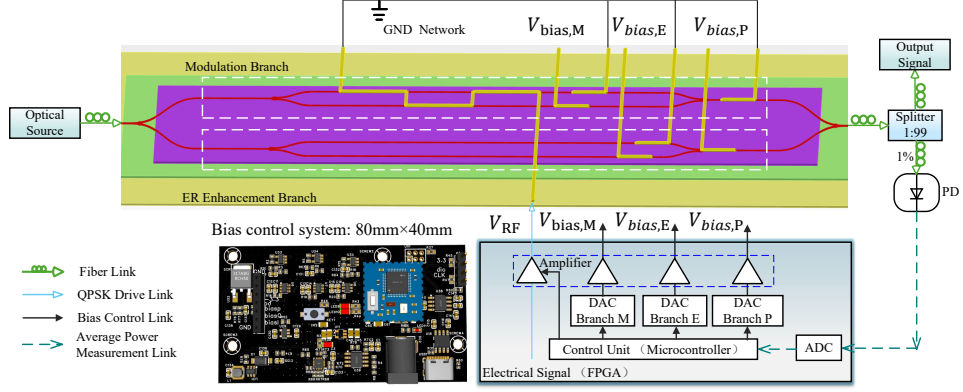


**Fig. 1** System configuration of the ER-Boost MZM and bias control unit. The modulation branch encodes the signal, while the ER enhancement branch introduces phase-controlled destructive interference to suppress leakage.

phase shift  $\phi_P$  is introduced at the recombination stage to enable phase-controlled interference between the two paths.

The resulting output optical field  $E_{\text{out}}$  as a function of the differential phases  $\phi_I$ ,  $\phi_Q$ , and  $\phi_P$  is expressed as<sup>1</sup>

$$E_{\text{out}}(\phi_M, \phi_E, \phi_P) = \iota E_{\text{in}} \left[ \gamma \left( \gamma_M \exp \left( j \frac{\phi_M}{2} \right) + (1 - \gamma_M) \exp \left( -j \frac{\phi_M}{2} \right) \right) + (1 - \gamma) \left( \gamma_E \exp \left( j \frac{\phi_E}{2} \right) + (1 - \gamma_E) \exp \left( -j \frac{\phi_E}{2} \right) \right) \exp(j\phi_P) \right], \quad (1)$$

where  $\iota$  is a proportionality constant accounting for insertion losses, and  $E_{\text{in}}$  is the input optical field amplitude. we define the phase shifts  $\phi_I$ ,  $\phi_Q$ , and  $\phi_P$  in terms of their respective bias voltages and signal components as follows:

$$\phi_M = \text{mod} \left( \frac{\pi(V_{\text{bias},M} + V_{\text{in}})}{V_\pi} + \phi_{s,M}, 2\pi \right), \quad (2a)$$

$$\phi_E = \text{mod} \left( \frac{\pi V_{\text{bias},E}}{V_\pi} + \phi_{s,E}, 2\pi \right), \quad (2b)$$

$$\phi_P = \text{mod} \left( \frac{\pi V_{\text{bias},P}}{V_\pi} + \phi_{s,P}, 2\pi \right). \quad (2c)$$

Here,  $V_{\text{bias},I}$ ,  $V_{\text{bias},Q}$ , and  $V_{\text{bias},P}$  are the independently tunable DC bias voltages applied to the M-branch, E-branch, and recombination phase control path, respectively. The term  $V_{\text{in}}$  denotes the high-speed RF modulation signal applied only to the M-branch, enabling intensity modulation. The parameter  $V_\pi$  is the half-wave voltage of the phase modulator, and  $\phi_{s,I}$ ,  $\phi_{s,Q}$ , and  $\phi_{s,P}$  represent static phase offsets caused by fabrication imperfections or waveguide mismatch. The corresponding optical output

power is given by<sup>2</sup>

$$\begin{aligned}
P_{\text{out}}(\phi_{\text{M}}, \phi_{\text{E}}, \phi_{\text{P}}) = & \iota^2 P_{\text{in}} \left[ \gamma^2 (\gamma_{\text{M}}^2 + (1 - \gamma_{\text{M}})^2 + 2\gamma_{\text{M}}(1 - \gamma_{\text{M}}) \cos \phi_{\text{M}}) \right. \\
& + (1 - \gamma)^2 (\gamma_{\text{E}}^2 + (1 - \gamma_{\text{E}})^2 + 2\gamma_{\text{E}}(1 - \gamma_{\text{E}}) \cos \phi_{\text{E}}) + 2\gamma(1 - \gamma) \left[ \gamma_{\text{M}}\gamma_{\text{E}} \cos \left( \frac{\phi_{\text{M}}}{2} - \frac{\phi_{\text{E}}}{2} - \phi_{\text{P}} \right) \right. \\
& + \gamma_{\text{M}}(1 - \gamma_{\text{E}}) \cos \left( \frac{\phi_{\text{M}}}{2} + \frac{\phi_{\text{E}}}{2} - \phi_{\text{P}} \right) + (1 - \gamma_{\text{M}})\gamma_{\text{E}} \cos \left( \frac{\phi_{\text{M}}}{2} + \frac{\phi_{\text{E}}}{2} + \phi_{\text{P}} \right) \\
& \left. \left. + (1 - \gamma_{\text{M}})(1 - \gamma_{\text{E}}) \cos \left( \frac{\phi_{\text{M}}}{2} - \frac{\phi_{\text{E}}}{2} + \phi_{\text{P}} \right) \right] \right], \tag{3}
\end{aligned}$$

where  $P_{\text{in}}$  denotes the input optical power.

## 2 Theoretical Analysis for the Initialization of the E-Branch

The initialization of the E-branch aims to determine its optimal bias phase  $\phi_{\text{E}}^*$  such that the residual interference in the output power is minimized. Starting from the general field model in Sec. 1, the total output power  $P_{\text{out}}$  can be decomposed into two components: a term independent of the recombination phase  $\phi_{\text{P}}$  and a single-harmonic term modulated by  $\phi_{\text{P}}$ . By collecting the cross-interference terms and applying angle-sum identities, the output power can be expressed as

$$P_{\text{out}}(\phi_{\text{M}}, \phi_{\text{E}}, \phi_{\text{P}}) = P_0(\phi_{\text{M}}, \phi_{\text{E}}) + 2\iota^2 P_{\text{in}} \gamma(1 - \gamma)(C \cos \phi_{\text{P}} + D \sin \phi_{\text{P}}), \tag{4}$$

where  $P_0(\phi_{\text{M}}, \phi_{\text{E}})$  collects the  $\phi_{\text{P}}$ -independent terms, and the coefficients  $C$  and  $D$  depend only on  $\gamma_{\text{M}}$ ,  $\gamma_{\text{E}}$ ,  $\phi_{\text{M}}$ , and  $\phi_{\text{E}}$ . This expression can be rewritten in the canonical cosine form

$$P_{\text{out}} = P_0 + 2\iota^2 P_{\text{in}} \gamma(1 - \gamma) \sqrt{C^2 + D^2} \cos(\phi_{\text{P}} - \delta), \quad \delta = \text{atan2}(D, C). \tag{5}$$

The term  $\sqrt{C^2 + D^2}$  quantifies the magnitude of  $\phi_{\text{P}}$ -induced interference, and hence determines the system's sensitivity to the recombination phase. Defining the modulation depth with respect to  $\phi_{\text{P}}$  as

$$\Delta P_{\phi_{\text{P}}} \triangleq \max_{\phi_{\text{P}}} P_{\text{out}} - \min_{\phi_{\text{P}}} P_{\text{out}} = 4\iota^2 P_{\text{in}} \gamma(1 - \gamma) \sqrt{C^2 + D^2}, \tag{6}$$

one can interpret  $\Delta P_{\phi_{\text{P}}}$  as a quantitative measure of the residual interference power between the modulation (M) and extinction (E) branches.

The two sub-interferometers can be equivalently described by complex phasors

$$\mathbf{A}_{\text{M}} = \sqrt{\gamma} \mathbf{a}_{\text{M}}(\gamma_{\text{M}}, \phi_{\text{M}}), \quad \mathbf{A}_{\text{E}} = \sqrt{1 - \gamma} \mathbf{a}_{\text{E}}(\gamma_{\text{E}}, \phi_{\text{E}}), \tag{7}$$

whose amplitudes represent the effective field strengths of the M- and E-paths, respectively. The interference amplitude is then given by

$$\sqrt{C^2 + D^2} = |\mathbf{a}_{\text{M}}| |\mathbf{a}_{\text{E}}|, \quad \Rightarrow \quad \Delta P_{\phi_{\text{P}}} = 4\iota^2 P_{\text{in}} \gamma(1 - \gamma) |\mathbf{a}_{\text{M}}| |\mathbf{a}_{\text{E}}|. \tag{8}$$

For a fixed modulation bias  $\phi_M$ , the sensitivity of the total output power to the recombination phase  $\phi_P$  is proportional to the field amplitude of the E-path. Therefore, the optimal E-branch bias satisfies

$$\phi_E^* \in \arg \min_{\phi_E} \Delta P_{\phi_P} \iff |\mathbf{a}_E(\gamma_E, \phi_E^*)| \text{ is minimized.} \quad (9)$$

At the operating point where  $\Delta P_{\phi_P}$  is minimized, the auxiliary E-path contribution is maximally suppressed. The recombination phase  $\phi_P$  no longer perturbs  $P_{\text{out}}$ , and the extinction ratio of the composite ER-Boost MZM is thereby enhanced to a level governed solely by the modulation branch contrast.

In practice,  $\phi_M$  is fixed by the modulation bias. The system performs a narrow-range sweep of  $\phi_P$  while iteratively adjusting  $\phi_E$  to minimize the measured ripple  $\Delta P_{\phi_P}$ . The phase  $\phi_E^*$  that minimizes this ripple provides the desired bias for optimal extinction enhancement.

### 3 Theoretical Analysis for Optimization of the M-branch

After the initialization of the E-branch, its output optical field can be effectively suppressed and thus approximated to zero. In this case, the ER-Boost MZM reduces to a single modulation branch, and the output optical power follows

$$P_{\text{out}}(V_{\text{in}}, V_{\text{bias,M}}) = \iota^2 \gamma P_{\text{in}} \left[ \gamma_M^2 + (1 - \gamma_M)^2 + 2\gamma_M(1 - \gamma_M) \cos\left(\frac{\pi(V_{\text{in}} + V_{\text{bias,M}})}{V_\pi} + \phi_{\text{s,M}}\right) \right]. \quad (10)$$

In practical high-speed transmission systems, it is challenging to directly measure the instantaneous optical power at the high- and low-intensity levels. This poses a significant obstacle in the design of control algorithms for the MZM. Because the ER, defined as the ratio of the maximum to minimum output power, requires measurement of both levels. To address this issue, we derive the relationship between the ER and the average output power. This allows the control unit to optimize the P2P driving and bias voltages based on real-time measurements of average output power, thereby achieving and maintaining a high ER in practical operation. For notational convenience, in this subsection, we focus on the control voltages for a single MZM and drop the superscript  $n$ , writing  $V_{\text{in}}$  and  $v_{\text{bias,M}}$ .

The output optical power at the low- and high-intensity levels corresponds to the logical ‘0’ and logical ‘1’ states of the driving signal, respectively. According to (10), the ER at the output of a single MZM can be expressed as

$$\xi(v_{\text{pp}}, V_{\text{bias,M}}) \equiv \xi(\phi_0, \phi_{\text{pp}}) = \frac{\gamma_M^2 + (1 - \gamma_M)^2 + 2\gamma_M(1 - \gamma_M) \cos(\phi_0 + \phi_{\text{pp}})}{\gamma_M^2 + (1 - \gamma_M)^2 + 2\gamma_M(1 - \gamma_M) \cos(\phi_0)}. \quad (11)$$

For convenience, the phase parameters are defined as

$$\phi_0 = \text{mod} \left( \phi_{s,M} + \frac{\pi(v_{\min} + V_{\text{bias},M})}{V_\pi}, 2\pi \right), \quad (12a)$$

$$\phi_{pp} = \text{mod} \left( \frac{\pi v_{pp}}{V_\pi}, 2\pi \right), \quad (12b)$$

where the driving voltage for logic ‘0’ is denoted by  $v_{\min}$ , and adding the P2P voltage swing  $v_{pp}$  to  $v_{\min}$  gives the driving voltage for logic ‘1’. In practice,  $v_{\min}$  is typically set to a small value, commonly zero, while the ER is optimized by adjusting  $v_{pp}$  and  $V_{\text{bias},M}$ . According to (11), the ER  $\xi$  is maximized when the MZM is biased at its null transmission point ( $\phi_0 = \pi$ ) and driven with a  $\pi$ -phase swing ( $\phi_{pp} = \pi$ ). These conditions correspond to setting  $(v_{\min} + V_{\text{bias},M}) = ((2i+1)\pi - \phi_{s,M})V_\pi/\pi$ ,  $i \in \mathbb{Z} \geq 0$ , and  $v_{pp} = V_\pi$ .

Assuming that the transmitted symbols are uniformly distributed, the average output optical power of a single MZM is given by

$$\begin{aligned} P_{\text{avg}}(v_{pp}, V_{\text{bias},M}) &\equiv P_{\text{avg}}(\phi_0, \phi_{pp}) \\ &= \iota^2 \gamma P_{\text{in}} \left[ \gamma_M^2 + (1 - \gamma_M)^2 + 2\gamma_M(1 - \gamma_M) \right. \\ &\quad \left. \times ((1 - \beta)\cos(\phi_0) + \beta \cos(\phi_0 + \phi_{pp})) \right]. \end{aligned} \quad (13)$$

where  $v_{pp}$  is the P2P voltage swing,  $V_{\text{bias},M}$  is the bias voltage, and their relations to the phase parameters,  $\phi_0$  and  $\phi_{pp}$ , are specified in (12). The parameter  $\beta \in (0, 0.5)$  represents the duty cycle, indicating the proportion of time the drive signal remains at the voltage for logic ‘1’.

The optimal values of the bias voltage  $v_{\text{bias},M}^*$  and the P2P voltage  $v_{pp}^*$  that maximize the ER can be jointly determined by locating the saddle point of the average output power function  $P_{\text{avg}}(\phi_0, \phi_{pp})$ , which is characterized by

$$[v_{\text{bias},M}^*, v_{pp}^*] = \arg \underset{V_{\text{bias},M}, v_{pp}}{\text{saddle}} P_{\text{avg}}(\phi_0, \phi_{pp}). \quad (14)$$

where the arg saddle operator denotes the process of jointly identifying the saddle point of the function  $P_{\text{avg}}(\phi_0, \phi_{pp})$ , that is, minimizing with respect to  $\phi_0$  and maximizing with respect to  $\phi_{pp}$ . The phase parameters  $\phi_0$  and  $\phi_{pp}$  are defined in (12) as functions of  $V_{\text{bias},M}$  and  $v_{pp}$ .

During the initialization stage, the optimal values of  $v_{pp}$  and  $v_{\text{bias},M}$  are determined by searching for the saddle point of  $P_{\text{avg}}$  with  $v_{pp}$  set near the nominal half-wave voltage. It is noteworthy that this method is well suited for hardware implementation without the need for direct sampling of instantaneous optical levels, as  $P_{\text{avg}}$  is easily measurable using a low-speed PD.

## References

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