Supplementary Information: Fracturing ranked surfaces

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CUTTING BONDS

In Fig. 1 we see the size dependence of the number of cutting bonds in $2D$ when bonds are removed with the constraint that cutting bonds are never removed. As discussed in the article for bridge bonds, a crossover is observed at $p = p_c$, between two fractal dimensions. In $2D$, for $p < p_c$ the fractal dimension of the cutting bonds d_{CB} is the same as observed for the bridge bonds for $p > p_c$, whereas at p_c it is $1/\nu$.

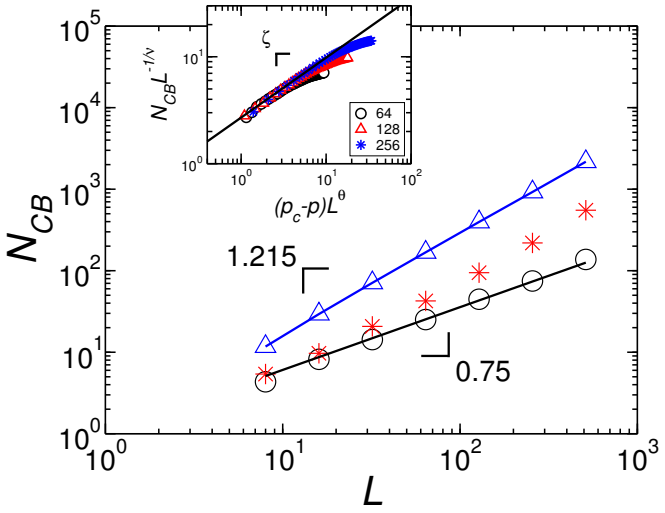

FIG. 1. Following the rank, bonds are sequentially removed except the cutting bonds. Size dependence of the number of cutting bonds, N_{CB}, for different fractions of occupied bonds p, namely $p = p_c = 0.5$ (circles), $p = p = 0.49$ (stars), and $p = 0.2$ (triangles). The solid lines are guides to the eye. At $p = p_c$, $N_{CB} \sim L^{1/\nu}$, where $\nu = 4/3$ is the critical exponent related with the correlation length. For $p < p_c$, the number of cutting bonds scales with $L^{d_{CB}}$. A crossover between the two regimes with the system size is observed (stars) for p in the neighborhood of p_c. Systems of size L^2 have been considered, with L ranging from 8 to 512. Results have been averaged over 10^3 samples for most system sizes and over 50 samples for the largest one. Error bars are smaller than the symbol size. The inset shows the number of cutting bonds, N_{CB}, as a function of occupied bonds, p, for two-dimensional lattices with L ranging from 64 to 256 (averaged over 10^2 samples). The scaling is given by Eq. (1) in the article, where the argument is then $(p_c - p)L^{\theta}$, with $\theta = 0.93$. We obtain $\zeta = 0.56 \pm 0.08$, which is compatible with the one for bridge bonds.

UNIVERSALITY OF THE FRACTAL DIMENSION OF BRIDGE BONDS

In Fig. 2 one sees results for the bridge-bond line in several different lattices, namely, square, star, triangular, and the honeycomb lattices. The data points for bridge sites on the square lattice are also included. For all considered lattices the obtained fractal dimension of the bridge line is 1.215 ± 0.005, a strong evidence for the universality of this fractal dimension.

CROSSOVER SCALING IN $3D$

Figure 3 shows the crossover scaling for the number of bridge bonds N_{BB} on a simple-cubic lattice with different sizes. Bonds are occupied except for the bridge bonds. We applied the scaling function proposed in Eq. (1) of the article with $\theta = 1.36$, giving $\zeta = 1.0 \pm 0.1$.

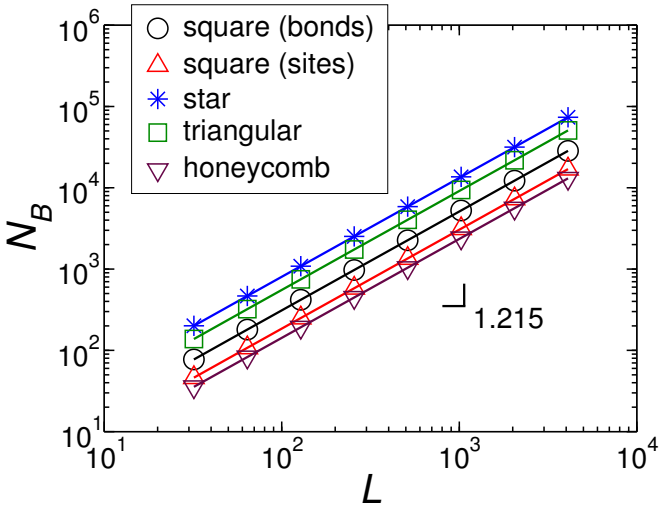

FIG. 2. Size dependence of the number of bridges, N_B, on several different lattices. The solid lines are guides to the eye, all with slope 1.215. The best fit to each data set gives the same fractal dimension of 1.215 ± 0.005. Systems of linear size L have been considered, with L ranging from 32 to 4096. Results have been averaged over 10^5 samples.

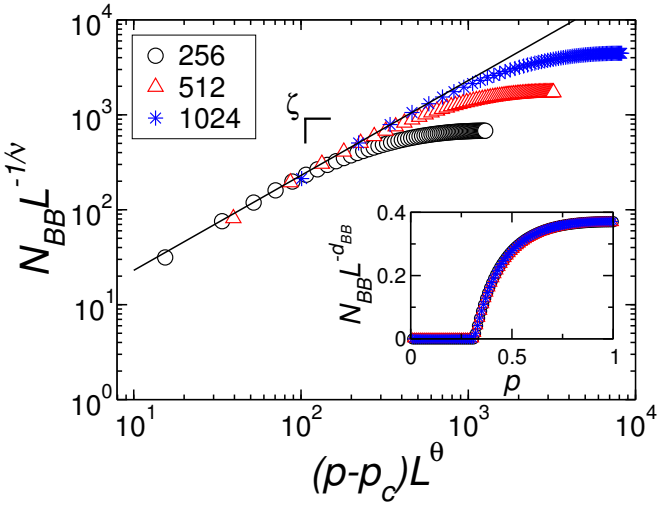

FIG. 3. Number of bridge bonds, N_{BB}, as a function of the fraction of occupied bonds, p, for $3D$ (simple-cubic lattices) with different sizes $L = \{256, 512, 1024\}$. The scaling function proposed in this article is applied, with $\theta = 1.36$, obtaining $\zeta = 1.0 \pm 0.1$. In the inset, N_{BB} has been rescaled by $L^{d_{BB}}$, where $d_{BB} = 2.50$. All results have been averaged over 10^4 samples.

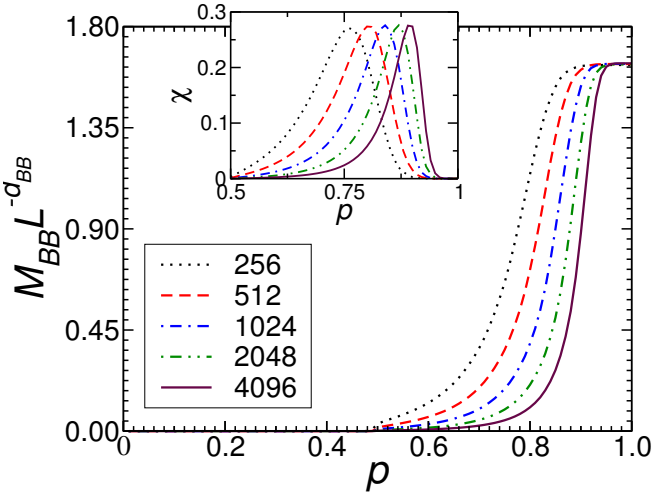

FIG. 4. Size of the largest cluster of sites on the edge of bridge bonds, M_{BB}, rescaled by $L^{d_{BB}}$, as a function of the fraction of occupied bonds p. The inset shows χ defined by Eq. (1). Results have been obtained on a square lattice with L ranging from 256 to 4096. All results have been averaged over 10^4 samples.

PERCOLATION OF BRIDGES

As discussed in the article, the set of bridge bonds merges towards a single connected line. In Fig. 4 we see the size of the largest cluster of sites on the edge of bridge bonds M_{BB}, rescaled by $L^{d_{BB}}$, as a function of p, for different system sizes. The inset shows χ, defined as,

$$\chi = \frac{1}{L^{2d_{BB}}} \left[\sum_i s_i^2 - M_{BB}^2 \right], \qquad (1)$$

where the sum runs over all clusters of sites on the edge of bridge bonds and s_i is their size. Alike percolation on a line, in the thermodynamic limit, the bridge-bond line only percolates at $p \to 1$.