SUPPLEMENTARY INFORMATION

Towards coherent spin precession in pure-spin current

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Nonlocal resistance in a lateral spin valve with dual injectors

To characterize the spin accumulation generated by nonlocal spin injection method, one employs one-dimensional spin diffusion equation[26]

$$\nabla^2(\sigma^{\uparrow}\mu^{\uparrow}+\sigma^{\downarrow}\mu^{\downarrow})=0\,, \tag{S1}$$

$$\nabla^2(\mu^{\uparrow}-\mu^{\downarrow})=\lambda^{-2}(\mu^{\uparrow}-\mu^{\downarrow})\,, \tag{S2}$$

where σ^{σ} is the spin-dependent conductivity, μ^{σ} is the spin-dependent electrochemical potential (ECP), upper script σ is the spin channel and λ is the spin diffusion length. When the current I is applied to the injector, the solutions for ECPs of the ferromagnetic (FM) and nonmagnetic (N) wires satisfy Eqs. (S1) and (S2). The ECP of the N wire in dual injector lateral spin valves (DLSV), as shown in Fig. S1, is $\mu_{\mathrm{N}}^{\sigma}(x)=\bar{\mu}_{\mathrm{N}}+\sigma\delta\mu_{\mathrm{N}}$, where $\bar{\mu}_{\mathrm{N}}=\left(eI/\sigma_{\mathrm{N}}A_{\mathrm{N}}\right)x$ for $x<0$, $\bar{\mu}_{\mathrm{N}}=0$ for $x>0$, $\sigma=+(-)$ for up (down) spin, $\delta\mu_{\mathrm{N}}=a_1e^{-|x+d_{12}|/\lambda_{\mathrm{N}}}+a_2e^{-|x|/\lambda_{\mathrm{N}}}+a_3e^{-|x-L|/\lambda_{\mathrm{N}}}$, a_1, a_2 and a_3 are the coefficient determining the ECP shifts due to the spin injection/absorption from FM1, FM2 and FM3, respectively, d_{12} and L are respectively the center-center separation between FM1-FM2 and FM2-FM3, and A_{N} is the cross-sectional area for the spin current. The ECP of the F wire is $\mu_{\mathrm{F}}^{\sigma}(z)=\bar{\mu}_{\mathrm{F}}+\sigma\delta\mu_{\mathrm{F}}$, where $\bar{\mu}_{\mathrm{F}}=(eI/\sigma_{\mathrm{F}}A_{\mathrm{J}})z+eV_1$ for FM1, $\bar{\mu}_{\mathrm{F}}=-(eI/\sigma_{\mathrm{F}}A_{\mathrm{J}})z+eV_2$ for FM2, $\bar{\mu}_{\mathrm{F}}=eV_3$ for FM3, $\delta\mu_{\mathrm{F}}=b_{\mathrm{j}}\left(\sigma_{\mathrm{F}}/\sigma_{\mathrm{F}}^{\sigma}\right)e^{-|z|/\lambda_{\mathrm{F}}}$ (j = 1, 2, 3 for FM1, FM2 and FM3), A_{J} is the junction area

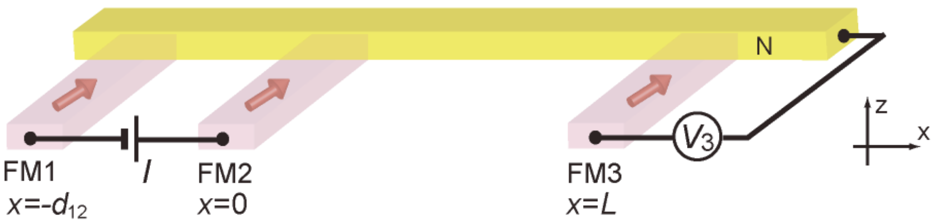

Figure S1 | Nonlocal spin injection measurement in lateral spin valve with dual injectors.

and V_1, V_2 and V_3 are the voltage drops at the FM/N interface. The injected spin-dependent current I_{I}^{σ} across the interface is given by $I_{\mathrm{I}}^{\sigma} = \left(G_{\mathrm{I}}^{\sigma}/e\right)\left(\mu_{\mathrm{F}}^{\sigma}\big|_{z=0^+} - \mu_{\mathrm{N}}^{\sigma}\big|_{z=0^-}\right)$, where G_{I}^{σ} is the interface conductance ($G_{\mathrm{I}}^{\uparrow} + G_{\mathrm{I}}^{\downarrow} = R_{\mathrm{I}}^{-1}$) and R_{I} is the interface resistance, in the frame work of the current-perpendicular-to-plane giant magneto-resistance (CPP-GMR) theory[29]. The polarization of ferromagnet and interface are $P_{\mathrm{F}} = \left|\sigma_{\mathrm{F}}^{\uparrow} - \sigma_{\mathrm{F}}^{\downarrow}\right| / \left(\sigma_{\mathrm{F}}^{\uparrow} + \sigma_{\mathrm{F}}^{\downarrow}\right)$ and $P_{\mathrm{I}} = \left|G_{\mathrm{I}}^{\uparrow} - G_{\mathrm{I}}^{\downarrow}\right| / \left(G_{\mathrm{I}}^{\uparrow} + G_{\mathrm{I}}^{\downarrow}\right)$, respectively.

The boundary condition is determined by a conservation of spin and charge current $j^{\uparrow,\downarrow} = -(\sigma^{\uparrow,\downarrow}/e)\nabla\mu^{\uparrow,\downarrow}$, and then we can deduce a_{i} and output voltage V_3 at the detector as follows

$$\begin{pmatrix} (r_{\mathrm{I1}} + r_{\mathrm{F1}} + 2) & -(r_{\mathrm{I1}} + r_{\mathrm{F1}} - 2)e^{-d_{12}/\lambda_{\mathrm{N}}} & -(r_{\mathrm{I1}} + r_{\mathrm{F1}} - 2)e^{(-d_{12}-L)/\lambda_{\mathrm{N}}} \\ e^{-d_{12}/\lambda_{\mathrm{N}}} & (1 + r_{\mathrm{I2}} + r_{\mathrm{F2}}) & e^{-L/\lambda_{\mathrm{N}}} \\ e^{(-L-d_{12})/\lambda_{\mathrm{N}}} & e^{-L/\lambda_{\mathrm{N}}} & (1 + r_{\mathrm{I3}} + r_{\mathrm{F3}}) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -\cos\theta_{13}\left(P_{\mathrm{I1}} r_{\mathrm{I1}} + P_{\mathrm{F1}} r_{\mathrm{F1}}\right) \\ \frac{1}{2}\cos\theta_{23}\left(P_{\mathrm{I2}} r_{\mathrm{I2}} + P_{\mathrm{F2}} r_{\mathrm{F2}}\right) \\ 0 \end{pmatrix} eIR_{\mathrm{N}}, \quad \text{(S3)}$$

$$eV_3 = -(P_{\mathrm{I3}} r_{\mathrm{I3}} + P_{\mathrm{F3}} r_{\mathrm{F3}})a_3, \quad \text{(S4)}$$

where $\cos\theta_{13} = \mathbf{e}^{\mathrm{FM1}} \bullet \mathbf{e}^{\mathrm{FM3}}$ and $\cos\theta_{23} = \mathbf{e}^{\mathrm{FM2}} \bullet \mathbf{e}^{\mathrm{FM3}}$ with the unit vector $\mathbf{e}^{\mathrm{FMj}}$ in the direction of the magnetization, $r_{\mathrm{Fj}} = [2/(1-P_{\mathrm{Fj}}^{\ 2})]R_{\mathrm{Fj}}/R_{\mathrm{N}}$ and $r_{\mathrm{Ij}} = [2/(1-P_{\mathrm{Ij}}^{\ 2})]R_{\mathrm{Ij}}/R_{\mathrm{N}}$ are respectively the normalized spin resistance of FM and the normalized interface resistance, P_{Fj} and P_{Ij} are respectively the spin polarization of jth FM and interface, $R_{\mathrm{N}} = \rho_{\mathrm{N}}\lambda_{\mathrm{N}}/A_{\mathrm{N}}$ and $R_{\mathrm{F}} = \rho_{\mathrm{F}}\lambda_{\mathrm{F}}/A_{\mathrm{J}}$ are respectively the spin resistance of N and FM, and ρ is the resistivity.

It should be mentioned that V_3 is decreased by the different angle θ between the accumulated spins **S** in N and the spin polarization **P** of the ferromagnet because the spins only in parallel to **P** are detected in the nonlocal measurement, as can be seen in Fig. S2 and Eq. (S3). The detected spin signal V_3/I is calculated as

$$\frac{V_3}{I}=\frac{R_{\mathrm{N}}}{2}(P_{\mathrm{I3}}r_{\mathrm{I3}}+P_{\mathrm{F3}}r_{\mathrm{F3}})e^{-L/\lambda_{\mathrm{N}}}\times \frac{(2+r_{\mathrm{I1}}+r_{\mathrm{F1}})(P_{\mathrm{I2}}r_{\mathrm{I2}}+P_{\mathrm{F2}}r_{\mathrm{F2}})\cos\theta_{23}-2e^{-d_{12}/\lambda_{\mathrm{N}}}(r_{2}+r_{\mathrm{F2}})(P_{\mathrm{I1}}r_{\mathrm{I1}}+P_{\mathrm{F1}}r_{\mathrm{F1}})\cos\theta_{13}+e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I1}}+r_{\mathrm{F1}})(P_{\mathrm{I2}}r_{\mathrm{I2}}+P_{\mathrm{F2}}r_{\mathrm{F2}})\cos\theta_{23}}{(2+r_{\mathrm{I1}}+r_{\mathrm{F1}})\left[(1+r_{\mathrm{I2}}+r_{\mathrm{F2}})(1+r_{\mathrm{I3}}+r_{\mathrm{F3}})-e^{-2L/\lambda_{\mathrm{N}}}\right]+e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I1}}+r_{\mathrm{F1}})\left[e^{-2L/\lambda_{\mathrm{N}}}(-1+r_{\mathrm{I2}}+r_{\mathrm{F2}})+(1+r_{\mathrm{I3}}+r_{\mathrm{F3}})\right]}. \tag{S5}$$

The overall change of the spin valve signal ΔR_{S} in the in-plane magnetic field dependence of V_3/I is expressed as

$$\begin{aligned}\Delta R_{\mathrm{S}}&=\left(V_3\big|_{\cos\theta_{13}=-1,\cos\theta_{23}=+1}-V_3\big|_{\cos\theta_{13}=+1,\cos\theta_{23}=-1}\right)/I\\&=R_{\mathrm{N}}(P_{\mathrm{F3}}r_{\mathrm{F3}}+P_{\mathrm{I3}}r_{\mathrm{I3}})e^{-L/\lambda_{\mathrm{N}}}\times\\&\frac{(2+r_{\mathrm{I1}}+r_{\mathrm{F1}})(P_{\mathrm{I2}}r_{\mathrm{I2}}+P_{\mathrm{F2}}r_{\mathrm{F2}})+2e^{-d_{12}/\lambda_{\mathrm{N}}}(r_{\mathrm{I2}}+r_{\mathrm{F2}})(P_{\mathrm{I1}}r_{\mathrm{I1}}+P_{\mathrm{F1}}r_{\mathrm{F1}})+e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I1}}+r_{\mathrm{F1}})(P_{\mathrm{I2}}r_{\mathrm{I2}}+P_{\mathrm{F2}}r_{\mathrm{F2}})}{(2+r_{\mathrm{I1}}+r_{\mathrm{F1}})\left[(1+r_{\mathrm{I2}}+r_{\mathrm{F2}})(1+r_{\mathrm{I3}}+r_{\mathrm{F3}})-e^{-2L/\lambda_{\mathrm{N}}}\right]+e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I1}}+r_{\mathrm{F1}})\left[e^{-2L/\lambda_{\mathrm{N}}}(-1+r_{\mathrm{I2}}+r_{\mathrm{F2}})+(1+r_{\mathrm{I3}}+r_{\mathrm{F3}})\right]}.\end{aligned} \tag{S6}$$

Assuming that all the electrodes and their interface are identical, Eq. (S6) is expressed as

$$\Delta R_{\mathrm{S}}=R_{\mathrm{N}}e^{-L/\lambda_{\mathrm{N}}}(P_{\mathrm{F}}r_{\mathrm{F}}+P_{\mathrm{I}}r_{\mathrm{I}})^2\times \frac{(1+e^{-d_{12}/\lambda_{\mathrm{N}}})\left[(2+r_{\mathrm{I}}+r_{\mathrm{F}})+e^{-d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I}}+r_{\mathrm{F}})\right]}{(2+r_{\mathrm{I}}+r_{\mathrm{F}})\left[(1+r_{\mathrm{I}}+r_{\mathrm{F}})^2-e^{-2L/\lambda_{\mathrm{N}}}\right]+e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2+r_{\mathrm{I}}+r_{\mathrm{F}})\left[(1+r_{\mathrm{I}}+r_{\mathrm{F}})+e^{-2L/\lambda_{\mathrm{N}}}(-1+r_{\mathrm{I}}+r_{\mathrm{F}})\right]}. \tag{S7}$$

When $d_{12} >> \lambda_{\mathrm{N}}$, Eq. (S7) reduces to the previous expression of ΔR_{S} for the conventional single injector LSV (SLSV) [26],

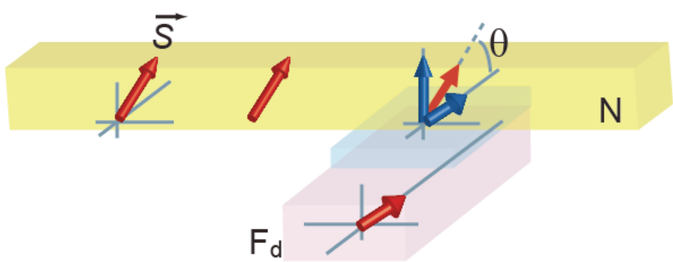

Figure S2 | Detection of accumulated spin **S** in nonmagnetic wire (N) by using ferromagnetic detector (F_d) with angle θ between its polarization **P** and **S**.

$$\Delta R_{\mathrm{S}} = R_{\mathrm{N}} e^{-L/\lambda_{\mathrm{N}}} \times \frac{(P_{\mathrm{F}} r_{\mathrm{F}} + P_{\mathrm{I}} r_{\mathrm{I}})^2}{\left(1 + r_{\mathrm{I}} + r_{\mathrm{F}}\right)^2 - e^{-2L/\lambda_{\mathrm{N}}}}. \tag{S8}$$

The enhancement factor of DLSV compared to SLSV is defined as

$$\begin{aligned}\alpha &\equiv \frac{\text{(S7)}}{\text{(S8)}} \\ &= \frac{\left[(1 + r_{\mathrm{I}} + r_{\mathrm{F}})^2 - e^{-2L/\lambda_{\mathrm{N}}}\right](1 + e^{-d_{12}/\lambda_{\mathrm{N}}})\left[(2 + r_{\mathrm{I}} + r_{\mathrm{F}}) + e^{-d_{12}/\lambda_{\mathrm{N}}}(-2 + r_{\mathrm{I}} + r_{\mathrm{F}})\right]}{(2 + r_{\mathrm{I}} + r_{\mathrm{F}})\left[(1 + r_{\mathrm{I}} + r_{\mathrm{F}})^2 - e^{-2L/\lambda_{\mathrm{N}}}\right] + e^{-2d_{12}/\lambda_{\mathrm{N}}}(-2 + r_{\mathrm{I}} + r_{\mathrm{F}})\left[(1 + r_{\mathrm{I}} + r_{\mathrm{F}}) + e^{-2L/\lambda_{\mathrm{N}}}(-1 + r_{\mathrm{I}} + r_{\mathrm{F}})\right]}.\end{aligned} \tag{S9}$$

α increases monotonically as $r_{\mathrm{I}} + r_{\mathrm{F}}$ increases and shows a maximum $\alpha_{\max} \equiv 1 + \exp(-2d_{12}/\lambda_{\mathrm{N}}) + 2\exp(-d_{12}/\lambda_{\mathrm{N}})$ for $r_{\mathrm{I}} + r_{\mathrm{F}} >> 1$.

When the interface resistance is larger enough to prevent the spin absorption effect into FM from N ($R_{\mathrm{I}} >> R_{\mathrm{N}}$), ΔR_{S} is expressed as

$$\Delta R_{\mathrm{S}} = \alpha_{\max} P_{\mathrm{I}}^2 R_{\mathrm{N}} e^{-L/\lambda_{\mathrm{N}}}. \tag{S10}$$

The spin signal is enhanced by a factor of $\alpha_{\max}$ compared to SLSV where the $1+ \exp(-2d_{12}/\lambda_{\mathrm{N}})$ and $2\exp(-d_{12}/\lambda_{\mathrm{N}})$ in $\alpha_{\max}$ come from spin injections from FM2 and FM1, respectively. The first term of 1 represents the direct diffusive spin flow to FM3 from FM2, and the next term of $\exp(-2d_{12}/\lambda_{\mathrm{N}})$ represents the flow to FM3 from FM2 via the reflection at the edge of the N wire near FM1. The last term of $2\exp(-d_{12}/\lambda_{\mathrm{N}})$ is related to spins injected from FM1.

When the magnetic field is applied perpendicular to spins in the N wire, the spin signal is modulated by not only Larmor precession but also the magnetization process, as depicted in Fig. S3. This is taken into account in Eq. (2) and assuming that $R_{\mathrm{I}} >> R_{\mathrm{N}}$ the modulated nonlocal resistance $R_{\mathrm{S}}^{\mathrm{Hanle}}$ is calculated by using Bloch-Torrey equation[22, 30]

$$\begin{aligned}R_{\mathrm{S}}^{\mathrm{Hanle}} &= \frac{1}{2} R_{\mathrm{N}} P_{\mathrm{I}}^2 \times \\ &\mathrm{Re}\left[\left(\lambda_{\omega}/\lambda_{\mathrm{N}}\right)\exp(-L/\lambda_{\omega})\left\{\cos\theta_{y2}\cos\theta_{y3} - 2\cos\theta_{y1}\cos\theta_{y3}\exp(-d_{12}/\lambda_{\omega}) + \cos\theta_{y2}\cos\theta_{y3}\exp(-2d_{12}/\lambda_{\omega})\right\}\right]\end{aligned} \tag{S11}$$

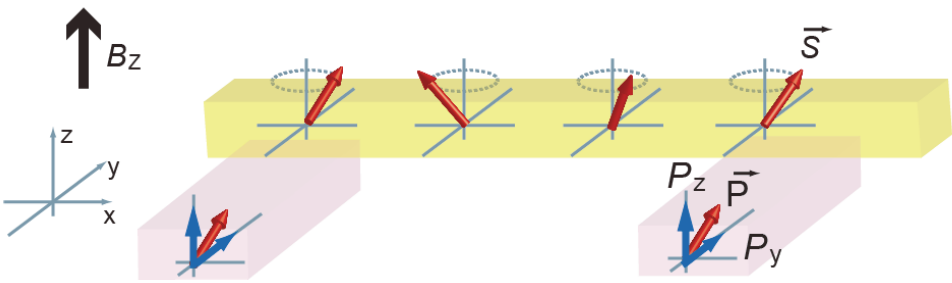

Figure S3 | Spin precession and magnetization under perpendicular magnetic field B_z. The magnetization of FMs is tilted up toward the z axis and then the polarization **P** has nonzero z component *Pz*. The spins injected into NM are also tilted up and their spins precess in the x-y plane.

and

$$\lambda_\omega = \frac{\lambda_\mathrm{N}}{\sqrt{1+i\omega_\mathrm{L}\tau_\mathrm{sf}}}, \tag{S12}$$

where ω_L is the Larmor frequency, τ_sf is the spin relaxation time, i is the imaginary number and $\cos\theta_{yj} = \mathbf{e}_\mathrm{y} \cdot \mathbf{e}^\mathrm{FMj}$. When the precession angle during the transport between FM1 and FM2 ϕ_{12} is negligible small, i.e., $d_{12} << L$, it is given by

$$R_\mathrm{S}^\mathrm{Hanle} \approx \left\{\cos\theta_{y2}\cos\theta_{y3} - 2\cos\theta_{y1}\cos\theta_{y3}\exp(-d_{12}/\lambda_\mathrm{N}) + \cos\theta_{y2}\cos\theta_{y3}\exp(-2d_{12}/\lambda_\mathrm{N})\right\}\times \frac{1}{2}R_\mathrm{N}P_\mathrm{I}^2\,\mathrm{Re}\left[\left(\lambda_\omega/\lambda_\mathrm{N}\right)\exp(-L/\lambda_\omega)\right]. \tag{S13}$$

In the anti-parallel configuration between FM1 and FM2 with $\cos\theta_{y1} = -\cos\theta_{y2}$, we obtain the simple expression,

$$R_\mathrm{S}^\mathrm{Hanle} \approx \frac{\alpha_\mathrm{max}}{2}R_\mathrm{N}P_\mathrm{I}^2\cos\theta_{y2}\cos\theta_{y3}\,\mathrm{Re}\left[\left(\lambda_\omega/\lambda_\mathrm{N}\right)\exp(-L/\lambda_\omega)\right]. \tag{S14}$$

Eqs. (S11)-(S14) can reduce to Eq. (S10) when the magnetization configuration of FM1 and FM2 is anti-parallel and B_Z=0.

Finally, we discuss the figure of merit of the coherency in the collective spin precession $\Delta R_{\mathrm{S}}^{\pi} / \Delta R_{\mathrm{S}}^{0}$, where $\Delta R_{\mathrm{S}}^{\pi}$ and $\Delta R_{\mathrm{S}}^{0}$ are respectively the amplitude of the spin valve signal after the π rotation and that before starting rotation at zero field. We can obtain $\Delta R_{\mathrm{S}}^{\pi}$ by using Eqs. (S9) and (S14), taking account of the magnetization process with the tilted angle θ_y,

$$\Delta R_{\mathrm{S}}^{\ \pi} \approx \alpha_{\max} R_{\mathrm{N}} P_{\mathrm{I}}^{2} \cos^{2} \theta_{y} \operatorname{Re}\left[\left(\lambda_{\omega} / \lambda_{\mathrm{N}} \right) \exp(-L / \lambda_{\omega}) \right], \tag{S15}$$

with $B = B_{Z}^{\pi}$ included in λ_{ω}. This expression is valid both for DLSV and SLSV. For SLSV, using $d_{12} >> \lambda_{\mathrm{N}}$, $\alpha_{\max}$ reduces to 1. Thus, when $\cos\theta_{y} = 1$, $\Delta R_{\mathrm{S}}^{\pi} / \Delta R_{\mathrm{S}}^{0}$ is simply expressed as

$$\Delta R_{\mathrm{S}}^{\ \pi} / \Delta R_{\mathrm{S}}^{\ 0} \approx \operatorname{Re}\left[\left(\lambda_{\omega} / \lambda_{\mathrm{N}} \right) \exp(-L / \lambda_{\omega}) \right] / \exp(-L / \lambda_{\mathrm{N}}), \tag{S16}$$

indicating that $\Delta R_{\mathrm{S}}^{\pi} / \Delta R_{\mathrm{S}}^{0}$ only depends on L / λ_{N} .

Additional References

29. Valet, T. & Fert, A. Theory of the perpendicular magnetoresistance in magnetic multilayers. *Phys. Rev. B* **48**, 7099-7113 (1993).

30. M. Johnson and R. H. Silsbee, Coupling of electronic charge and spin at a ferromagnetic -paramagnetic metal interface, *Phys. Rev. B* **37**, 5312 (1988).