Mechanical Resonances in Helically Coiled Carbon Nanowires

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Harmonic Detection of Resonance (HDR) under a Scanning Electron Microscope

The jig is a custom made device capable of micro-manipulating the cantilever HCNW w.r.t the W-tip to a desired orientation. The piezo-controllers enable precise alignment in X,Y & Z directions.

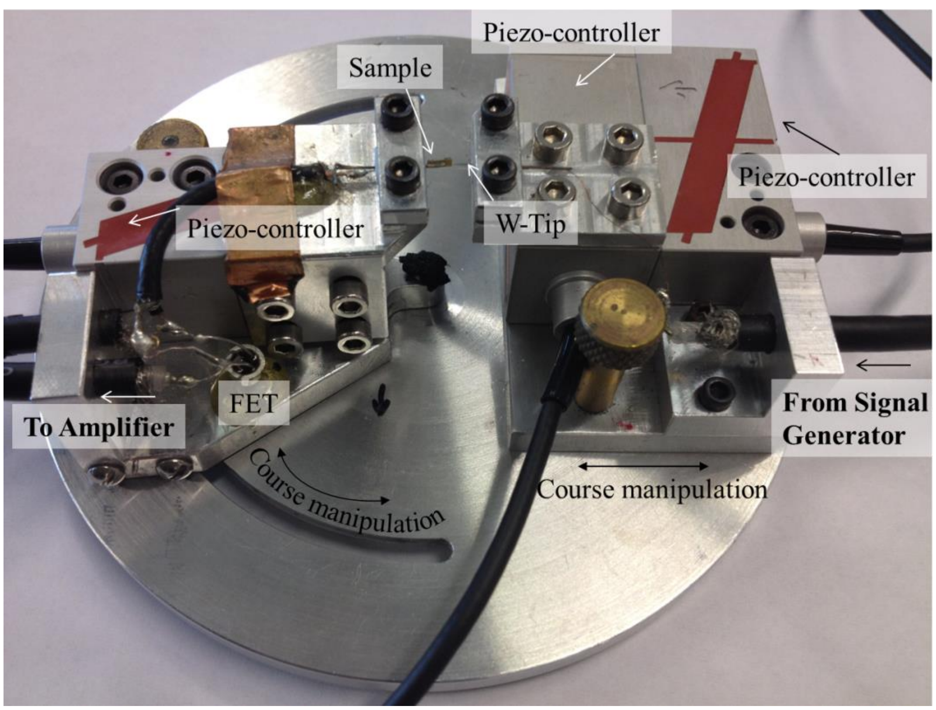

Figure S1: The jig used to hold the HCNW-W Tip in the SEM. Voltages to the jig are controlled though the BNC connectors on the SEM chamber.

Detailed description of the Analytical model

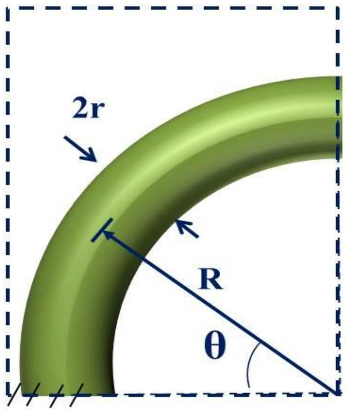

Figure S2: Dimensions of a quarter of a turn of HCNW

Wahl[1] showed that the deflection angle φ_q of a quarter turn of the coil is a sum of the deformation due to bending (φ_b) and torsional strain (φ_t) under moment M. The component of φ due to the bending strain (φ_b) is

$$\varphi_b = \frac{1}{EI}\int_0^{\pi/2} M\sin\theta \frac{D}{2} d\theta \sin\theta = \frac{MD}{2EI}\int_0^{\pi/2} \sin^2\theta d\theta = \frac{\pi MD}{8EI} \quad (1),$$

where I is the area moment of inertia, $M\sin\theta$ is the component of M along the axis of the wire, and the additional $\sin\theta$ is the ratio of φ to the component of bending in the plane of the wire, and θ is the angle along the quarter turn of the coil. Similarly the component of φ due to the torsional strain (φ_t) due to the moment M is:

$$\varphi_t = \frac{1}{GJ}\int_0^{\pi/2} M\cos\theta \frac{D}{2} d\theta \cos\theta = \frac{MD}{2GJ}\int_0^{\pi/2} \sin^2\theta d\theta = \frac{\pi MD}{8GJ} \quad (2),$$

where J is the polar moment of inertia, $M\cos\theta$ is the component of the moment M along the axis of the wire, and the additional $\cos\theta$ is the ratio of φ to the component of bending in the plane of the wire.

At larger pitch angles, the bending and twisting components of the HCNW become convoluted i.e. the twisting portions experience an additional bending. Hence, to compensate for this effect, we introduce a factor $\cos^2\alpha$ to the denominator of φ_b *(for increased bending)* and to the numerator of φ_t (for reduced twisting). Thus the equations (1) and (2) become

$$\varphi_b = \frac{\pi MD}{8EI\cos^2\alpha} \;;\; \varphi_t = \frac{\pi MD\cos^2\alpha}{8JG} \quad (3).$$

Thus, for a complete turn the total deflection angle is

$$\varphi = 4\varphi_q = 4(\varphi_b + \varphi_t) = \frac{\pi MD}{2EI\cos^2\alpha} + \frac{\pi MD}{2EI\cos^2\alpha} \qquad (4),$$

where the first term is bending ($4\varphi_b$) and the second term corresponds to shear deformation ($4\varphi_t$).

The spring constant *k* for a complete turn of the coil is defined as *F/z* where *F* is the force acting on a single turn, and *z* is the displacement resulting from *F*. *F* is related to the moment by *F* = *M/p*, where *p* is the pitch of coil. From Fig. 3d one can see that

$$\mathrm{z} = \xi - \xi\cos\phi \qquad (5).$$

Using the small angle approximation, $p = \xi\phi$ (6).

Using equations (5) and (6) we obtain the following

$$z = \frac{p}{\phi}(1 - \cos\phi).$$

The series expansion of $\cos\phi = 1 - \frac{1}{2}\phi^2 + \frac{1}{24}\phi^4 - ...$

Under small angle approximation we neglect the fourth and higher order terms. Thus we get

$z = \frac{p}{\phi}\left(\frac{\phi^2}{2}\right)$, that gives

$$z = p\left(\frac{\phi}{2}\right) \qquad (7).$$

Combining these relations we obtain:

$$\frac{1}{k} = \frac{zp}{M} = \frac{\varphi}{M}\left(\frac{p}{2}^2\right) \qquad (8).$$

On substituting *φ (equation 3)* in equation (8), we get tensile and shear spring constants for one turn.

$$\frac{1}{k_b} = \frac{p^2 D}{Er^4\cos^2\phi} \qquad (9),$$

$$\frac{1}{k_t} = \frac{p^2 D \cos^2 \phi}{2Gr^4} \qquad (10).$$

Finally, to obtain the combined spring constant we use the relation for adding springs in series:

$$\frac{1}{k} = \frac{1}{k_b} + \frac{1}{k_t}$$

Note that k is inversely proportional to the cube of the length for cantilever beams. Hence, an n^3 term was incorporated to accommodate the total number of the turns in the coil. The relation between number of coils n and k was verified using COMSOL as follows.

We modeled an HCNW in COMSOL and systematically varied n (and thus the length), keeping other geometrical parameters constant. We then calculated the mass (m) of this COMSOL generated HCNW by calculating its volume within COMSOL and multiplying it with input material density. We obtained f for each of these HCNWS. Using the formula

$f=1/2\pi(k/m_{eff})^{1/2}$, m_{eff} being the effective mass, we were able to calculate the corresponding k.

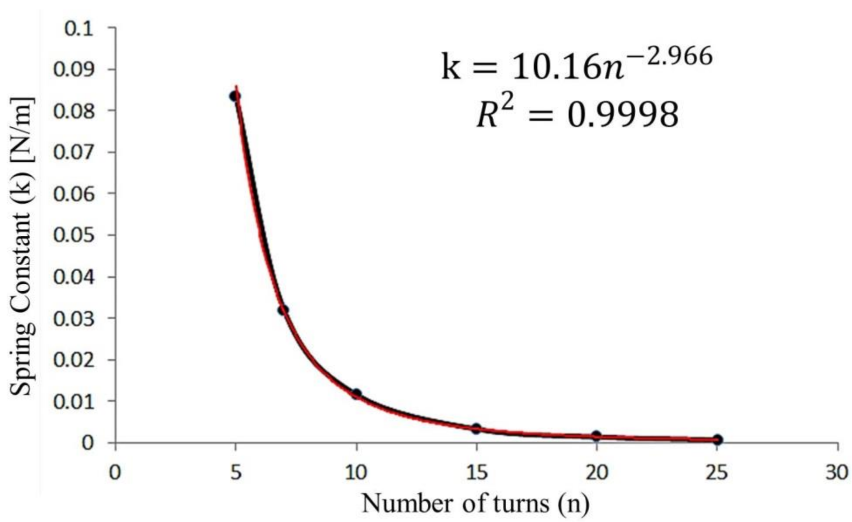

Figure S3: Spring Constant k Vs number of turns n plot.

Thus, we get

$$\frac{n^3}{k_{turn}} = n^3\left[\frac{1}{k_b} + \frac{1}{k_t}\right] = \frac{1}{k_n}$$

$$\frac{1}{k_n} = \frac{n^3 p^2 D}{Er^4 \cos^2 \phi} + \frac{n^3 p^2 D \cos^2 \phi}{2Gr^4} \qquad (11).$$

The model was validated using COMSOL and was scaled by a factor of s =1.28 to compensate for the assumptions in the model.

As we know, $f = \frac{1}{2\pi}\sqrt{\frac{k_n}{m_{eff}}}$, thus analytical resonance frequency f_{ANA} is given by

$$f_{ANA} = \frac{1}{2\pi}\frac{r^2}{p}\left(1.28n^3Dm_{eff}\left[\frac{\cos^2\alpha}{2G}+\frac{1}{E\cos^2\alpha}\right]\right)^{-1/2}.$$

This closed form expression yields an expression for f with 12% accuracy as compared the COMSOL simulated resonance frequency.

Circularly polarized resonance

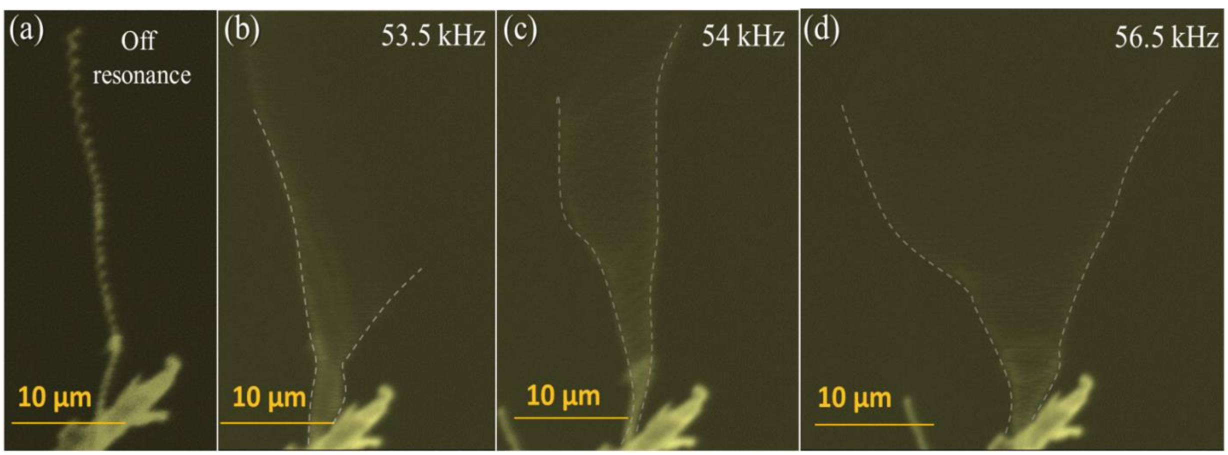

Figure S4: The circular resonance behavior can be seen in HCNW3 (wire radius 103 nm; diameter 330 nm and pitch 1.014 μm) in (a) off resonance, (b) Ω = 53.5 kHz, (c) Ω = 54 kHz, and (d) Ω = 56.5 kHz, where Ω is the driving frequency. On applying relatively higher driving voltages, the nanocoil actuates in an in-plane transverse mode. On further sweeping the driving frequency, its motion is transformed into an elliptical mode (c) which becomes close to a circular motion when it hits the resonance frequency.

COMSOL Simulation results

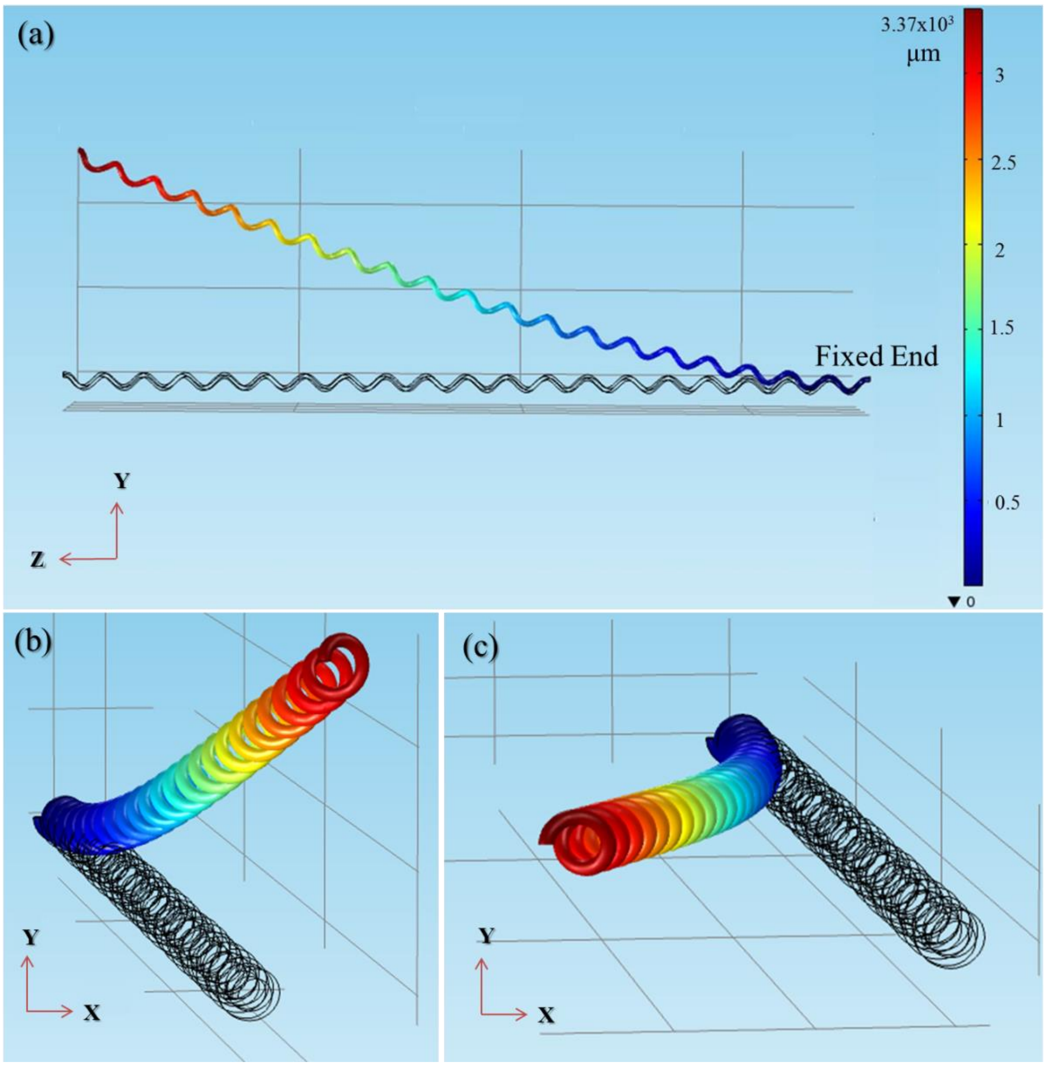

Figure S5: COMSOL Simulated plots of representative helical coil geometry. (a) Off resonance (b) First transverse mode –Y polarization (b) First transverse mode- X Polarization.

References

1 Wahl, A. M. *Mechanical Springs*. (McGraw-Hill Book Company, 1963).