Concurrent enhancement of percolation and synchronization in adaptive networks (Supplementary Information)

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I. CASES OF DIFFERENT TIME SCALE BETWEEN OSCILLATORS' DYNAMICS AND STRUCTURAL EVOLUTION

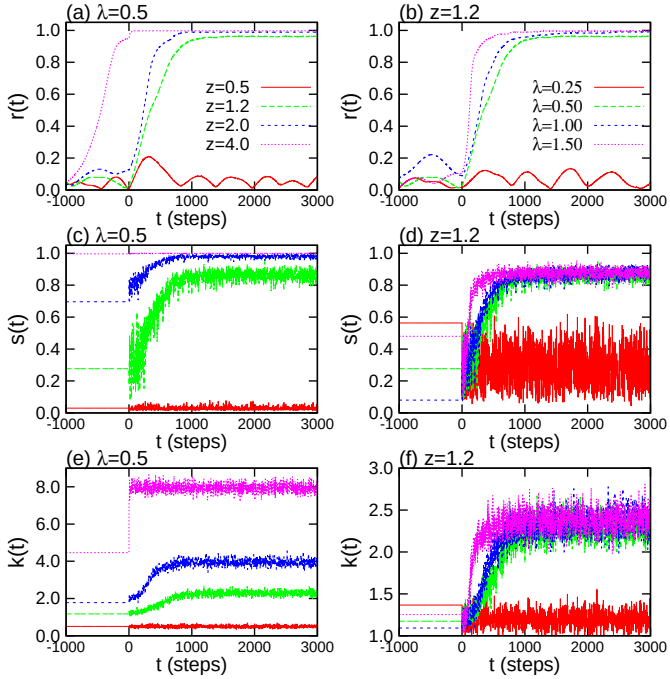

FIG. 1. Time evolution of $r(t)$ (a-b), $s(t)$ (c-d) and of the network's average degree $k(t)$ (e-f) for $P = 0.5$ and $N = 300$. (a, c, and e) $\lambda = 0.5$; (b, d, and f) $z = 1.2$. Color codes in the legends of (a) and (b).

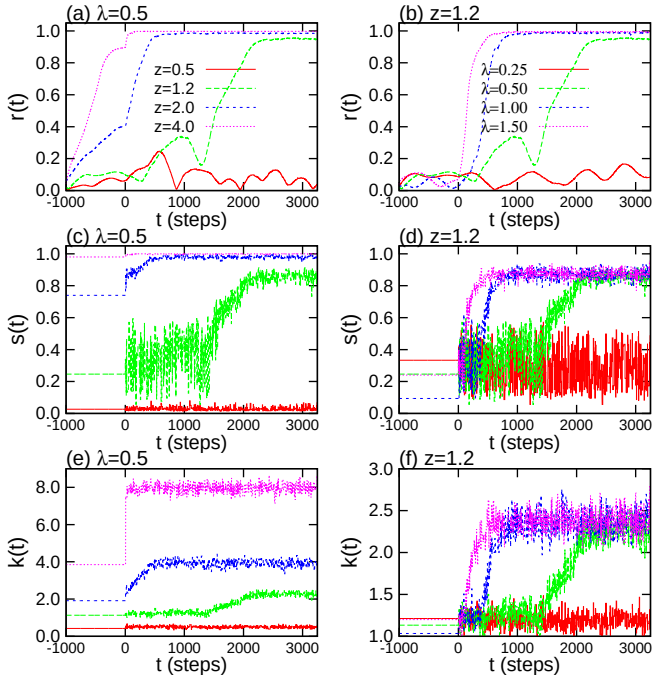

FIG. 2. Time evolution of $r(t)$ (a-b), $s(t)$ (c-d) and of the network's average degree $k(t)$ (e-f) for $P = 0.2$ and $N = 300$. (a, c, and e) $\lambda = 0.5$; (b, d, and f) $z = 1.2$. Color codes in the legends of (a) and (b).

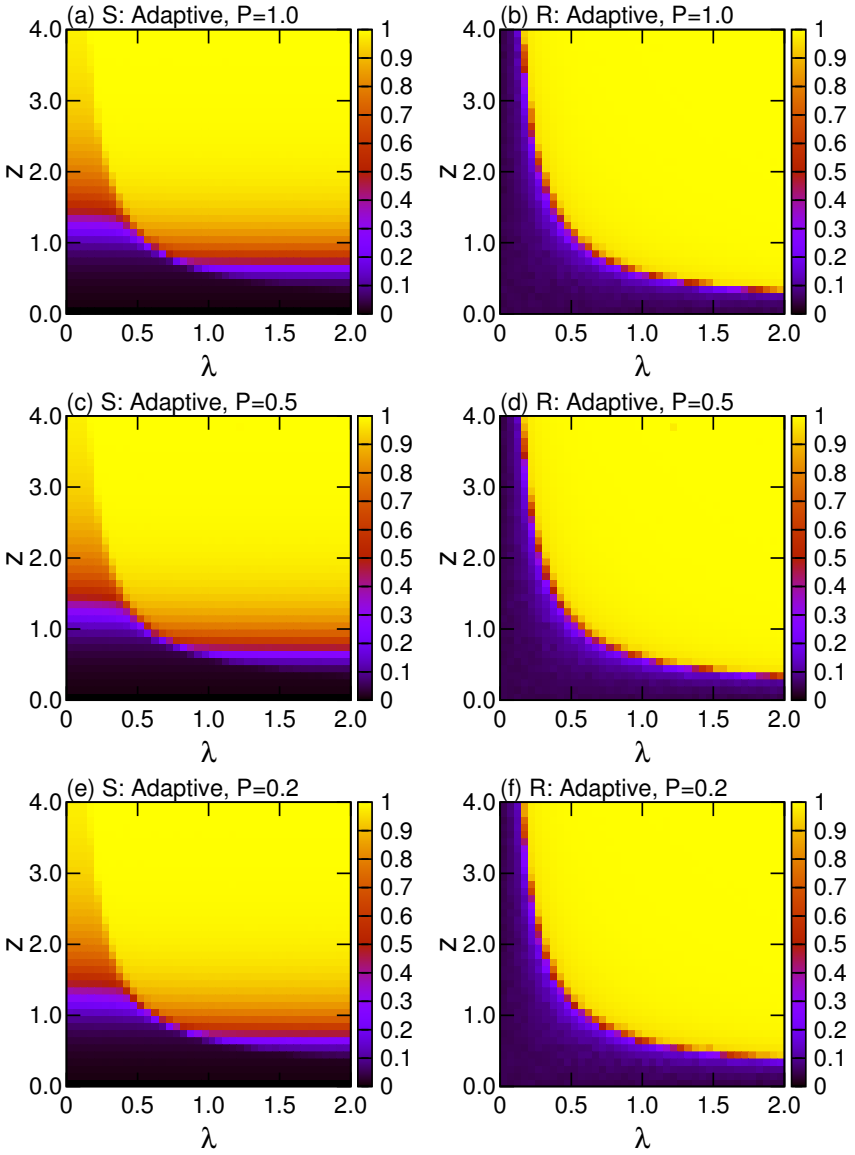

FIG. 3. Phase diagrams of adaptive models with $N = 300$ and varying P ($P = 1.0$, 0.5, and 0.2). Panels refer to the percolation indicator S (a,c,e) and the synchronization indicator R (b,d,f). For each z and λ, data refer to ensemble averages over 40 different realizations.

II. CASES OF DIFFERENT SYSTEM SIZES

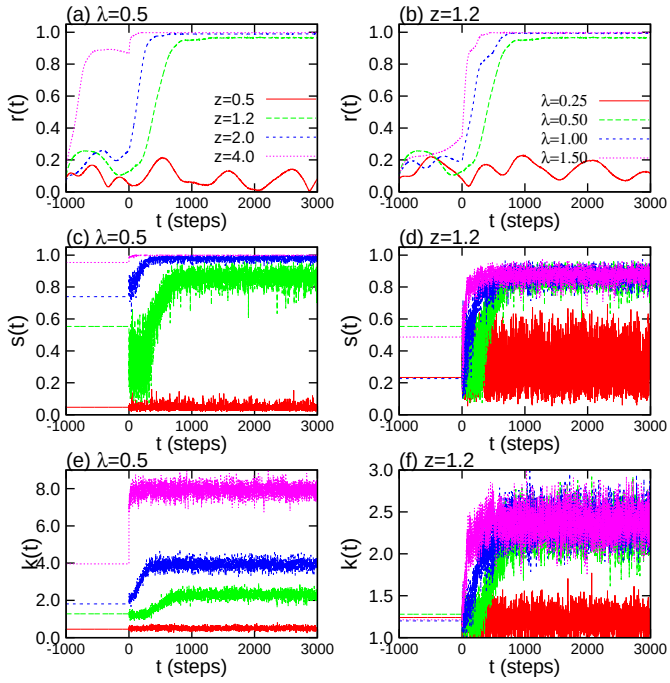

FIG. 4. Time evolution of $r(t)$ (a-b), $s(t)$ (c-d) and of the network's average degree $k(t)$ (e-f) for $P = 1$ and $N = 150$. (a, c, and e) $\lambda = 0.5$; (b, d, and f) $z = 1.2$. Color codes in the legends of (a) and (b).

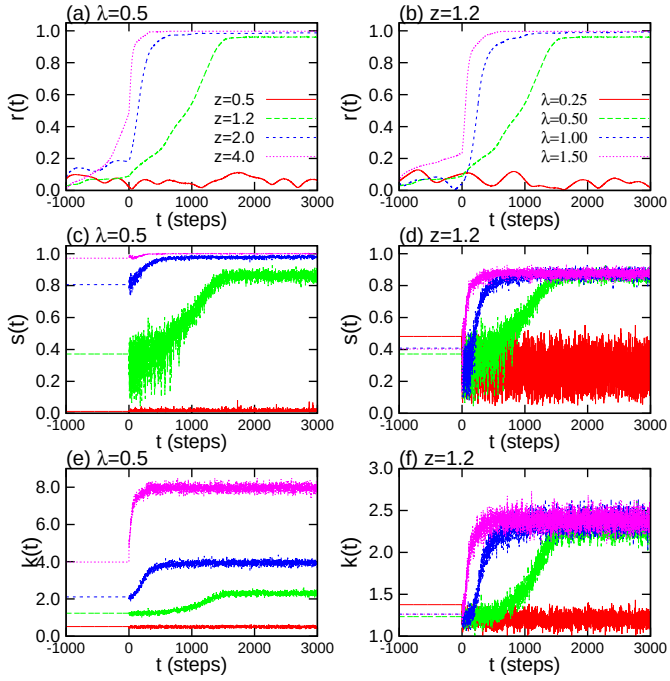

FIG. 5. Time evolution of $r(t)$ (a-b), $s(t)$ (c-d) and of the network's average degree $k(t)$ (e-f) for $P = 1$ and $N = 600$. (a, c, and e) $\lambda = 0.5$; (b, d, and f) $z = 1.2$. Color codes in the legends of (a) and (b).

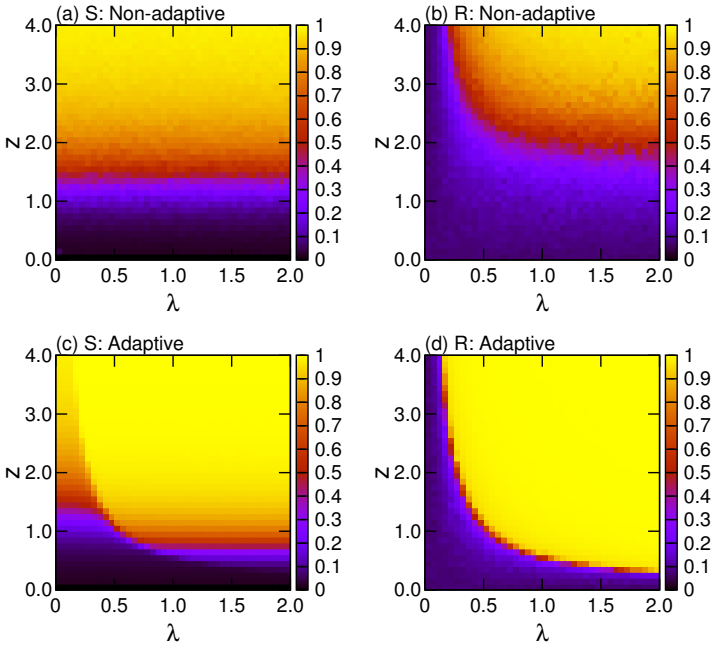

FIG. 6. Phase diagrams of the non adaptive (a,b) and adaptive(c,d) models with $N = 150$ and $P = 1$. Panels refer to the percolation indicator S (a,c) and the synchronization indicator R (b,d). For each z and λ, data refer to ensemble averages over 40 different realizations.

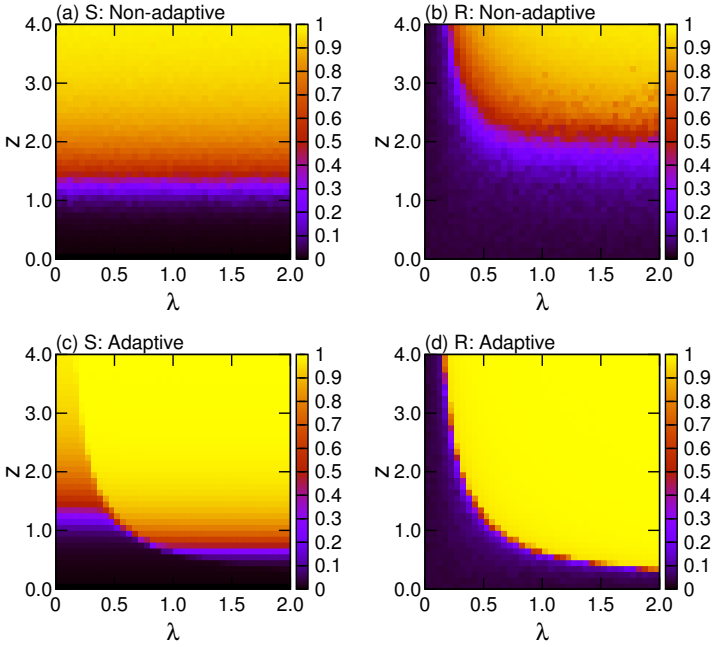

FIG. 7. Phase diagrams of the non adaptive (a,b) and adaptive(c,d) models with $N = 600$ and $P = 1$. Panels refer to the percolation indicator S (a,c) and the synchronization indicator R (b,d). For each z and λ, data refer to ensemble averages over 20 different realizations.

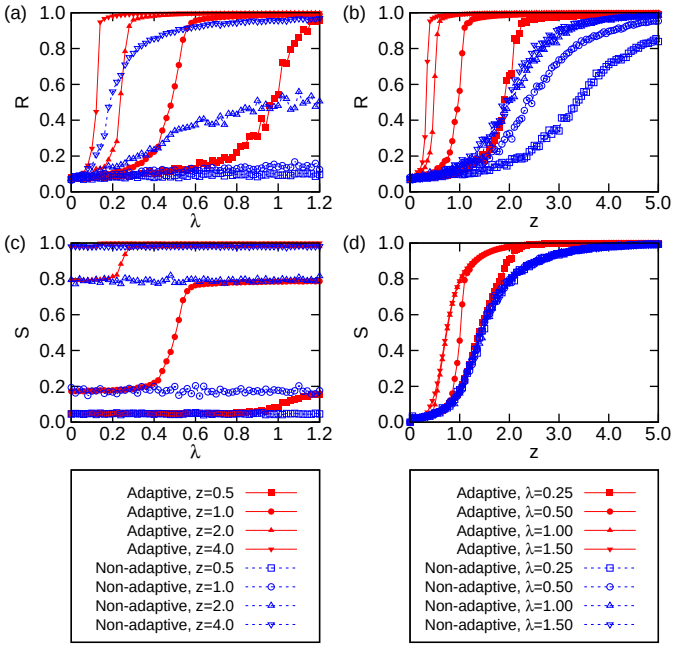

FIG. 8. R (a,b) and S (c,d) for adaptive and non-adaptive networks with $N = 150$ and $P = 1.0$. (a) R *vs.* λ at different z values; (b) R *vs.* z at different λ values; (c) S *vs.* λ at different z values; (d) S *vs.* z at different λ values. Legends (in the bottom panels) have to be referred to for the understanding of the used parameters' values. Data refer to ensemble averages over 40 realizations.

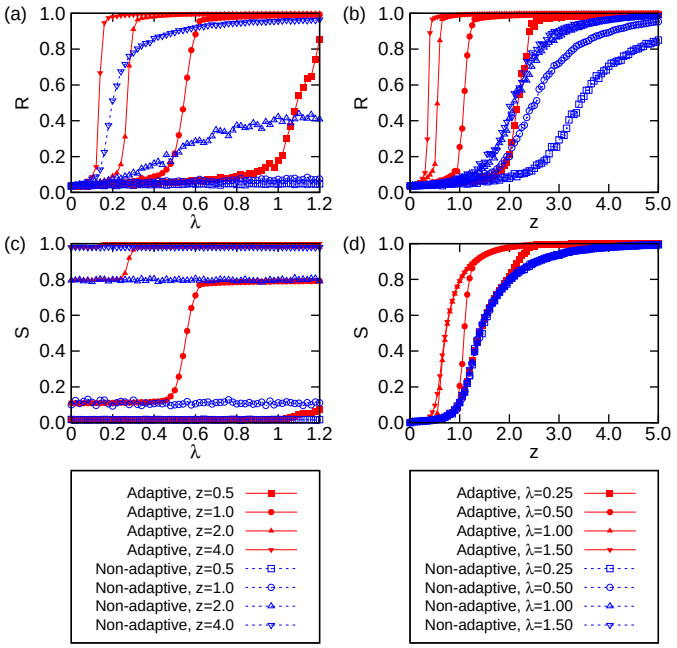

FIG. 9. R (a,b) and S (c,d) for adaptive and non-adaptive networks with $N = 600$ and $P = 1.0$. (a) R *vs.* λ at different z values; (b) R *vs.* z at different λ values; (c) S *vs.* λ at different z values; (d) S *vs.* z at different λ values. Legends (in the bottom panels) have to be referred to for the understanding of the used parameters' values. Data refer to ensemble averages over 40 realizations.

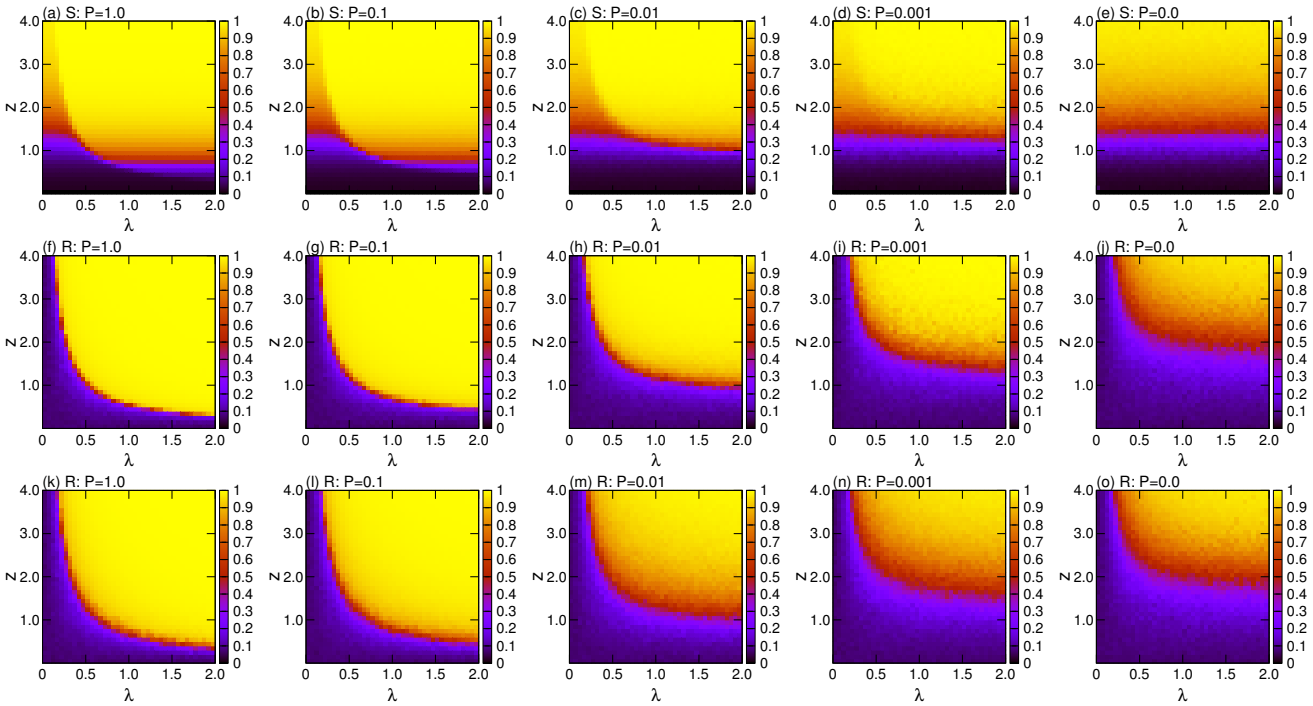

FIG. 10. S (top row) and R (middle row) in the parameter space (z, λ) for the adaptive network of size $N = 150$ with different coupling probability P. Bottom row reports, instead, R (in the same parameter space) for blinking networks with different coupling probability P. Once again, data refer to ensemble average over 40 different realizations for each z and λ.

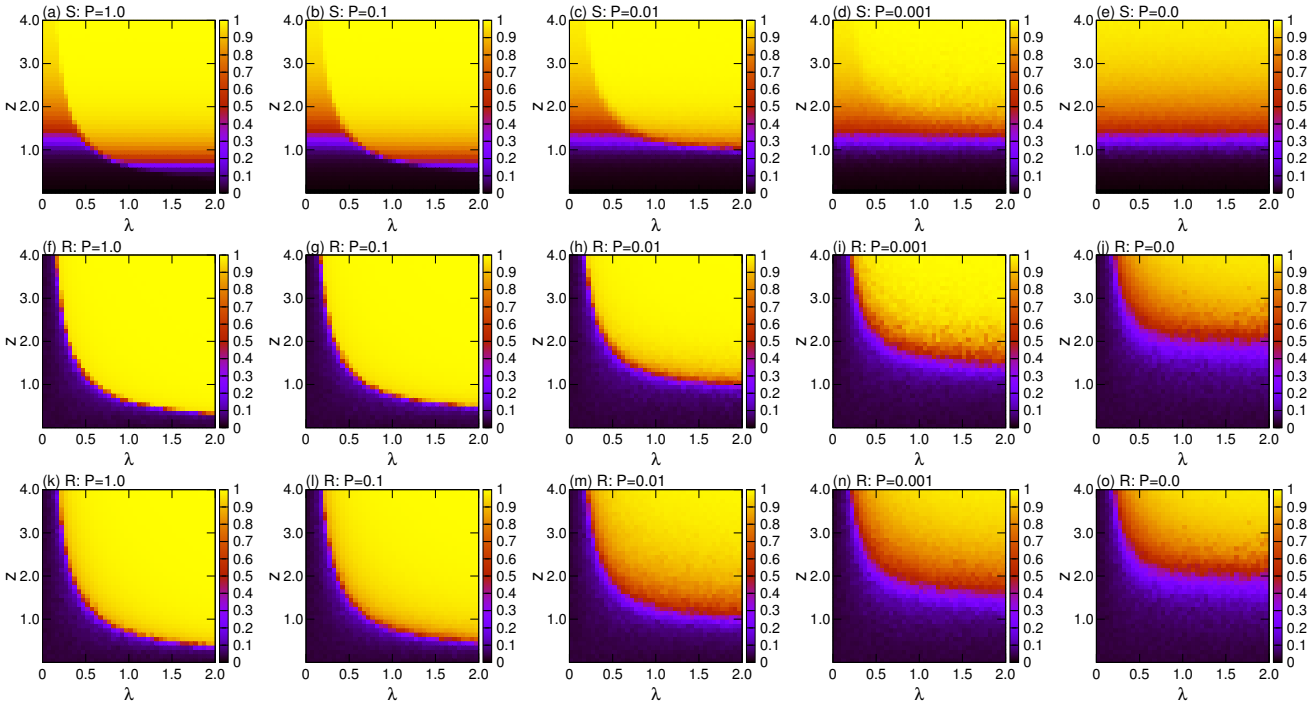

FIG. 11. S (top row) and R (middle row) in the parameter space (z, λ) for the adaptive network of size $N = 600$ with different coupling probability P. Bottom row reports, instead, R (in the same parameter space) for blinking networks with different coupling probability P. Once again, data refer to ensemble average over 20 different realizations for each z and λ.

III. CASES OF DIFFERENT STEP-SIZES IN RUNGE-KUTTA METHOD

So far, all the results in the main manuscript and in the supplementary information are obtained from Runge-Kutta method with step-size $\Delta t = 0.02$. We also consider the cases of different step-sizes of $\Delta t = 0.05$ and 0.1 for comparison.

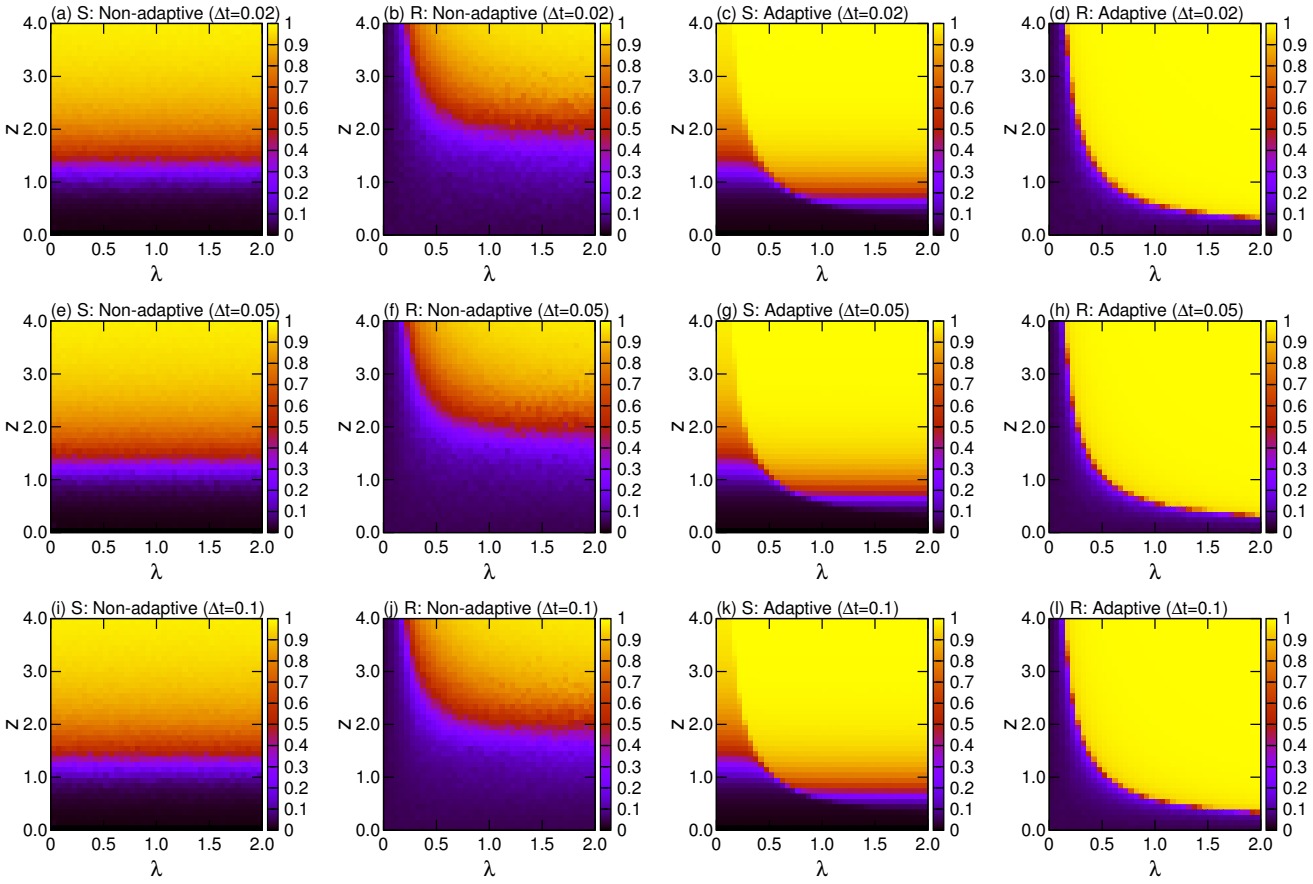

FIG. 12. Phase diagrams of the non-adaptive (a,b,e,f,i,j) and adaptive(c,d,g,h,k,l) models. Panels refer to the percolation indicator S (a,c,e,g,i,k) and the synchronization indicator R (b,d,f,h,j,l). Panels from (a) to (d) are from simulations with $\Delta t = 0.02$. Panels from (e) to (h) are from simulations with $\Delta t = 0.05$. Panels from (i) to (l) are from simulations with $\Delta t = 0.10$. For each z and l, data refer to ensemble averages over 40 different realizations