

# Supplementary Material for : Interplay between market structure changes and future volatility outbursts

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## S1 Forecasting performance of $\langle z \rangle(T_a)$

In this section we present further analyses on  $z(T_a, T_b)$ , namely  $P^+$  and ROC. From Tabs. S1-S2 we can see that  $\langle z \rangle(T_a)$  has almost always an higher  $P^+$  than  $\langle ES \rangle(T_a)$  in the NYSE dataset, whereas on average it has lower values than  $\langle ES \rangle(T_a)$  in the LSE dataset (with 7 out of 16 values of  $P^+$  greater than the corresponding  $\langle ES \rangle(T_a)$  probabilities). On the other hand, the ROC analysis in Fig. S1 and Tabs. S3 - S4 shows that the predictor  $\langle ES \rangle(T_a)$  performs better than  $\langle z \rangle(T_a)$  in the NYSE dataset and worse in the LSE dataset. Fig. S2 shows  $\langle z \rangle(T_a)$  performances in time.

Table S1: **NYSE dataset: Probability of successful forecasting  $P^+$  using  $\langle z \rangle(T_a)$  as a predictor  $\langle z \rangle(T_a)$  or  $\langle z(T_a, T_b) \rangle$** , for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis.

|          |      | L       |         |         |         |
|----------|------|---------|---------|---------|---------|
|          |      | 10      | 25      | 50      | 100     |
| $\theta$ | 250  | 0.592*  | 0.587** | 0.632** | 0.652** |
|          | 500  | 0.732** | 0.707** | 0.670** | 0.671** |
|          | 750  | 0.786** | 0.736** | 0.697** | 0.759** |
|          | 1000 | 0.886** | 0.882** | 0.861** | 0.860   |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

Table S2: **LSE dataset: Probability of successful forecasting  $P^+$  using  $\langle z \rangle(T_a)$  as a predictor**, for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis.

|          |      | L       |         |        |         |
|----------|------|---------|---------|--------|---------|
|          |      | 10      | 25      | 50     | 100     |
| $\theta$ | 250  | 0.723** | 0.342   | 0.598* | 0.522   |
|          | 500  | 0.701** | 0.650** | 0.621* | 0.589   |
|          | 750  | 0.674** | 0.464   | 0.256  | 0.666** |
|          | 1000 | 0.236   | 0.245   | 0.441  | 0.689** |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

Table S3: **NYSE dataset: Area under the curve (AUC) using  $\langle z \rangle(T_a)$  as a predictor**, measured from the ROC curve in Fig. S1. Values greater than 0.5 indicate that the classifier performs better than chance.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.803 | 0.700 | 0.625 | 0.496 |
|          | 500  | 0.834 | 0.761 | 0.639 | 0.406 |
|          | 750  | 0.633 | 0.549 | 0.436 | 0.135 |
|          | 1000 | 0.413 | 0.393 | 0.352 | 0.107 |

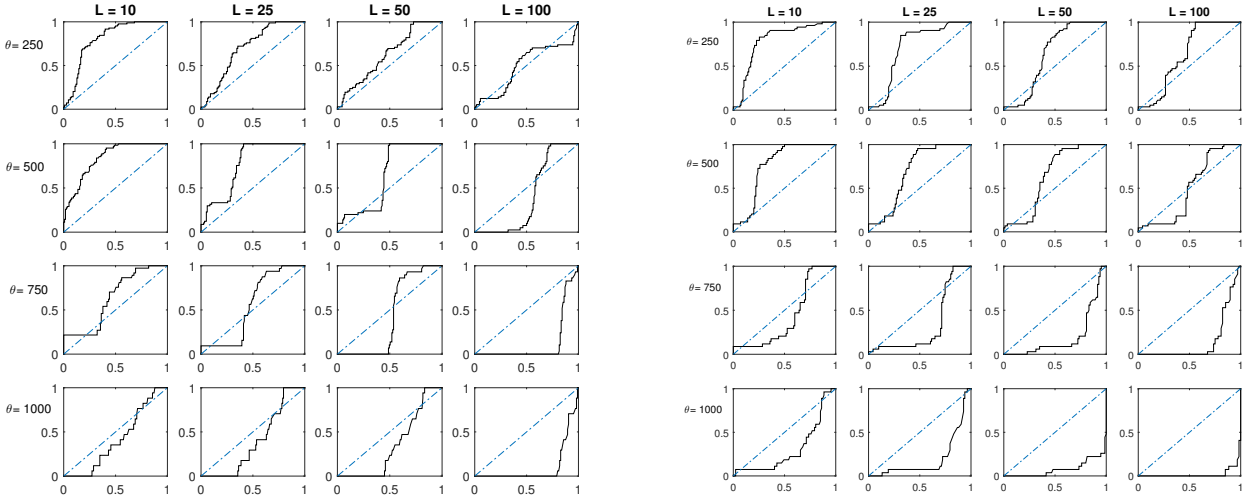


Figure S1: **Receiver operating characteristic (ROC) curve for the NYSE and LSE dataset, by using  $\langle z \rangle(T_a)$  as a predictor.** True positive rate (TPR) against False positive rate (FPR) as the discriminant threshold  $p_{max}$  of the classifier is varied, for each combination of parameters  $\theta$  and  $L$  in the NYSE dataset. The closer the curve is to the upper left corner of each graph, the better is the classifier compared to chance.

Table S4: **LSE dataset: Area under the curve (AUC) using  $\langle z \rangle(T_a)$  as a predictor,** measured from the ROC curve in Fig. S1. Values greater than 0.5 indicate that the classifier performs better than chance.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.790 | 0.714 | 0.637 | 0.636 |
|          | 500  | 0.765 | 0.694 | 0.638 | 0.496 |
|          | 750  | 0.434 | 0.338 | 0.200 | 0.150 |
|          | 1000 | 0.309 | 0.200 | 0.079 | 0.022 |

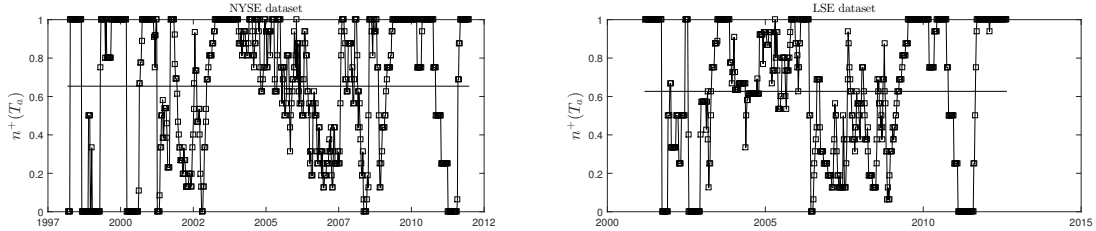


Figure S2: **Fraction of successful predictions as a function of time, using  $\langle z \rangle(T_a)$  as a predictor.** NYSE (left graph) and LSE dataset (right graph). Horizontal lines represent the average over the entire period.

## S2 Forecasting performance of $q(T_a)$

In this section we report some further analyses on the performances of  $q(T_a)$  as a predictor in the logistic regression.

In Fig. S3 we show the autocorrelation of  $q(T_a)$  for different lags and different values of  $\theta$ . As we can see,  $q(T_a)$  displays a strong autocorrelation.

In Tabs S5-S6 we report the probabilities  $P^+$  obtained by using  $q(T_a)$  as a predictor in the logistic regression. These values are the null probabilities which have been used in Tabs. 5-6 of the paper to compute the p-values.

In Fig. S4 the ROC curves for  $q(T_a)$  as a predictor are shown, whereas in Tabs S7 - S8 we report the corresponding AUC values. As we can see  $q(T_a)$  performs quite poorly in terms of ROC analysis as well, underperforming the random predictor for almost any choice of the parameters. Specifically, in terms of AUC the  $\langle ES \rangle(T_a)$  predictor outperforms  $q(T_a)$  for all the combinations of parameters in the NYSE dataset, whereas in the LSE dataset this is true in 8 out of 16 combinations of parameters: the rest are those parameters combination for which both  $\langle ES \rangle(T_a)$  and  $q(T_a)$  underperform the random predictor.

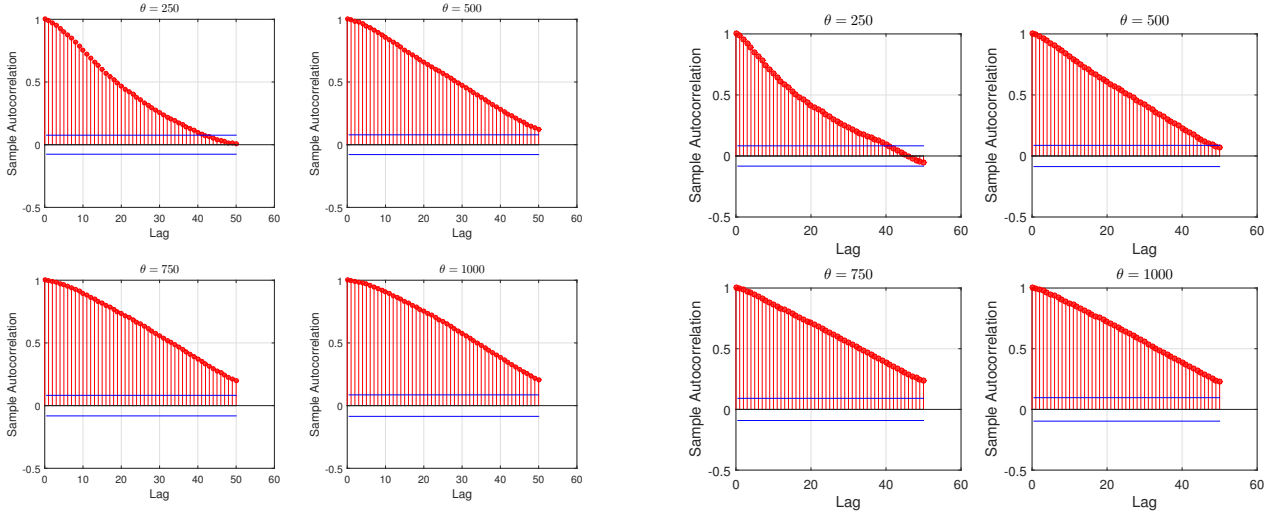


Figure S3: Autocorrelation of  $q(T_a)$  at different lags, for each value of  $\theta$ . Left panel: NYSE dataset. Right panel: LSE dataset.

Table S5: **NYSE dataset: Probability of successful forecasting  $P^+$  using  $q(T_a)$  as a predictor**, for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.487 | 0.468 | 0.420 | 0.373 |
|          | 500  | 0.455 | 0.433 | 0.403 | 0.417 |
|          | 750  | 0.509 | 0.493 | 0.503 | 0.602 |
|          | 1000 | 0.594 | 0.643 | 0.691 | 0.785 |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

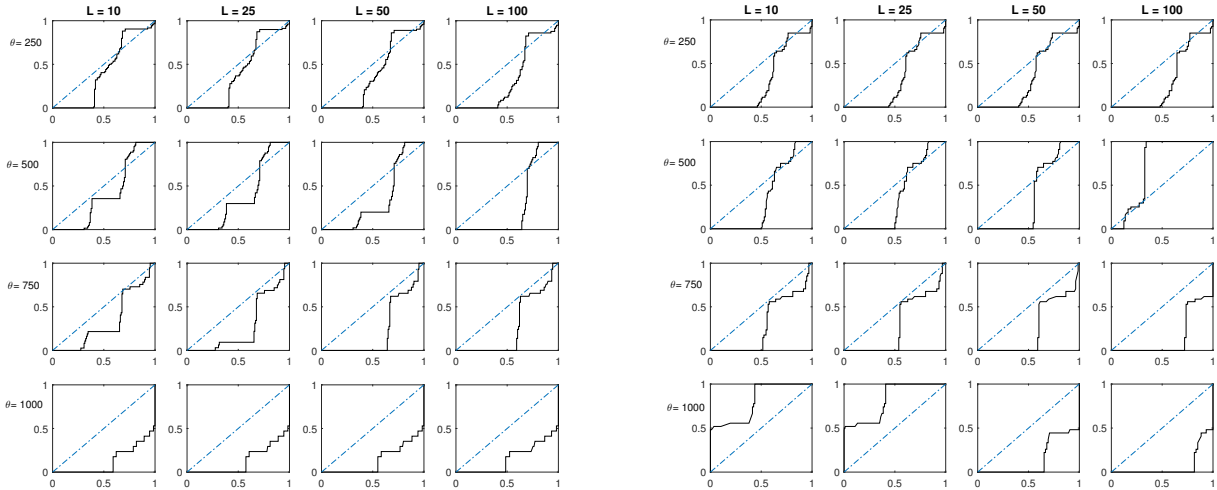


Figure S4: **Receiver operating characteristic (ROC) curve for the NYSE and LSE dataset, by using  $q(T_a)$  as a predictor**. True positive rate (TPR) against False positive rate (FPR) as the discriminant threshold  $p_{max}$  of the classifier is varied, for each combination of parameters  $\theta$  and  $L$  in the NYSE dataset. The closer the curve is to the upper left corner of each graph, the better is the classifier compared to chance.

Table S6: **LSE dataset: Probability of successful forecasting  $P^+$  using  $q(T_a)$  as a predictor**, for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.373 | 0.357 | 0.438 | 0.404 |
|          | 500  | 0.412 | 0.388 | 0.335 | 0.379 |
|          | 750  | 0.406 | 0.282 | 0.293 | 0.326 |
|          | 1000 | 0.239 | 0.247 | 0.237 | 0.686 |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

Table S7: **NYSE dataset: Area under the curve (AUC) using  $q(T_a)$  as a predictor**, measured from the ROC curve in Fig. S4. Values greater than 0.5 indicate that the classifier performs better than chance.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.421 | 0.410 | 0.393 | 0.344 |
|          | 500  | 0.406 | 0.386 | 0.353 | 0.294 |
|          | 750  | 0.336 | 0.283 | 0.248 | 0.282 |
|          | 1000 | 0.128 | 0.133 | 0.141 | 0.160 |

Table S8: **LSE dataset: Area under the curve (AUC) using  $q(T_a)$  as a predictor**, measured from the ROC curve in Fig. S4. Values greater than 0.5 indicate that the classifier performs better than chance.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.323 | 0.335 | 0.363 | 0.305 |
|          | 500  | 0.362 | 0.376 | 0.378 | 0.719 |
|          | 750  | 0.300 | 0.302 | 0.257 | 0.157 |
|          | 1000 | 0.806 | 0.821 | 0.150 | 0.074 |

Finally, in Tabs S9-S10 we report the  $P^+$  probabilities obtained by using a weighted average version of  $q(T_a)$ ,  $\langle q \rangle(T_a)$ , as predictor. As we can see the values are very close to those in Tabs S5-S6 obtained by using simply  $q(T_a)$  as predictor; overall the performance of  $q(T_a)$  is not improved. We interpret this as a consequence of the strong autocorrelation of  $q(T_a)$ , which makes the average  $\langle q \rangle(T_a)$  very close to  $q(T_a)$ , and therefore the two predictors very similar.

Table S9: **NYSE dataset: Probability of successful forecasting  $P^+$  using  $\langle q \rangle(T_a)$  as a predictor**, for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis.

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.481 | 0.441 | 0.419 | 0.373 |
|          | 500  | 0.449 | 0.427 | 0.415 | 0.364 |
|          | 750  | 0.509 | 0.493 | 0.509 | 0.426 |
|          | 1000 | 0.601 | 0.643 | 0.654 | 0.231 |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

Table S10: **LSE dataset: Probability of successful forecasting  $P^+$  using  $\langle q(T_a) \rangle$  as a predictor**, for different combinations of parameters  $\theta$  and  $L$ . Out-of-sample analysis. 7-8 for ES

|          |      | L     |       |       |       |
|----------|------|-------|-------|-------|-------|
|          |      | 10    | 25    | 50    | 100   |
| $\theta$ | 250  | 0.367 | 0.389 | 0.363 | 0.564 |
|          | 500  | 0.328 | 0.316 | 0.335 | 0.448 |
|          | 750  | 0.242 | 0.274 | 0.293 | 0.663 |
|          | 1000 | 0.239 | 0.247 | 0.267 | 0.686 |

\*\*  $p < 0.001$ , \*  $p < 0.01$ ,

### S3 Relation between $\langle ES \rangle(T_a)$ and $\sigma(T_a)$

In Fig. S5 we report the scatter plot of  $\sigma(T_a)$  against  $\langle ES \rangle(T_a)$ . As we can see, no clear pattern appears which could explain the relationship between  $\langle ES \rangle(T_a)$  and  $q(T_a)$  reported in the paper.

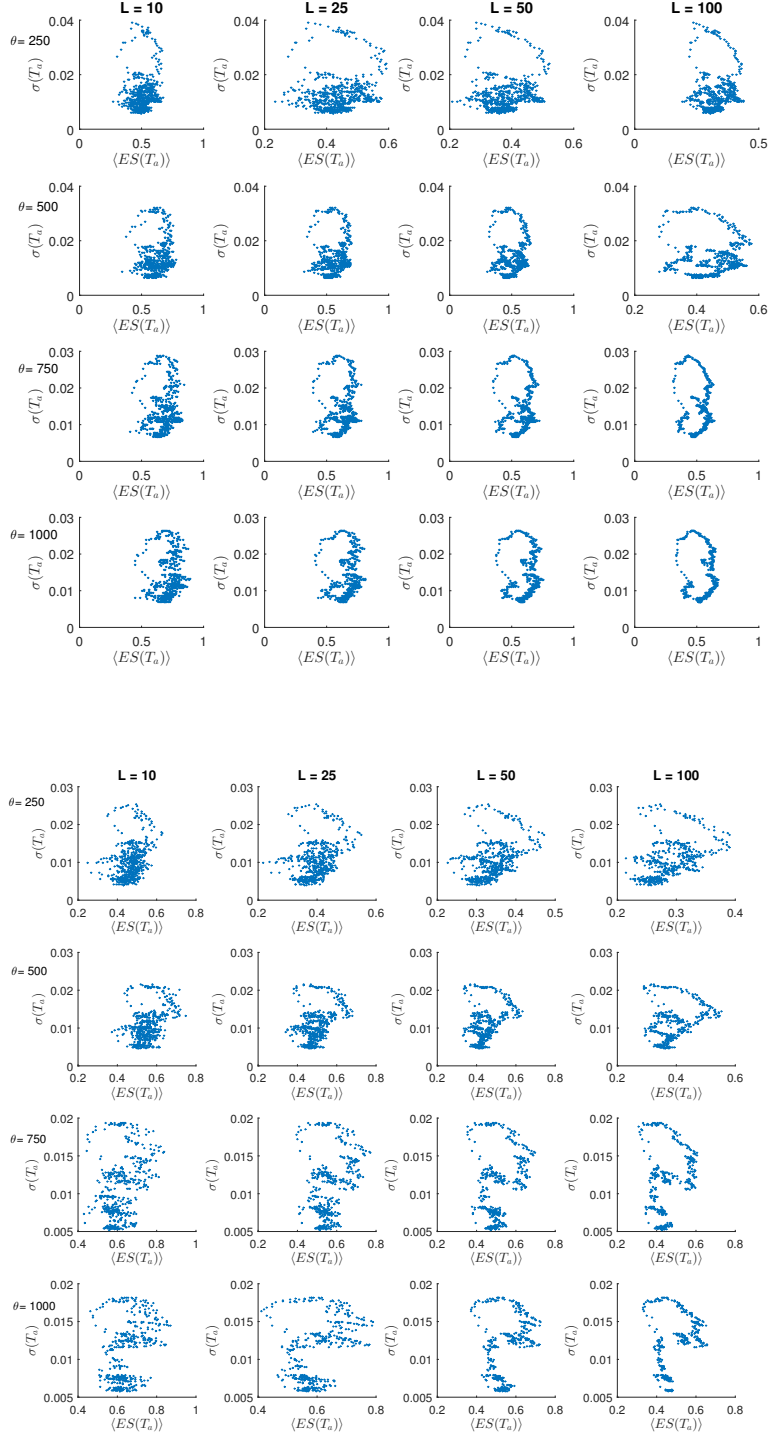


Figure S5: Scatter plot of  $\sigma(T_a)$  against  $\langle ES \rangle(T_a)$ , for each combination of parameters  $L$  and  $\theta$ , for the NYSE (left) and LSE (right) datasets.