Appendix

A: The Social Planner's Problem

Let us define the monetary competitive general equilibrium of the Economy described in section 2:

Definition 1 (Monetary Competitive General Equilibrium) *A value of the initial money supplied by the Central Bank M_0, a sequence of values for the interest rate margin $\{m_t\}_{t=0}^{\infty}$ the financial sector works with, a sequence of money supply increases $\{G_t\}_{t=0}^{\infty}$, a sequence of legal reserves coefficients $\{s_t\}_{t=0}^{\infty}$, sequences of prices $\{\hat{p}_t\}_{t=0}^{\infty}$, $\{\hat{w}_t\}_{t=0}^{\infty}$, $\{\hat{i}_t\}_{t=0}^{\infty}$, and $\{\hat{r}_t^P\}_{t=0}^{\infty}$, sequences of household's decisions $\{\hat{C}_t\}_{t=0}^{\infty}$, $\{\hat{M}_t^D\}_{t=0}^{\infty}$, $\{\hat{D}_t^S\}_{t=0}^{\infty}$, and $\{\hat{l}_t^S\}_{t=0}^{\infty}$, sequences of financial sector's decisions $\{\hat{K}_t^P\}_{t=0}^{\infty}$, $\{\hat{K}_t^F\}_{t=0}^{\infty}$, and $\{\hat{D}_t^D\}_{t=0}^{\infty}$, sequences of production sector's decisions $\{\hat{K}_t^P\}_{t=0}^{\infty}$, and $\{\hat{l}_t^D\}_{t=0}^{\infty}$, and sequences of percentages of money supply becoming productive capital $\{\hat{\rho}_t\}_{t=0}^{\infty}$, such that:*

- *Given the sequences of prices $\{\hat{p}_t\}_{t=0}^{\infty}$, $\{\hat{w}_t\}_{t=0}^{\infty}$, and $\{\hat{i}_t\}_{t=0}^{\infty}$, the sequences of household's decisions $\{\hat{C}_t\}_{t=0}^{\infty}$, $\{\hat{M}_t^D\}_{t=0}^{\infty}$, $\{\hat{D}_t^S\}_{t=0}^{\infty}$, and $\{\hat{l}_t^S\}_{t=0}^{\infty}$, solve the consumer's problem (1).*
- *Given the sequences of prices $\{\hat{p}_t\}_{t=0}^{\infty}$, $\{\hat{r}_t^P\}_{t=0}^{\infty}$, $\{\hat{i}_t\}_{t=0}^{\infty}$, the sequence of legal reserves coefficients $\{s_t\}_{t=0}^{\infty}$, the sequences of margins $\{m_t\}_{t=0}^{\infty}$, and the sequence of percentages of money supply becoming productive capital $\{\hat{\rho}_t\}_{t=0}^{\infty}$, the sequences of the financial sector's decisions $\{\hat{K}_t^P\}_{t=0}^{\infty}$, $\{\hat{K}_t^F\}_{t=0}^{\infty}$, and $\{\hat{D}_t^D\}_{t=0}^{\infty}$ solve the financial sector's problem (2) and verify the zero profits condition (3).*
- *Given the sequences of prices $\{\hat{p}_t\}_{t=0}^{\infty}$, $\{\hat{r}_t^P\}_{t=0}^{\infty}$, and $\{\hat{w}_t\}_{t=0}^{\infty}$, the sequences of the production sector's decisions $\{\hat{K}_t^P\}_{t=0}^{\infty}$, and $\{\hat{l}_t^S\}_{t=0}^{\infty}$, solve the production sector's problem (4).*
- *Given the value of the initial money supply M_0^S, the sequence of money supply increases $\{G_t\}_{t=0}^{\infty}$, and the sequence of legal reserves coefficients $\{s_t\}_{t=0}^{\infty}$, the sequences of household's decisions, financial sector's decisions, production sector's decisions, and percentages of money supply becoming productive capital, verify the market-clearing conditions*

$$\hat{D}_t^D = \hat{D}_t^S, \qquad \hat{l}_t^D = \hat{l}_t^S, \qquad \hat{M}_t^D = M_t^S = \rho_0 M_0^S + \sum_{j=0}^{t} \rho_t G_j,$$

$$\hat{C}_t + \hat{K}_{t+1}^P + \hat{K}_{t+1}^F \leq (\hat{K}_t^P)^{\alpha} (\hat{l}_t^D)^{1-\alpha} + (\hat{K}_t^P + \hat{K}_t^F)(1-\delta),$$
$$t = 0, 1, \ldots, \infty.$$

It is an extended case of the usual definition of a monetary competitive general equilibrium. We will now prove that all the equilibrium sequences of the monetary general equilibrium can be obtained from those in the equivalent social planner's problem formulation in definition 1. Moreover, in real terms, the equilibrium sequences of the monetary general equilibrium and the equivalent social planner's problem formulation coincide.

Let us consider the representative consumer's problem (1) in the monetary equilibrium. It is obvious that the solution for this problem also has to be a solution of

$$\max_{C_t, l_t^S} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$$

$$s.t. \qquad \sum_{t=0}^{\infty} \frac{p_t}{\prod_{j=0}^{t}(1+i_j)} C_t \geq \sum_{t=0}^{\infty} \frac{p_t w_t}{\prod_{j=0}^{t}(1+i_j)} l_t^S + D_0^S,$$
$$0 \leq l_t^S \leq \overline{h}, \quad C_t \geq 0,$$
$$t = 0, 1, \ldots, \infty, \qquad D_0^S \text{ historically given.}$$

Under the standard assumptions on the Bernoulli utility function $\mathcal{U}$, if the representative agent captures the behavior of rational agents [see for instance Gutiérrez (2004)], interior solutions are ensured, and the first order necessary and sufficient conditions are

$$\beta^t \frac{d\mathcal{U}(C_t)}{dC_t} = \lambda \frac{p_t}{\prod_{j=0}^{t}(1+i_j)},$$

$$\sum_{t=0}^{\infty} \frac{p_t}{\prod_{j=0}^{t}(1+i_j)} C_t = \sum_{t=0}^{\infty} \frac{p_t w_t}{\prod_{j=0}^{t}(1+i_j)} l_t^S + D_0^S,$$

$$l_t^S = 1,$$

where λ is the Lagrange multiplier. These conditions are just those of problem

$$\max_{C_t, l_t^S} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$$

$$s.t. \sum_{t=0}^{\infty} \frac{p_t}{\prod_{j=0}^{t}(1+i_j)} C_t \leq \sum_{t=0}^{\infty} \frac{p_t w_t}{\prod_{j=0}^{t}(1+i_j)} l_t^S + D_0^S,$$

$$0 \leq l_t^S \leq \overline{h}, \quad C_t \geq 0,$$

$$t = 0, 1, \ldots, \infty, \qquad D_0^S \text{ historically given},$$

in the Arrow-Debreu equilibrium, which is equivalent to the Radner formulation

$$\max_{C_t, l_t^S, d_{t+1}^S} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$$

$$s.t. \qquad C_t + d_{t+1}^S \leq w_t l_t^S + d_t^S(1 + r_t^H - \delta),$$

$$0 \leq l_t^S \leq \overline{h}, \quad C_t, d_{t+1}^S \geq 0,$$

$$t = 0, 1, \ldots, \infty, \qquad d_0^S \text{ historically given},$$

where

$$\frac{p_t}{p_{t+1}}(1 + i_{t+1}) = (1 + r_{t+1}^H - \delta),$$

and d_t, r_t^H and δ are those defined in section 2. The interested reader can consult Mas-Colell, Whinston, and Green (1995), where an exhaustive proof is given. As the solution for this problem is unique due to the concavity properties of the Bernoulli utility function, this real formulation is equivalent -in real terms- to the representative consumer's problem in the monetary equilibrium.

Let us now consider the financial sector's problem in the monetary general equilibrium definition and the zero profits condition

$$p_t(K_t^P + K_t^F)(1 - \delta) + p_t K_t^P r_t^P = D_t^D(1 + i_t).$$

Dividing by p_t, this condition becomes

$$(K_t^P + K_t^F)(1 - \delta) + K_t^P r_t^P = \frac{1}{p_t} D_t^D (1 + i_t) =$$

$$\frac{D_t^D}{p_{t-1}} \frac{p_{t-1}}{p_t}(1 + i_t) = d_t^D(1 + r_t^H - \delta).$$

Let us now consider the right side of the constraint in the household's problem under the Radner formulation. Since in the monetary general equilibrium markets clear,

$$w_t l_t^S + d_t^S(1 + r_t^H - \delta) = w_t l_t^D + d_t^D(1 + r_t^H - \delta),$$

and substituting $d_t^D(1+r_t^H-\delta)$ according to the above result, we get

$$w_t l_t^D + d_t^D(1+r_t^H-\delta) = w_t l_t^D + (K_t^P + K_t^F)(1-\delta) + K_t^P r_t^P$$

Also, from the first order necessary conditions in the production sector's problem,

$$r_t^P = \alpha (K_t^P)^{\alpha-1}(l_t^D)^{1-\alpha}, \qquad w_t = (1-\alpha)(K_t^P)^{\alpha}(l_t^D)^{-\alpha},$$

and then

$$w_t l_t^D + K_t^P r_t^P = (K_t^P)^{\alpha}(l_t^D)^{1-\alpha},$$

and the right side of the constraint in the household's problem becomes

$$w_t l_t^D + (K_t^P + K_t^F)(1-\delta) + K_t^P r_t^P = (K_t^P)^{\alpha}(l_t^D)^{1-\alpha} + (K_t^P + K_t^F)(1-\delta).$$

Finally, given that $K_t^P + K_t^F = d_t$ and $K_t^P = (1 - s_t + \rho_t g_t)d_t$, from the household's problem in real terms, we reach the following equivalent social planner's problem

$$\max_{C_t, l_t, d_{t+1}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$$

$$\begin{aligned} s.t. \qquad & C_t + d_{t+1} \leq [(1 - s_t + \rho_t g_t)d_t]^{\alpha}(l_t)^{1-\alpha} + d_t(1-\delta), \\ & 0 \leq l_t \leq \overline{h}, \quad C_t, d_{t+1} \geq 0, \\ & t = 0, 1, \ldots, \infty, \qquad d_0 \text{ historically given.} \end{aligned}$$

Let us now define the variable $\gamma_t = (1 - s_t + \rho_t g_t)$. Then the above problem can be written

$$\max_{C_t, l_t, d_{t+1}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$$

$$\begin{aligned} s.t. \qquad & C_t + d_{t+1} \leq [\gamma_t d_t]^{\alpha}(l_t)^{1-\alpha} + d_t(1-\delta), \\ & 0 \leq l_t \leq \overline{h}, \quad C_t, d_{t+1} \geq 0, \\ & t = 0, 1, \ldots, \infty, \qquad d_0 \text{ historically given.} \end{aligned}$$

In this problem γ_t is an exogenous variable out of control, and then there exists a unique policy function providing the solution, which takes the general form $d_{t+1}(d_t, \gamma_{t+1})$. On this point, the interested reader can consult Stokey and Lucas with Prescott (1989).

Let us now consider the zero profits condition. This situation, with an evident logical meaning from the economic perspective, is the only possible logical mathematical outcome and determines the endogenous nature of ρ_t. Indeed, since $K_t^P + K_t^F = d_t^D$, $K_t^P = (1 - s_t + \rho_t g_t)d_t^D$, and $d_t^D = \frac{D_t^D}{p_{t-1}}$, it is straightforward that the financial sector problem is equivalent to

$$\max_{d_t^D} \quad d_t^D(1-\delta) + (1 - s_t + \rho_t g_t)d_t^D r_t^P - d_t^D(1+i_t)\frac{p_{t-1}}{p_t}.$$

Let us denote the inflation rate by π_t, and, for the sake of notational simplicity, express the real interest rate paid to household deposits as $r_t^H - \delta$. Then

$$(1+i_t)\frac{p_{t-1}}{p_t} = \frac{1+i_t}{1+\pi_t} = (1 + r_t^H - \delta),$$

and the financial sector profits can be written as

$$d_t^D(1-\delta) + (1 - s_t + \rho_t g_t)d_t^D r_t^P - d_t^D(1 + r_t^H - \delta) =$$

$$d_t^D[(1-\delta) + (1 - s_t + \rho_t g_t)r_t^P - (1 + r_t^H - \delta)] = d_t^D[(1 - s_t + \rho_t g_t)r_t^P - r_t^H].$$

Assuming that $d_t^D > 0$, the only logical situation,

$$d_t^D[(1 - s_t + \rho_t g_t)r_t^P - r_t^H] \gtreqless 0 \Leftrightarrow (1 - s_t + \rho_t g_t)r_t^P - r_t^H \gtreqless 0,$$

Obviously, the financial sector problem would only have a defined solution when

$$(1 - s_t + \rho_t g_t)r_t^P - r_t^H = 0,$$

equivalent to the zero profits condition

$$p_t(K_t^P + K_t^F)(1 - \delta) + p_t K_t^P r_t^P = D_t^D(1 + i_t).$$

From the zero profits condition $(1 - s_t + \rho_t g_t)r_t^P - r_t^H = 0$, we deduce the endogenous nature of ρ_t, whose equilibrium value is given by the expression

$$\rho_t = \frac{1}{g_t}[\frac{r_t^H}{r_t^P} - 1 + s_t] = \frac{1}{g_t}[\frac{r_t^P - m_t}{r_t^P} - 1 + s_t] = \frac{1}{g_t}[\frac{m_t}{r_t^P} + s_t].$$

In addition, from the zero profits condition (4), we obtain

$$(1 - s_t + \rho_t g_t)r_t^P - r_t^H = 0, \qquad \gamma_t r_t^P = r_t^H, \qquad \gamma_t = \frac{r_t^H}{r_t^P}.$$

We also know that, by definition

$$\gamma_t = \frac{K_t^P}{d_t}, \qquad d_t = K_t^P + K_t^H, \qquad K_t^H > 0,$$

we conclude that $\gamma_t < 1$ and therefore that $r_t^H < r_t^P$. If m_t is the margin fixed by the Central Bank, $r_t^H = r_t^P - m_t$, and we get

$$\gamma_t = \frac{r_t^H}{r_t^P} = \frac{r_t^P - m_t}{r_t^P} = 1 - \frac{m_t}{r_t^P}.$$

According to the production sector's problem, $r_t^P = \alpha(\gamma_t d_t)^{\alpha-1}$, and we conclude that, at the equilibrium, $\hat{\gamma}_t$ must verify

$$\hat{\gamma}_t = 1 - \frac{m_t}{\alpha(\hat{\gamma}_t \hat{d}_t)^{\alpha-1}} \quad \forall t.$$

We can easily deduce the sequences solving the equivalent social planner's problem formulation from the sequences solving the original monetary general equilibrium. As we have proved, the equivalent social planner's problem formulation provides the sequences $\{\hat{\gamma}_t\}_{t=0}^{\infty}$, $\{\hat{C}_t\}_{t=0}^{\infty}$, $\{\hat{l}_t\}_{t=0}^{\infty}$ and $\{\hat{d}_t\}_{t=0}^{\infty}$. According to the definitions of $\hat{K}_t^P$, $\hat{K}_t^F$, and $\hat{y}_t$, we obtain

$$\hat{K}_t^P = \hat{\gamma}_t \hat{d}_t, \qquad \hat{K}_t^F = \hat{d}_t(1 - \hat{\gamma}_t), \qquad \hat{y}_t = (\hat{K}_t^P)^\alpha \hat{l}_t^{1-\alpha}.$$

According to the production sector's problem,

$$\hat{w}_t = \frac{\partial \hat{y}_t}{\partial \hat{l}_t} = (1 - \alpha)(\hat{\gamma}_t \hat{d}_t)^\alpha \hat{l}_t^{-\alpha}, \qquad \hat{r}_t^P = \frac{\partial \hat{y}_t}{\partial \hat{K}_t^P} = \alpha(\hat{\gamma}_t \hat{d}_t)^{\alpha-1} \hat{l}_t^{1-\alpha}.$$

From the definition of γ_t, we obtain

$$\hat{\rho}_t = \frac{\hat{\gamma}_t + s_t - 1}{g_t}.$$

Regarding prices, given that our model is a cash-in-advance *in everything* model where the available money for spending is $\rho_0 M_0 + \sum_{j=0}^{t} \rho_t G_j$, from the money market clearing condition

$$\frac{M_t^D}{p_t} = \frac{p_t C_t + D_{t+1}}{p_t} = C_t + \frac{D_{t+1}}{p_t} = C_t + d_{t+1}$$

we get

$$\hat{p}_t = \frac{\hat{\rho}_0 M_0 + \sum_{j=0}^{t} \hat{\rho}_t G_j}{\hat{C}_t + \hat{d}_{t+1}}.$$

Finally, according to the zero profits condition $\gamma_t = \frac{r_t^H}{r_t^P}$, and then the equilibrium nominal interest rate is

$$\hat{i}_t = (1 + \hat{\gamma}_t \hat{r}_t^P - \delta)\frac{\hat{p}_t}{\hat{p}_{t-1}} - 1.$$

Obviously, $\hat{l}_t = \overline{h}\ \forall t$, and the equivalence is proved. $\square$

B: Estimation of the VEC Model

Our VEC model, specified in section 3.4, was estimated by imposing *restricted linear trend coefficient* in the cointegration relations. As explained in section 3.2, this is due to the detection of potential trends in the series after applying the usual tests to study stationarity. As figure 7 shows, these trends are apparent.

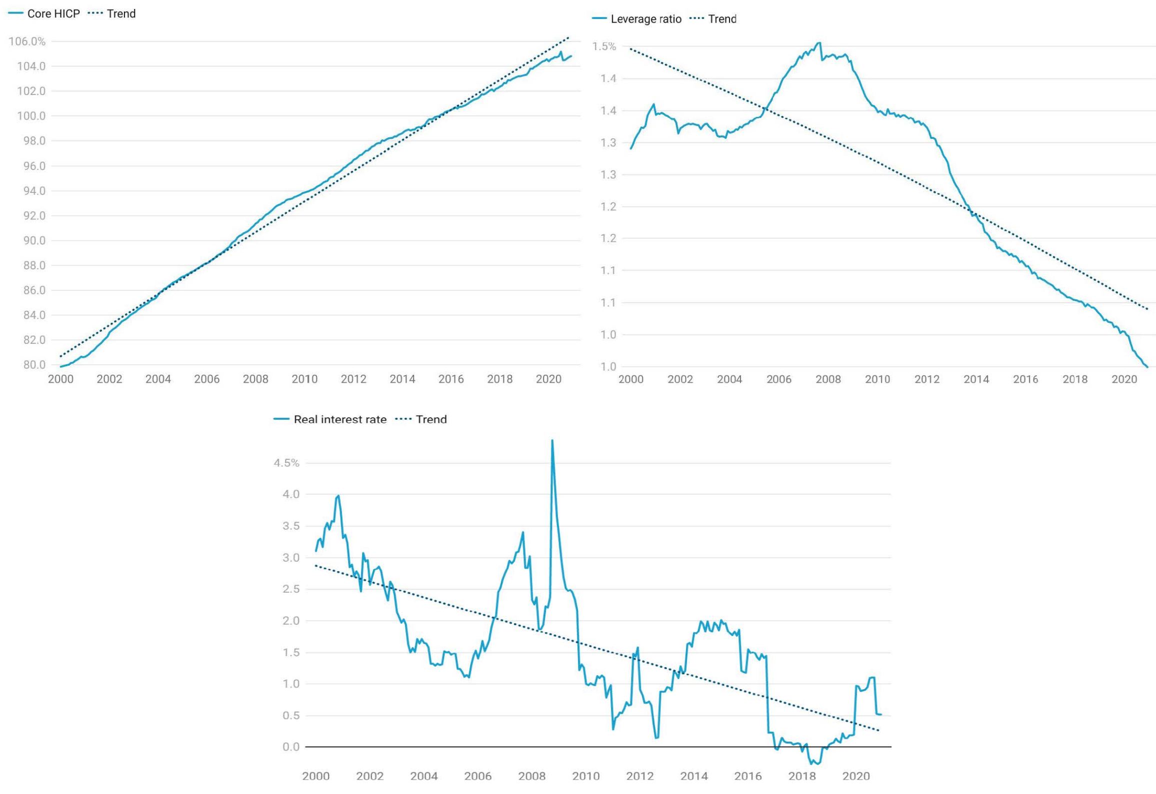

Figure S1: Eurozone, 2000:2022. HICP core (upper-left); Leverage ratio (upper-right); Real interest rate (bottom).

The VECM model has been tested for autocorrelation by LM tests, which indicate no serial correlation in the residuals. We also applied the White heteroscedasticity test, proving that the series are homoscedastic (p-value: 0.7106). Furthermore, all the inverse roots are inside the unit circle, ensuring model stability.