

Table S1 - Performance comparison in null simulation using t-type test

		Shrinkage Paramter							
		Normal test				<i>t</i> -test			
		0.1	1	10	100	0.1	1	10	100
Individuals	SNPs								
500	20	0.066	0.066	0.066	0.075	0.066	0.066	0.066	0.066
1000	1000	0.050	0.049	0.050	0.046	0.049	0.048	0.049	0.046

False positive rates at the nominal significance threshold $\alpha = 0.05$ in null datasets, comparing the performance of the approximate test when the normal or *t*-distribution is used as the null distribution of the test statistic.

Table S2 - Performance comparison in simulated data with continuous outcomes using t-type test

			Shrinkage Paramter							
			Normal test				<i>t</i> -test			
			0.1	1	10	100	0.1	1	10	100
Individuals	SNPs									
500	20	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.045	0.045	0.061	0.133	0.038	0.038	0.047	0.118
	100	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.056	0.054	0.071	0.141	0.046	0.044	0.063	0.134
1000	20	TPR	0.900	1.000	1.000	1.000	0.900	1.000	1.000	1.000
		FPR	0.043	0.043	0.087	0.137	0.034	0.034	0.071	0.119
	100	TPR	0.900	1.000	1.000	1.000	0.900	1.000	1.000	1.000
		FPR	0.051	0.052	0.060	0.108	0.048	0.050	0.056	0.099
	1000	TPR	0.700	0.700	1.000	1.000	0.700	0.700	1.000	1.000
		FPR	0.060	0.058	0.055	0.076	0.059	0.058	0.055	0.075
	20	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.048	0.048	0.048	0.113	0.036	0.036	0.031	0.096
5000	100	TPR	0.900	0.900	1.000	1.000	0.900	0.900	1.000	1.000
		FPR	0.055	0.052	0.062	0.100	0.052	0.047	0.059	0.096
	1000	TPR	0.700	0.700	1.000	1.000	0.700	0.700	1.000	1.000
		FPR	0.046	0.046	0.045	0.060	0.046	0.045	0.045	0.060

Performance comparison of the approximate test using a normal or *t*-distribution as the null distribution of

the test statistic in simulated genotype data with continuous phenotypes. Reported are proportion of true positive and false positive results at significance threshold $\alpha = 0.05$. **TPR**=True Positive Rate, **FPR**=False positive rate. Results for each simulation scenario are averaged over ten replicates.

Table S3 - Performance comparison in null simulation including comparison to univariate tests of significance

Individuals	SNPs	Outcomes	Univariate test	Shrinkage Paramter							
				Approximate test				Permutation test			
				0.1	1	10	100	0.1	1	10	100
500	20	Continuous	0.106	0.066	0.066	0.066	0.075	0.066	0.066	0.066	0.075
		Binary	0.052	0.021	0.021	0.041	0.067	0.027	0.027	0.033	0.052
1000	1000	Continuous	0.050	0.050	0.049	0.050	0.046	0.051	0.052	0.050	0.046
		Binary	0.026	0.118	0.092	0.066	0.053	0.054	0.056	0.053	0.051

Performance comparison in null simulation including comparison to univariate tests of significance. False positive rates at the nominal significance threshold $\alpha = 0.05$ in null datasets. In each scenario, results are averaged over ten replicates.

Table S4 - Performance comparison with continuous outcomes including comparison to univariate tests of significance

Individuals	SNPs		Shrinkage Paramter								
			Univariate test	Approximate test				Permutation test			
				0.1	1	10	100	0.1	1	10	100
500	20	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.341	0.045	0.045	0.061	0.133	0.015	0.015	0.017	0.095
	100	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.331	0.056	0.054	0.071	0.141	0.015	0.018	0.024	0.074
	1000	TPR	1.000	0.100	0.500	0.900	1.000	0.000	0.200	0.800	1.000
		FPR	0.219	0.038	0.045	0.049	0.080	0.007	0.006	0.010	0.029
	ALL	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.100	0.318	0.071	0.068	0.069	0.019	0.019	0.020	0.020
1000	20	TPR	1.000	0.900	1.000	1.000	1.000	0.900	1.000	1.000	1.000
		FPR	0.348	0.043	0.043	0.087	0.137	0.013	0.013	0.034	0.096
	100	TPR	1.000	0.900	1.000	1.000	1.000	0.900	0.900	1.000	1.000
		FPR	0.432	0.051	0.052	0.060	0.108	0.023	0.023	0.019	0.062
	1000	TPR	1.000	0.700	0.700	1.000	1.000	0.400	0.500	0.900	1.000
		FPR	0.295	0.060	0.058	0.055	0.076	0.007	0.008	0.010	0.020
	ALL	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.114	0.166	0.155	0.110	0.071	0.015	0.015	0.015	0.017
5000	20	TPR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		FPR	0.541	0.048	0.048	0.048	0.113	0.006	0.006	0.006	0.053
	100	TPR	1.000	0.900	0.900	1.000	1.000	0.800	0.900	1.000	1.000
		FPR	0.498	0.055	0.052	0.062	0.100	0.003	0.001	0.007	0.055
	1000	TPR	1.000	0.700	0.700	1.000	1.000	0.700	0.700	0.900	1.000
		FPR	0.397	0.046	0.046	0.045	0.060	0.006	0.007	0.008	0.014
	ALL	TPR	1.000	0.400	0.500	0.900	1.000	0.300	0.900	0.900	1.000
		FPR	0.216	0.026	0.027	0.029	0.042	0.007	0.007	0.007	0.009

Performance comparison between a univariate test, a permutation test and the approximate test in simulated genotype data with continuous phenotypes. Reported are proportion of true positive and false positive results at significance threshold $\alpha = 0.05$. **TPR**=True Positive Rate, **FPR**=False positive rate. Results for each simulation scenario are averaged over ten replicates.

Table S5 - Performance comparison with binary outcomes including comparison to univariate tests of significance

Individuals	SNPs		Univariate test	Shrinkage Paramter							
				Approximate test				Permutation test			
				0.1	1	10	100	0.1	1	10	100
500	20	TPR	1.000	0.300	0.500	0.900	0.900	0.400	0.600	0.900	0.900
		FPR	0.134	0.023	0.036	0.068	0.142	0.078	0.078	0.099	0.174
	100	TPR	1.0000	0.100	0.100	0.500	0.900	0.200	0.200	0.400	0.900
		FPR	0.134	0.024	0.037	0.046	0.087	0.050	0.052	0.058	0.115
	1000	TPR	1.000	0.200	0.300	0.500	0.700	0.100	0.100	0.400	0.700
		FPR	0.063	0.103	0.096	0.071	0.054	0.046	0.045	0.047	0.056
	2000	TPR	1.000	0.000	0.300	0.500	0.700	0.200	0.300	0.300	0.700
		FPR	0.052	0.008	0.056	0.081	0.063	0.052	0.049	0.048	0.055
	ALL	TPR	1.000	0.000	0.000	0.600	0.900	—	—	—	—
		FPR	0.033	0.000	0.000	0.014	0.068	—	—	—	—
5000	20	TPR	1.000	0.700	0.800	1.000	1.000	0.700	0.800	1.000	1.000
		FPR	0.294	0.024	0.024	0.030	0.096	0.090	0.083	0.089	0.154
	100	TPR	1.000	0.400	0.400	0.900	1.000	0.200	0.300	0.900	1.000
		FPR	0.271	0.027	0.028	0.041	0.078	0.071	0.067	0.078	0.110
	1000	TPR	1.000	0.200	0.300	0.600	1.000	0.100	0.200	0.600	1.000
		FPR	0.176	0.047	0.046	0.041	0.053	0.053	0.052	0.052	0.062
	2000	TPR	1.000	0.000	0.200	0.500	1.000	0.000	0.100	0.400	1.000
		FPR	0.136	0.074	0.067	0.056	0.057	0.053	0.052	0.053	0.058

Performance comparison between a univariate test, a permutation test and the approximate test in simulated genotype data with continuous phenotypes. Reported are proportion of true positive and false positive results at significance threshold $\alpha = 0.05$. **TPR**=True Positive Rate, **FPR**=False positive rate. Results for each simulation scenario are averaged over ten replicates.