Additional File 4.

One set of T column eigenvector of the covariance matrix in (6) is given by

$$\left[\mathbf{e}_1 \quad \cdots \quad \mathbf{e}_T\right] = \left[\begin{array}{c:ccc} 1 & -1 & \cdots & -1 \\ \hdashline 1 & & & \\ \vdots & & \mathbf{I}_{(T-1)\times(T-1)} & \\ 1 & & & \end{array}\right], \tag{S1}$$

where $\mathbf{I}_{(T-1)\times(T-1)}$ is an (T-1)-dimensional identity matrix. Orthonormal eigenvectors can be further obtained by applying the Gramm-Schmidt procedure which orthogonalize each eigenvector ($\mathbf{e}_i$) with respect to all the other eigenvectors to give $\mathbf{e}_{i\perp}$:

$$\mathbf{e}_{i\perp} = \frac{\mathbf{e}_i - \sum_{j=1}^{i-1}\left(\mathbf{e}_i \circ \mathbf{e}_{j\perp}\right)\mathbf{e}_{j\perp}}{\left|\mathbf{e}_i - \sum_{j=1}^{i-1}\left(\mathbf{e}_i \circ \mathbf{e}_{j\perp}\right)\mathbf{e}_{j\perp}\right|} \tag{S2}$$

Finally, we can obtain one set of orthonormal eigenvectors of (S1) as follows:

$$\left[\mathbf{e}_{1\perp} \quad \mathbf{e}_{2\perp} \quad \mathbf{e}_{3\perp} \quad \cdots \quad \mathbf{e}_{T\perp}\right] = \begin{bmatrix} \frac{1}{\sqrt{T}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \cdots & \frac{1}{\sqrt{T(T-1)}} \\ \frac{1}{\sqrt{T}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \cdots & \frac{1}{\sqrt{T(T-1)}} \\ \frac{1}{\sqrt{T}} & 0 & -\frac{2}{\sqrt{6}} & \cdots & \frac{1}{\sqrt{T(T-1)}} \\ \frac{1}{\sqrt{T}} & 0 & 0 & \cdots & \frac{1}{\sqrt{T(T-1)}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{T}} & 0 & 0 & \cdots & -\frac{T-1}{\sqrt{T(T-1)}} \end{bmatrix} \tag{S3}$$

For ease of notation, we used $\mathbf{e}_i$ instead of $\mathbf{e}_{i\perp}$ in the main text to denote the orthonormal eigenvectors. By randomly permuting the rows (performing row-switching transformations), we can obtain alternative orthonormal eigenspaces of the covariance matrix in (6).