

(I) Optimal control model

$u(t)$ = fraction of resources devoted for biomass production (M).

$1-u(t)$ = fraction of resources devoted for production of bottleneck protein (P).

The extended Baranyi's model [19]:

$$\frac{dP}{dt} = (1-u) v P$$

$$\frac{dM}{dt} = u \mu \frac{P}{1+P} M$$

We optimize $u(t)$ for the fastest possible cell doubling in the new medium (minimal time to doubling biomass), given an initial enzyme levels P_0 .

(II) The optimal solution is 'bang-bang' control

$u(t) = 0$ till time $t = \tau$, then $u(t)=1$.

The optimal lag1 phase duration is:

$$\tau = \frac{1}{v} \ln\left(\frac{v}{v_c}\right), \quad v_c = \frac{\mu P_0}{\ln(2)}$$

