

RESEARCH

Supplementary material for Electrical conductivity of the mantle of Mars from MGS magnetic observations

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Sample of title note

Abstract

Mars Global Surveyor magnetic data were analyzed to obtain the electromagnetically-induced response of the Martian mantle. The time-varying part of the MGS magnetic data analysis was obtained by removing a model of the static field. Only night data were selected and binned on a regular grid. The binned time series were processed using a proxy of the transient magnetic field. Two proxies were designed, one based on the MGS data themselves and one built from the Advanced Composition Explorer (ACE) dataset. The internal induced response of the planet was obtained at periods ranging from more than 1 day to 200 days. The induced responses were inverted to infer the mantle's electrical conductivity down to about 1300 km. The comparison of our conductivity profiles with previous theoretical studies confirms an aerotherm close to the cold boundary of thermal models of the Mars interior. A mantle transition zone was determined between 1000 and 1200 km deep.

Keywords: Electrical conductivity; Electromagnetic induction; Mars mantle; Mars Global Surveyor; Magnetic field

Method

We present a summary of [1] approach used in the accompanying article. In the quasi-static approximation [e.g., 2], and assuming an insulating medium at the

satellite altitude, the magnetic field may be written as the gradient of a scalar potential. Because the field is divergence-less, the potential verifies the Laplace equation. The solution contains both the external and the internal field potentials. Potentials of the transient magnetic field were derived on the basis of the spherical harmonic. Hence, we obtain the magnetic field $\mathbf{B}^N(t, r, \theta, \phi)$:

$$\mathbf{B}^N(t, r, \theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[\underbrace{\varepsilon_l^N \alpha_l^m(t) \left(\frac{r}{a}\right)^{l-1}}_{\text{External}} + \underbrace{\iota_l^N \beta_l^m(t) \left(\frac{a}{r}\right)^{l+2}}_{\text{Internal}} \right] Y_l^{Nm}(\theta, \phi) \quad (1)$$

where r is the orbital radius of the satellite, a the planet radius and, the angles θ and ϕ are the co-latitude and longitude, respectively. The functions $Y_l^{Nm}(\theta, \phi)$ are the generalized spherical harmonic functions with the generalized associated Legendre polynomials: $P_l^{Nm}(\cos(\theta))$. The value N (+1, 0, -1) defines the three vectorial magnetic components, $B^+ = (-B_\theta + i.B_\phi)/\sqrt{2}$, $B^- = (B_\theta + i.B_\phi)/\sqrt{2}$ and $B^0 = B_r$ where B_r , B_θ and B_ϕ are the radial, north-south and east-west components of the measured magnetic field. The coefficients ε_l^N and ι_l^N are given by [e.g., 3]:

$$\begin{aligned} \varepsilon^+ &= \varepsilon^- = \iota^+ = \iota^- = -\sqrt{\frac{l(l+1)}{2}} \\ \varepsilon^0 &= -l \\ \iota^0 &= l + 1 \end{aligned} \quad (2)$$

In eq. 1 the values $\alpha_l^m(t)$ and $\beta_l^m(t)$ are the SH coefficients of the magnetic potentials of the external and internal fields, respectively. The internal potential β_l^m is convolved to the external source term and is a function of the electrical conductivity in the planet. We define the electromagnetically-induced (EI) response

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of the planet $Q(\mathbf{r}, t)$ [e.g., 4]. It links $\beta(\mathbf{r}, t)$ to $\alpha(\mathbf{r}, t)$ through the relationship:

$$\beta(\mathbf{r}, t) = Q(\mathbf{r}, t) * \alpha(\mathbf{r}, t) \quad (3)$$

where $*$ stands for the convolution product.

The processing of eq. 3 generally involves SH expansion and Fourier transform to determine the SH and time spectral coefficients of the EI response $Q_l^m(\omega)$ at SH degree and order l, m and frequency ω . For a spherically symmetric structure, the EI response does not vary with order m and simply reduces to:

$$Q_l(\omega) = \beta_l^m(\omega) / \alpha_l^m(\omega) \quad (4)$$

The purpose of EI analysis is to infer $Q_l(\omega)$ values at several frequencies and invert this data to obtain an electrical conductivity profile. In order to solve eq. 3, [1] suggests binning the satellite data on a regular grid and simultaneously dealing with the gaps in the time series with a proxy providing the necessary time spectral content of the transient exosphere field.

For a source field with some geometry $\mathbf{C}_{ext}(\mathbf{r})$ with a dominant temporal variation $S(t)$, the external magnetic field is to the first approximation equal to $\mathbf{C}_{ext}(\mathbf{r}) \cdot S(t)$. The EI internal magnetic field is convolved to this source field. The total transient field is therefore:

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{C}(\mathbf{r}, t) * S(t) \quad (5)$$

where $\mathbf{C}(\mathbf{r}, t)$ includes both the inducing field and the induced response.

Writing eq. 5 as such implies that the external magnetic field has a dominant term that fluctuates as $S(t)$. [1] shows that the approximation works well for earth satellite magnetic data. The analysis recovers correctly the dominant uniform earth external field with no a priori hypothesis on its geometry. When the time series of the magnetic field $\mathbf{B}(\mathbf{r}, t)$ is not measured continuously, we replace $S(t)$ by a proxy $\hat{S}(t)$ proportional to it:

$$\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{C}}(\mathbf{r}, t) * \hat{S}(t) \quad (6)$$

where $\hat{\mathbf{C}}(\mathbf{r}, t)$ is proportional to $\mathbf{C}(\mathbf{r}, t)$. The function $\mathbf{C}(\mathbf{r}, t)$ or $\hat{\mathbf{C}}(\mathbf{r}, t)$ may be expanded in SH as in eq. 1 and Fourier transformed to obtain new potential coefficients $\hat{\beta}(\mathbf{r}, \omega)$ and $\hat{\alpha}(\mathbf{r}, \omega)$ proportional to $\beta(\mathbf{r}, \omega)$ and $\alpha(\mathbf{r}, \omega)$ respectively within the same constant so that eq. 4 is verified for $\hat{\beta}_l^m(\omega)$ and $\hat{\alpha}_l^m(\omega)$.

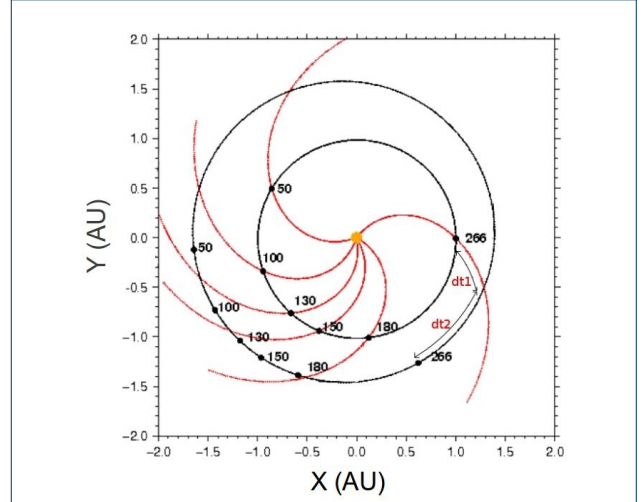


Figure 1 Positions of Mars and Earth (black) along their orbit, for several days, day 1 = 99/01/01, the distances X and Y are expressed in astronomical units. Parker spirals are shown in red passing through the Earth's position to Mars orbit. At day 266, dt1 and dt2 used to time-shift ACE data, are represented.

Time shifting of ACE data

To time shift ACE data to the Mars position, a technique similar to [5] was used. The trajectory of particles in the IMF is represented by an arithmetic spiral (see Fig. 1):

$$\frac{r}{b} - 1 - \ln\left(\frac{r}{b}\right) = \frac{V_m}{b\omega}(\Phi - \Phi_0) \quad (7)$$

where b is the solar corona limit, Φ_0 is the longitude origin of the spiral, and ω is the sun angular velocity. The position of the spiral is defined by the radius r and the longitude Φ . The position of the spiral crossing the ACE trajectory was calculated and the spiral arm between ACE and Mars was digitized to improve the translation model. We applied the spiro-radial time shift $dt1$ along a pseudo Parker spiral passing through the ACE position to Mars orbit (eq. 7) and the rotational time shift $(dt2 = (\Phi_{Mars} - \Phi_{ACE})/\omega_{Sun})$.

Self-proxy method on Earth

To complete our research on the resolution of the global EM induction study on Mars, we tested an approach using the satellite magnetic dataset itself; the so-called *self-proxy* method.

The method was first tested on Earth using Ørsted data. Data were binned over half-orbit. Each point was then an average over each pole flyby and was considered to be located close to the equator. fig. 2 presents our preferred profile of electrical conductivity using this proxy to complete the signal. The profile obtained was very similar to those from previous studies [1, 6].

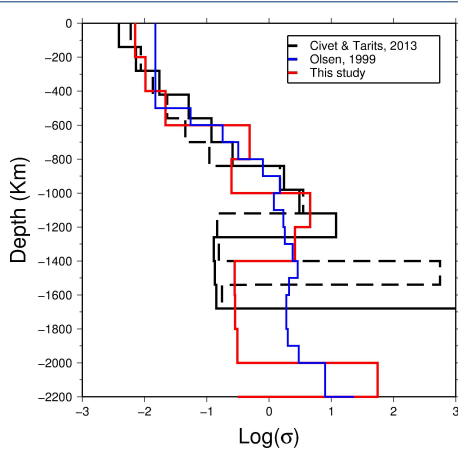


Figure 2 Electrical conductivity profile obtained from Ørsted data (red) using the self-proxy approach. Black and blue lines represent a previous study from [1, 6].

Time window	Component	Pearson r (%)	Kendall r (%)	Spearman r (%)
1	B_x	11.51	0.06	9.37
	B_y	59.45	42.30	59.85
	B_z	11.62	8.15	12.52
2	B_x	23.57	18.08	26.13
	B_y	56.32	42.81	61.21
	B_z	16.10	7.86	11.20
3	B_x	41.33	29.74	42.89
	B_y	72.60	55.09	75.58
	B_z	23.51	7.66	10.90
4	B_x	-0.08	1.91	2.83
	B_y	58.46	42.46	60.15
	B_z	0.42	2.70	4.48

Table 1 Calculated linear (Pearson) and non-parametric (Kendall and Spearman) correlation coefficients (r) between the 200-day time series of ACE data and MGS data, averaged over half-orbits [7].

Correlations between Martian proxies

Tab. 1 presents the correlation study between the ACE and the MGS proxies described in the accompanying article. Time series of both proxies were first re-sampled daily to minimize the effects of shorter periods. A classical Pearson (linear) correlation was applied [7, chap. 14.5] as well as a non parametric correlation [7, chap. 14.6] using *Spearman's* and *Kendall's* methods.

Tables

Fits of inversion

Inversions were carried out for all three SHs, $l = 1 - 3$, $m = 0$, all periods and all windows for a given proxy model. The comparison between the theoretical internal potential coefficients from the inversion and the observations is presented for proxy-MGS-2 in Fig. 3-a and for proxy-ACE in Fig. 3-b in the form of the

modulus and phase of the complex coefficient β_l^m as a function of period in days.

Data

A set of data obtained using proxy-MGS2 from a time series of 210 days is presented in Table. 2.

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Period	l	m	α		β	
140.00	1	-1	—	—	—	—
140.00	1	0	0.0324	-0.1039	-0.0064	-0.0014
140.00	1	1	—	—	—	—
140.00	2	-2	—	—	—	—
140.00	2	-1	—	—	—	—
140.00	2	0	—	—	—	—
140.00	2	1	—	—	—	—
140.00	2	2	—	—	—	—
140.00	3	-3	-0.0495	-0.0017	-0.0186	0.0236
140.00	3	-2	—	—	—	—
140.00	3	-1	0.0197	0.0271	0.0053	0.0047
140.00	3	0	-0.0320	-0.0105	-0.0004	0.0157
140.00	3	1	—	—	—	—
140.00	3	2	—	—	—	—
140.00	3	3	—	—	—	—
52.50	1	-1	—	—	—	—
52.50	1	0	-0.0455	0.0105	-0.0056	0.0094
52.50	1	1	0.0353	0.0180	-0.0014	0.0181
52.50	2	-2	—	—	—	—
52.50	2	-1	—	—	—	—
52.50	2	0	0.0426	-0.0171	-0.0218	0.0013
52.50	2	1	-0.0051	-0.0300	0.0140	-0.0067
52.50	2	2	—	—	—	—
52.50	3	-3	—	—	—	—
52.50	3	-2	—	—	—	—
52.50	3	-1	—	—	—	—
52.50	3	0	0.0169	0.0065	0.0040	-0.0008
52.50	3	1	-0.0128	-0.0053	0.0073	0.0021
52.50	3	2	-0.0085	-0.0109	0.0034	-0.0059
52.50	3	3	—	—	—	—
26.25	1	-1	—	—	—	—
26.25	1	0	0.0102	-0.0128	-0.0045	0.0006
26.25	1	1	—	—	—	—
26.25	2	-2	0.0053	0.0065	0.0034	-0.0001
26.25	2	-1	0.0107	0.0025	-0.0035	-0.0062
26.25	2	0	0.0724	0.0063	-0.0102	0.0093
26.25	2	1	—	—	—	—
26.25	2	2	—	—	—	—
26.25	3	-3	—	—	—	—
26.25	3	-2	—	—	—	—
26.25	3	-1	0.0111	-0.0052	0.0077	-0.0019
26.25	3	0	—	—	—	—
26.25	3	1	—	—	—	—
26.25	3	2	—	—	—	—
26.25	3	3	—	—	—	—

Table 2 SH coefficients after filtering expressed in nT from a 210-day time series and calculated using proxy-MGS-2. — symbols denotes non-physical values that were removed. The period is expressed in days, SH coefficients are shown for degree l and order m .

Period	l	m	α		β	
14.48	1	-1	—	—	—	—
14.48	1	0	0.0029	0.0031	0.0003	0.0004
14.48	1	1	—	—	—	—
14.48	2	-2	—	—	—	—
14.48	2	-1	0.0001	0.0012	—	0.0002
14.48	2	0	-0.0023	-0.0063	0.0008	0.0002
14.48	2	1	-0.0007	-0.0009	0.0002	0.0003
14.48	2	2	—	—	—	—
14.48	3	-3	0.0004	-0.0001	0.0001	0.0003
14.48	3	-2	—	—	—	—
14.48	3	-1	—	—	—	—
14.48	3	0	-0.0003	-0.0005	-0.0004	-0.0002
14.48	3	1	0.0005	0.0004	0.0002	0.0003
14.48	3	2	—	—	—	—
14.48	3	3	—	—	—	—
8.57	1	-1	0.0060	-0.0109	-0.0006	-0.0042
8.57	1	0	—	—	—	—
8.57	1	1	—	—	—	—
8.57	2	-2	—	—	—	—
8.57	2	-1	—	—	—	—
8.57	2	0	-0.0107	0.0070	0.0052	-0.0006
8.57	2	1	-0.0006	0.0149	-0.0081	0.0029
8.57	2	2	—	—	—	—
8.57	3	-3	—	—	—	—
8.57	3	-2	—	—	—	—
8.57	3	-1	—	—	—	—
8.57	3	0	—	—	—	—
8.57	3	1	0.0007	-0.0062	-0.0034	-0.0015
8.57	3	2	-0.0053	0.0031	-0.0038	-0.0003
8.57	3	3	—	—	—	—
5.32	1	-1	—	—	—	—
5.32	1	0	0.0035	-0.0091	0.0008	-0.0001
5.32	1	1	—	—	—	—
5.32	2	-2	—	—	—	—
5.32	2	-1	—	—	—	—
5.32	2	0	-0.0141	0.0108	0.0011	0.0027
5.32	2	1	0.0008	-0.0043	—	0.0004
5.32	2	2	—	—	—	—
5.32	3	-3	—	—	—	—
5.32	3	-2	—	—	—	—
5.32	3	-1	—	—	—	—
5.32	3	0	—	—	—	—
5.32	3	1	—	—	—	—
5.32	3	2	—	—	—	—
5.32	3	3	—	—	—	—

Table 3 continuation of Tab. 2

Period	l	m	α		β	
3.39	1	-1	—	—	—	—
3.39	1	0	—	—	—	—
3.39	1	1	—	—	—	—
3.39	2	-2	—	—	—	—
3.39	2	-1	—	—	—	—
3.39	2	0	0.0010	0.0001	-0.0001	0.0006
3.39	2	1	0.0001	-0.0056	0.0025	-0.0013
3.39	2	2	—	—	—	—
3.39	3	-3	-0.0006	-0.0018	-0.0002	-0.0008
3.39	3	-2	—	—	—	—
3.39	3	-1	-0.0009	0.0015	0.0008	-0.0006
3.39	3	0	—	—	—	—
3.39	3	1	-0.0004	0.0030	0.0008	0.0018
3.39	3	2	—	—	—	—
3.39	3	3	—	—	—	—
2.19	1	-1	0.0004	0.0007	—	—
2.19	1	0	-0.0009	-0.0028	0.0003	-0.0003
2.19	1	1	—	—	—	—
2.19	2	-2	-0.0001	-0.0005	-0.0002	-0.0001
2.19	2	-1	—	—	—	—
2.19	2	0	-0.0003	-0.0009	-0.0002	-0.0001
2.19	2	1	—	0.0043	-0.0002	0.0005
2.19	2	2	-0.0012	0.0005	-0.0001	0.0003
2.19	3	-3	-0.0004	-0.0001	0.0001	—
2.19	3	-2	—	—	—	—
2.19	3	-1	-0.0016	0.0009	0.0006	-0.0002
2.19	3	0	0.0008	-0.0002	—	0.0002
2.19	3	1	-0.0027	-0.0020	-0.0009	-0.0011
2.19	3	2	0.0001	0.0001	-0.0001	—
2.19	3	3	0.0001	-0.0002	-0.0001	—
1.42	1	-1	—	—	—	—
1.42	1	0	-0.0005	0.0010	0.0004	-0.0002
1.42	1	1	—	—	—	—
1.42	2	-2	—	—	—	—
1.42	2	-1	—	—	—	—
1.42	2	0	—	—	—	—
1.42	2	1	—	—	—	—
1.42	2	2	-0.0007	0.0004	—	-0.0002
1.42	3	-3	—	—	—	—
1.42	3	-2	—	—	—	—
1.42	3	-1	—	—	—	—
1.42	3	0	—	—	—	—
1.42	3	1	—	—	—	—
1.42	3	2	—	—	—	—
1.42	3	3	0.0001	-0.0002	-0.0001	—

Table 4 continuation of Tab. 2

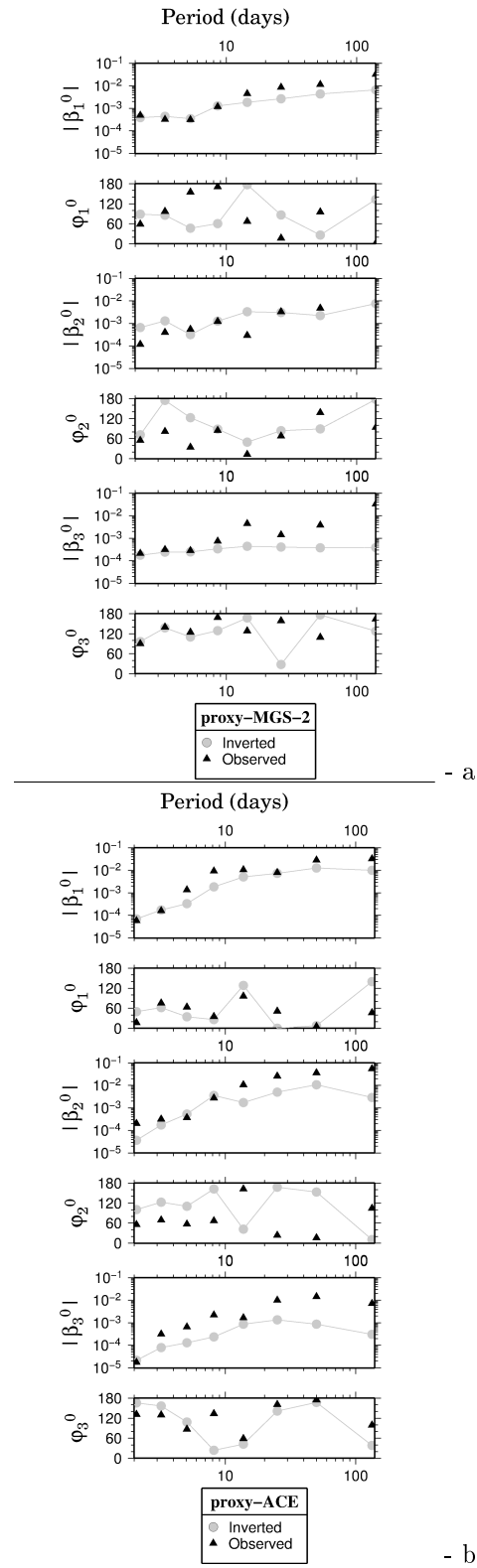


Figure 3 Fit of result for proxy-MGS-2 (a) and proxy-ACE (b) : modulus and phase of the internal magnetic potential SH coefficient at $l = 1 - 3$ and $m = 0$ as a function of period in days. The black triangles are the data β_l^m and the connected gray triangles are the model responses equal to $Q_l \alpha_l^m$.