[3] Fitting distributions to expert group quantiles

Three alternative distributions (Dirichlet, Beta and Generalized Trapezoidal) were compared to find the best fit to the joint DM percentile estimates. The Beta distribution varies smoothly over a fixed interval (cf. the triangular distribution) and is commonly used to represent uncertainty in probability estimates (Granger Morgan et al., 1992; Vose, 2000). The standard Beta distribution on the interval [0,1] is characterized by two shape parameters α and β, (Evans et al., 2000) and can approximate a wide range of distributional shapes When α and $\beta>1$, the distribution is unimodal, with a density of 0 at the maximum and minimum extents. A uniform distribution is given by $\alpha = \beta = 1$. When α (or β) <1 the density function decreases (or increases) monotonically from zero. The Dirichlet Distribution is the multinomial generalisation of the Beta Distribution, and can used to model nodes with more than two states (Heckerman et al., 1995). Olkin and Liu (2003) suggest a bivariate Beta Distribution as an alternative.

The trapezoidal distribution is a generalization of the triangular distribution, and similarly covers a fixed interval, but with an additional central stage. Unlike the standard triangular and trapezoidal distributions, the generalized trapezoidal (GT) distribution can represent nonlinear behaviour in the tails and a linear incline (or decline) in the central stage van Dorp and Kotz (2003); van Dorp et al. (2007). It can therefore approximate a wider range of distributional shapes.

Distribution parameters were obtained by finding the best fit to the elicited percentiles, using SolvOpt (Kuntsevich and Kappel, 1997), a solver for non-smooth optimization. The solver is used to minimize a least-squares function: the square of the difference between the solution at each of the three percentiles, and the elicited values for the percentiles. Constraints on the distribution parameters (described in the main article) were imposed by a penalty function. The Dirichlet distribution gave the overall best-fit and was chosen for the final network (see figure 8 in main article).

In fitting a probability distribution to a small number of elicited percentiles (in this case 3 values), there will be a number of locally optimal solutions. To find the best approximation to the globally optimal solution, the solver was run multiple times, starting from randomly sampled initial values. In this case between 5000 - 50,000 restarts were performed for each distribution to find an acceptable fit. The initial values were constrained to ensure they were appropriate for the distribution, and yield a sensible shape (e.g. to ensure a unimodal distribution over the required range).

Evans, M., Hastings, N., Peacock, B., 2000. Statistical Distributions. Measurement Science and Technology 12 (1), 10.1088/0957-0233/12/1/702, 170 pp.

Granger Morgan, M., Henrion, M., (with Small, M.), 1992. Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis. Cambridge University Press. ISBN: 978-0521427449, 332pp.

Heckerman, D., Geiger, D., Chickering, D. M. 1995. Learning Bayesian networks: The combination of knowledge and statistical data. Machine Learning 20 (3), 197–243.

Kuntsevich, A., Kappel, F., 1997. Solvopt (Solver for local optimization problems). Software. Url: http://www.uni-graz.at/imawww/kuntsevich/solvopt/ (accessed 26/06/13)

Olkin, I., Liu, R. 2003. A bivariate beta distribution. Statistics and Probability Letters 62 (4), 407 – 412, doi: 10.1016/S0167-7152(03)00048-8.

van Dorp, J., Kotz, S., 2003. Generalized trapezoidal distributions. Metrika 58 (1), 85–97.

van Dorp, J. R., Rambaud, S. C., Pérez, J. G., Pleguezuelo, R. H., September 2007. An Elicitation Procedure for the Generalized Trapezoidal Distribution with a Uniform Central Stage. Decision Analysis 4 (3), 156–166, doi: 10.1287/deca.1070.0093.

Vose, D., 2000. Risk analysis : a quantitative guide. Wiley. ISBN: 047199765X, 9780471997658, 430pp.