

Chinese Journal of Mechanical Engineering

**A Finite and Instantaneous Screw Based Approach
for Topology Design and Kinematic Analysis of
5-Axis Parallel Kinematic Machines**

Tao SUN, Shuofei YANG, Tian HUANG, Jian S. DAI

Tianjin University



天津大学
Tianjin University

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



Example: Metrom



Conclusions

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



Example: Metrom

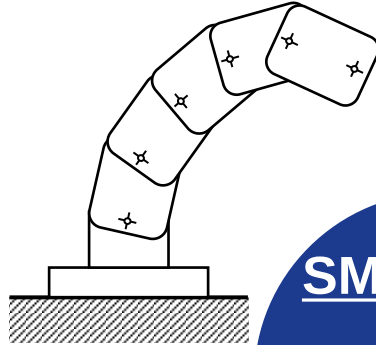


Conclusions

1. Background

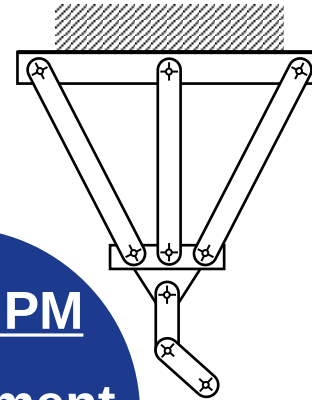
Serial Mechanism

- ✓ Simple Structure
- ✓ Low Cost
- ✓ Easy Control
- ✓ Large Workspace



Parallel Mechanism

- ✓ High Stiffness
- ✓ High Accuracy
- ✓ High Speed
- ✓ Excellent Dynamics



SM Vs. PM
Complement
Each Other



The top missions of Modern Mechanisms are to reveal the structure construction principles of mechanical systems, to invent new mechanisms, to investigate the mechanism analysis and design methods for based upon specific performances.

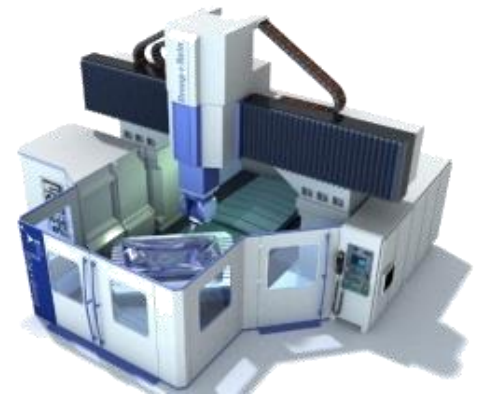
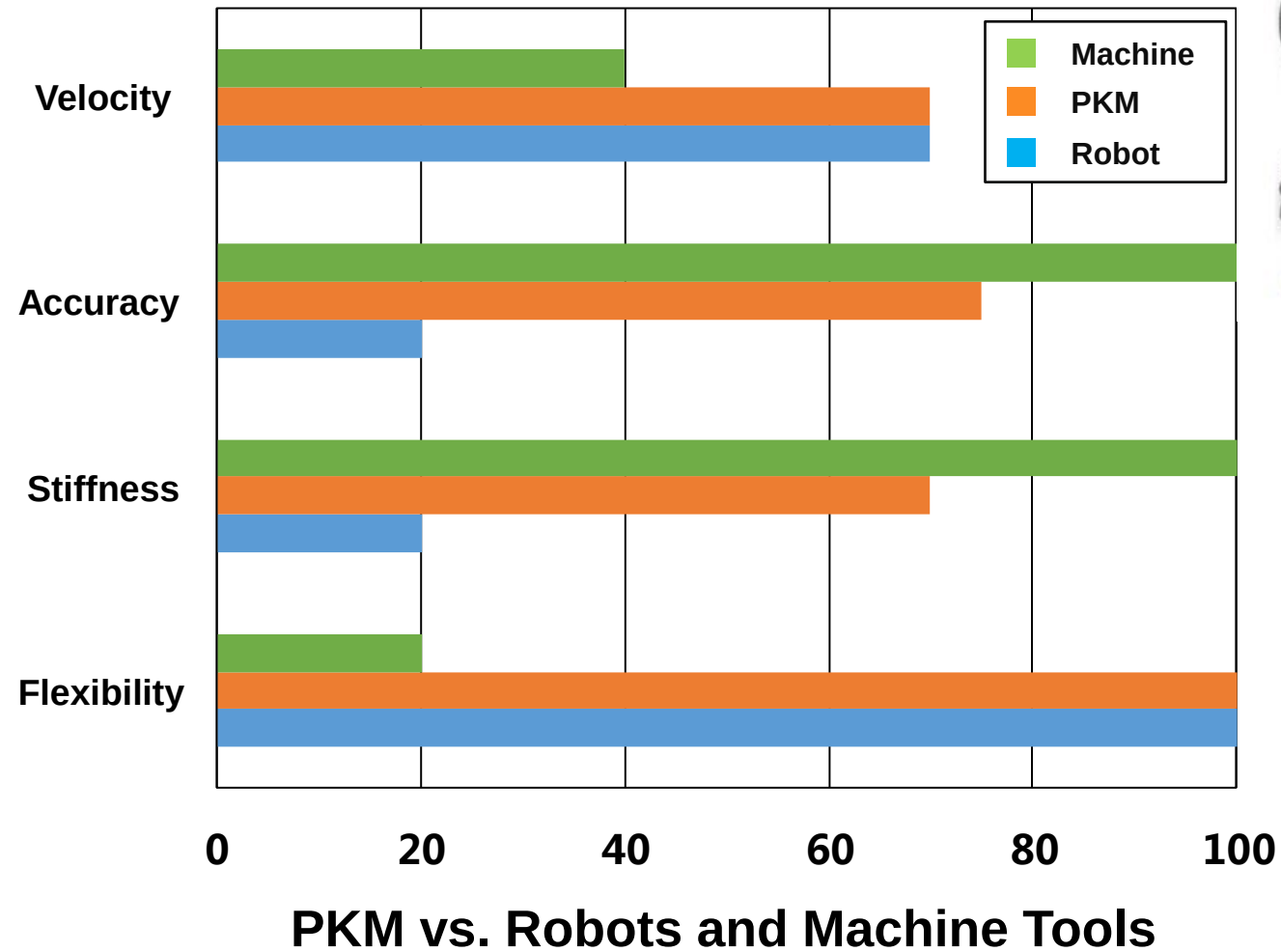
----- NSFC Report -----

1. Background

Parallel Kinematic Machine:

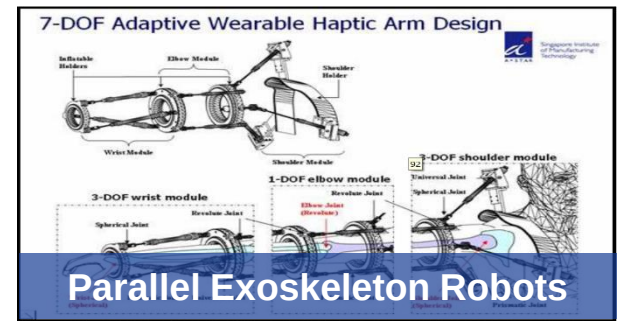
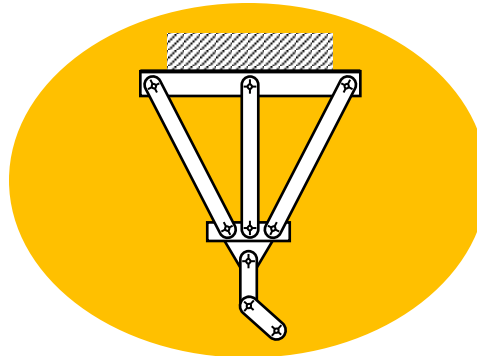
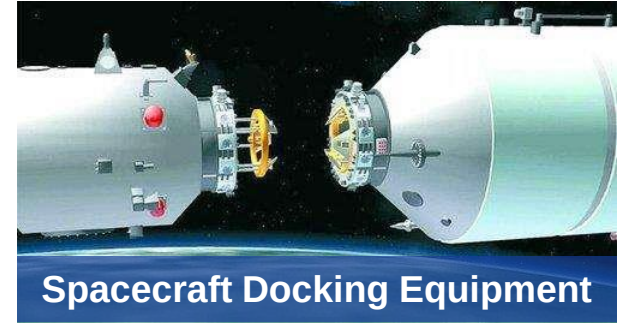
Combination of Traditional Machine and Articulated Robot

Source : www.pkmtricept.com



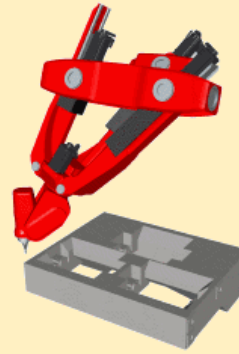
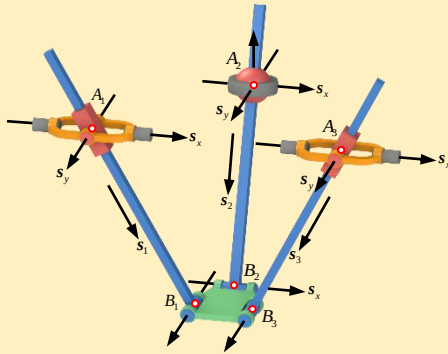
1. Background

Parallel mechanisms based mechanical systems have been widely applied in aerospace, automobile, medical and bionic fields.



1. Background

Design of Parallel Mechanisms



**Topology
Design**

**Performance
Design**

**Prototype and
Experiments**

- ◆ To establish topology model in an algebraic way
- ◆ To establish kinematic, stiffness, accuracy and dynamic models in an unified way
- ◆ To connect topology model and performance model

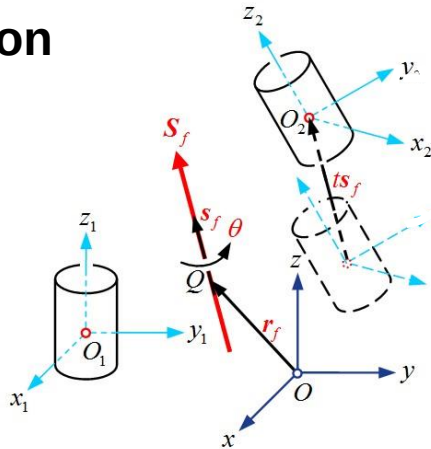
Challenge: Unified mathematical tool is required in whole design process

1. Background

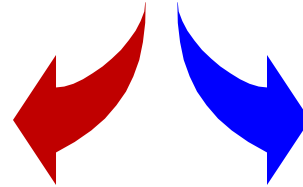
Displacement

Translation
Rotation

Finite
motion



Rigid body
motion

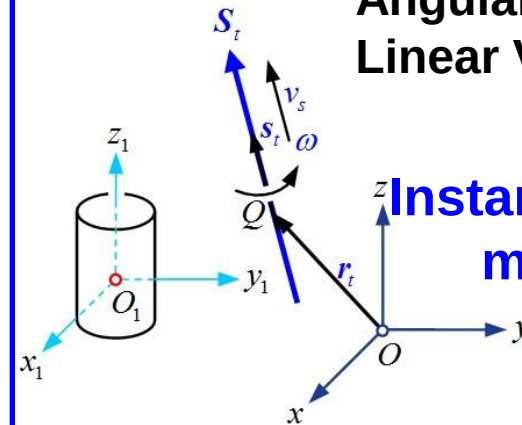


Relation

Velocity

Angular Velocity
Linear Velocity

Instantaneous
motion



Integration



- ◆ Finite motion description and calculation
- ◆ Topology design

- ◆ Performance modeling and analysis
- ◆ Parameter design

1. Background

Two mathematical tool: **Matrix Lie Group & Matrix Lie Algebra**
Dual Quaternion & Pure Dual Quaternion

Matrix Lie Group

$$g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Finite motion

$$g = e^{\hat{\xi} \frac{\theta}{\omega}}$$

Product of
Exponential

Matrix Lie Algebra

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$$

Instantaneous motion

Dual Quaternion

$$D = \cos \frac{\hat{\theta}}{2} + \sin \frac{\hat{\theta}}{2} S$$

$$D = e^{d \frac{\theta}{\omega}}$$

Product of
Exponential

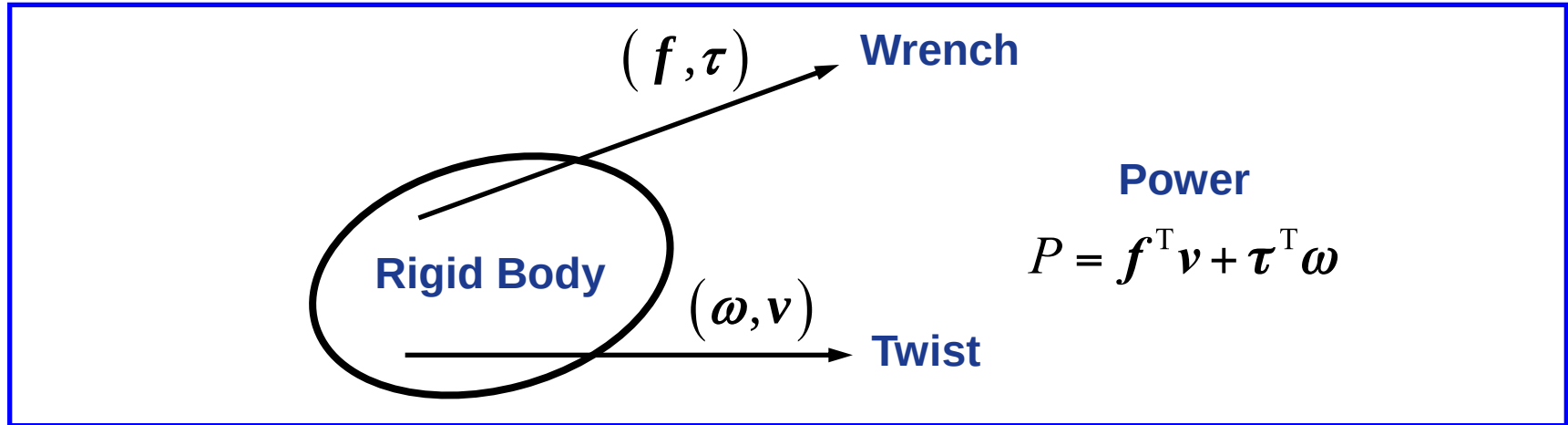
Pure Dual Quaternion

$$d = \omega + \varepsilon v$$

Disadvantages:

- ❑ There are redundant items in matrix group or dual quaternion representations of finite motions, leading to the complexity in the composition computations.
- ❑ The methods are lack of algebraic intersection algorithms. The computation of intersection relies on intuition and experience.

1. Background



$$S_t = \begin{pmatrix} \omega \\ v \end{pmatrix}$$

Instantaneous Screw

Directly Correspond to
Plücker Coordinates

Constraint Wrench is
Reciprocal to Twist

Power is the Reciprocal
Product of Twist and Wrench

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$$

Matrix Lie Algebra

Skew-Symmetric Matrix of
Plücker Coordinates

**No Wrench
Representation**

Cannot Express Power

$$d = \omega + \varepsilon v$$

Pure Dual Quaternion

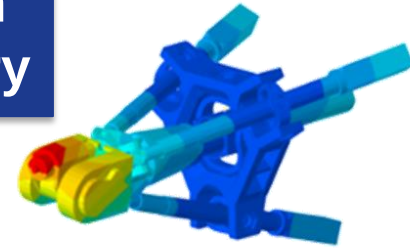
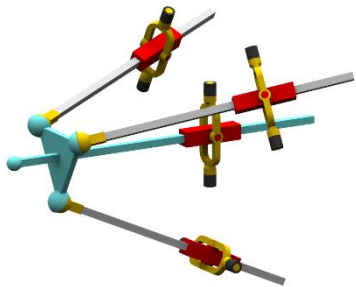
Pure Dual Quaternion of
Plücker Coordinates

**No Wrench
Representation**

Cannot Express Power

1. Background

Unified Topology and Performance Design of Parallel Mechanisms Under Screw Theory



Topology Design

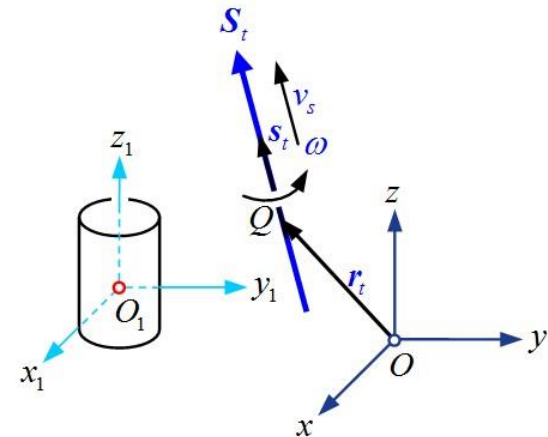
- Constraint Screw Method
- Differential Manifold Method
- Line Geometry and Atlas Method
- Virtual Chain Method
- POC Method
- G_F Set Method
- Linear Transformation Method
- Differential Geometry Method



Finite Screw

Performance Design

$$\mathbf{S}_t = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix} = \omega \begin{pmatrix} \mathbf{s}_t \\ \mathbf{r}_t \times \mathbf{s}_t \end{pmatrix} + v_s \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_t \end{pmatrix}$$



Instantaneous Screw

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



Example: Metrom



Conclusions

2. Finite Screw and Instantaneous Screw

Finite Screw

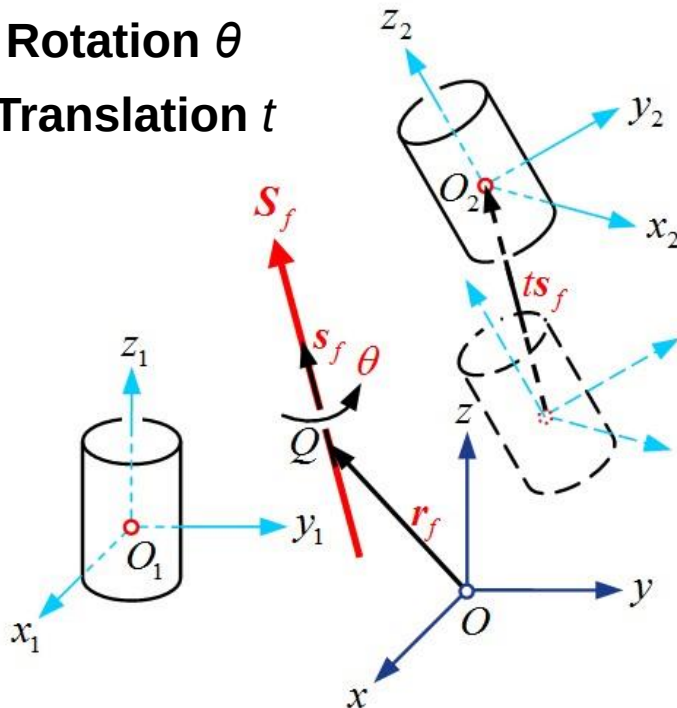
$$\mathbf{S}_f = ? \begin{pmatrix} \mathbf{s}_f \\ \mathbf{r}_f \times \mathbf{s}_f \end{pmatrix} + ? \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_f \end{pmatrix}$$

6×1 **Vector** Form ?

(1) Chasles Axis

(2) Rotation θ

(3) Translation t



Instantaneous Screw

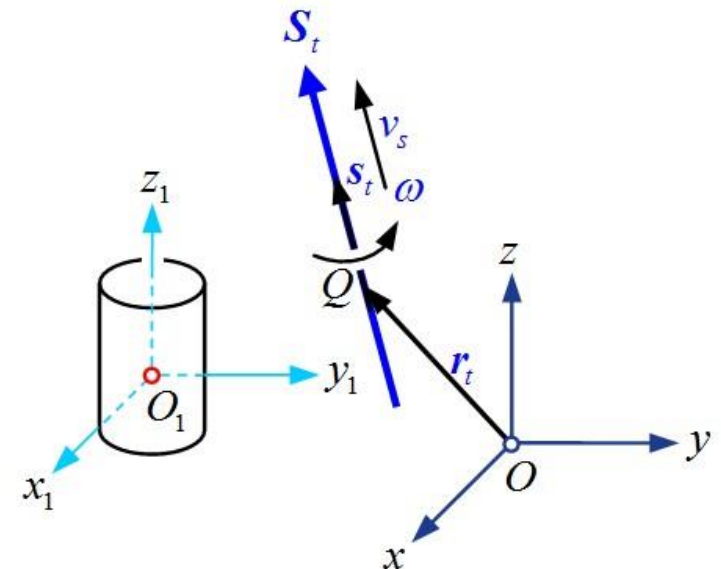
$$\mathbf{S}_t = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix} = \omega \begin{pmatrix} \mathbf{s}_t \\ \mathbf{r}_t \times \mathbf{s}_t \end{pmatrix} + v_s \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_t \end{pmatrix}$$

6×1 **Vector** Form

Axis : Position and Orientation

Velocity : Angular and Linear

Fore : Pure Force and Couple



2. Finite Screw and Instantaneous Screw

Finite Screw

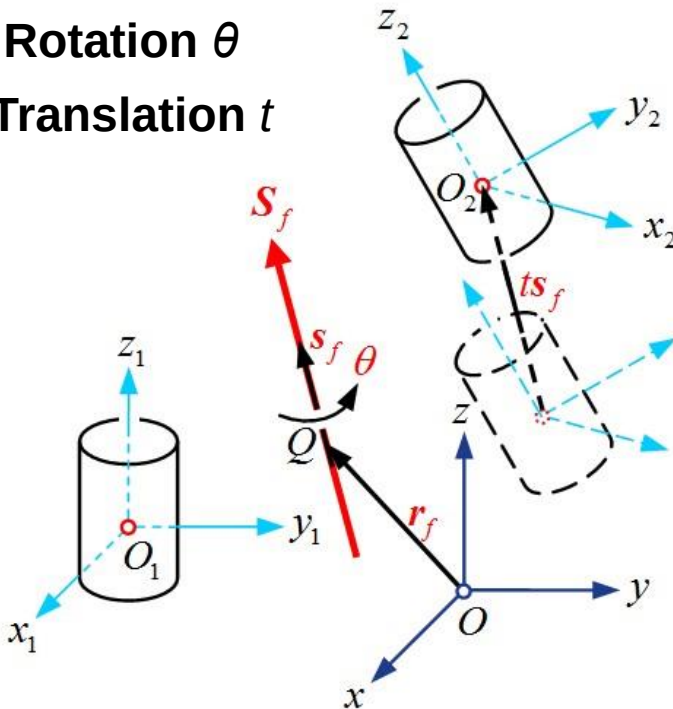
$$\mathbf{S}_f = ? \begin{pmatrix} \mathbf{s}_f \\ \mathbf{r}_f \times \mathbf{s}_f \end{pmatrix} + ? \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_f \end{pmatrix}$$

6×1 **Quasi-vector** Form

(1) Chasles Axis

(2) Rotation θ

(3) Translation t



1969 Dimentberg F. M. proposed the guess and basic concept

1992 Parkin I. A. defined Quasi-Pitch

1994 Huang C.T. defined Amplitude



2010s Discuss mathematical sources of the form

Why?

2. Finite Screw and Instantaneous Screw

Dual
Quaternion

$$\mathbf{D} = \left(\cos \frac{\theta}{2} - \varepsilon \frac{t}{2} \sin \frac{\theta}{2} \right) + \left(\sin \frac{\theta}{2} \mathbf{s} + \varepsilon \left(\sin \frac{\theta}{2} (\mathbf{r} \times \mathbf{s}) + \frac{t}{2} \cos \frac{\theta}{2} \mathbf{s} \right) \right)$$

Redundant information

Divide by $\cos \frac{\theta}{2}$

Includes all info.
of finite motion

$$\frac{1}{2} \cos \frac{\theta}{2} \left(2 \tan \frac{\theta}{2} (\mathbf{s} + \varepsilon (\mathbf{r} \times \mathbf{s})) + \varepsilon t \mathbf{s} \right)$$

Write dual vector into Quasi-vector

Finite screw
Quasi-vector form

$$\mathbf{S}_f = 2 \tan \frac{\theta}{2} \begin{pmatrix} \mathbf{s} \\ \mathbf{r} \times \mathbf{s} \end{pmatrix} + t \begin{pmatrix} \mathbf{0} \\ \mathbf{s} \end{pmatrix} \quad \Bigg| \quad \mathbf{S}_t = \omega \begin{pmatrix} \mathbf{s}_t \\ \mathbf{r}_t \times \mathbf{s}_t \end{pmatrix} + v_s \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_t \end{pmatrix}$$

2. Finite Screw and Instantaneous Screw

Composition

Describe all info. of finite motions analytically after composition

$$\begin{aligned} \mathbf{S}_{f,ab} &= \mathbf{S}_{f,a} \Delta \mathbf{S}_{f,b} \\ &= \frac{1}{1 - \tan \frac{\theta_a}{2} \tan \frac{\theta_b}{2} \mathbf{s}_a^T \mathbf{s}_b} \left(\mathbf{S}_{f,a} + \mathbf{S}_{f,b} + \frac{1}{2} \mathbf{S}_{f,b} \times \mathbf{S}_{f,a} - \tan \frac{\theta_a}{2} \tan \frac{\theta_b}{2} \left(t_b \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_a \end{pmatrix} + t_a \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_b \end{pmatrix} \right) \right) \end{aligned}$$

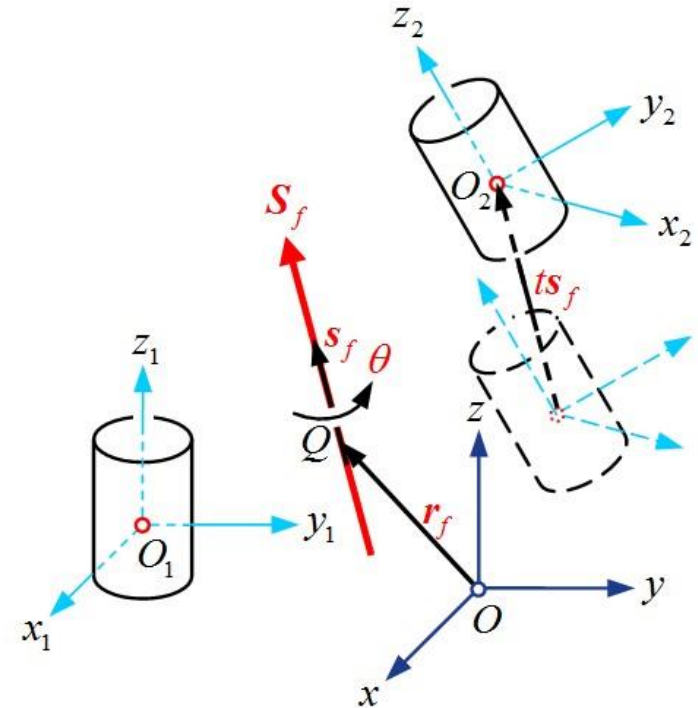
Associativity

Composition of three finite motions:

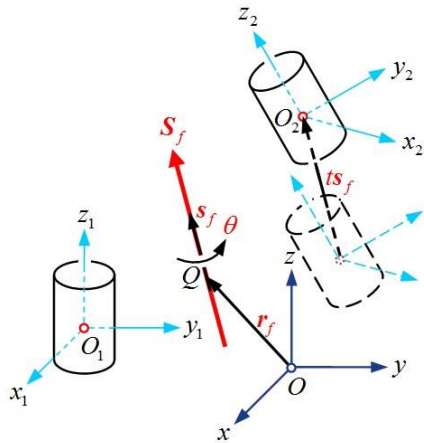
$$\mathbf{S}_{f,123} = \mathbf{S}_{f,1} \Delta \mathbf{S}_{f,2} \Delta \mathbf{S}_{f,3}$$

Associativity law:

$$(\mathbf{S}_{f,1} \Delta \mathbf{S}_{f,2}) \Delta \mathbf{S}_{f,3} = \mathbf{S}_{f,1} \Delta (\mathbf{S}_{f,2} \Delta \mathbf{S}_{f,3})$$



2. Finite Screw and Instantaneous Screw



Basic Elements in Chasles' Theorem:

1) Chasles's Axis $\begin{pmatrix} s^T & (r \times s)^T \end{pmatrix}^T$

2) Rotational Angle θ

3) Translational Distance t

$$S_f = 2 \tan \frac{q}{2} \begin{pmatrix} s_f \\ r_f \times s_f \end{pmatrix} + t \begin{pmatrix} 0 \\ s_f \end{pmatrix}$$

Finite Screw

Directly Reflects Elements of Chasles's Theorem

Screw Triangle Product

Easy to be Analytically Composited and Computed

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Matrix Lie Group

Hard to Extract Elements of Chasles' Theorem

Baker-Campbell-Haursdorff Formula

Hard to be Analytically Composited and Computed

$$D = q + \epsilon q_0$$

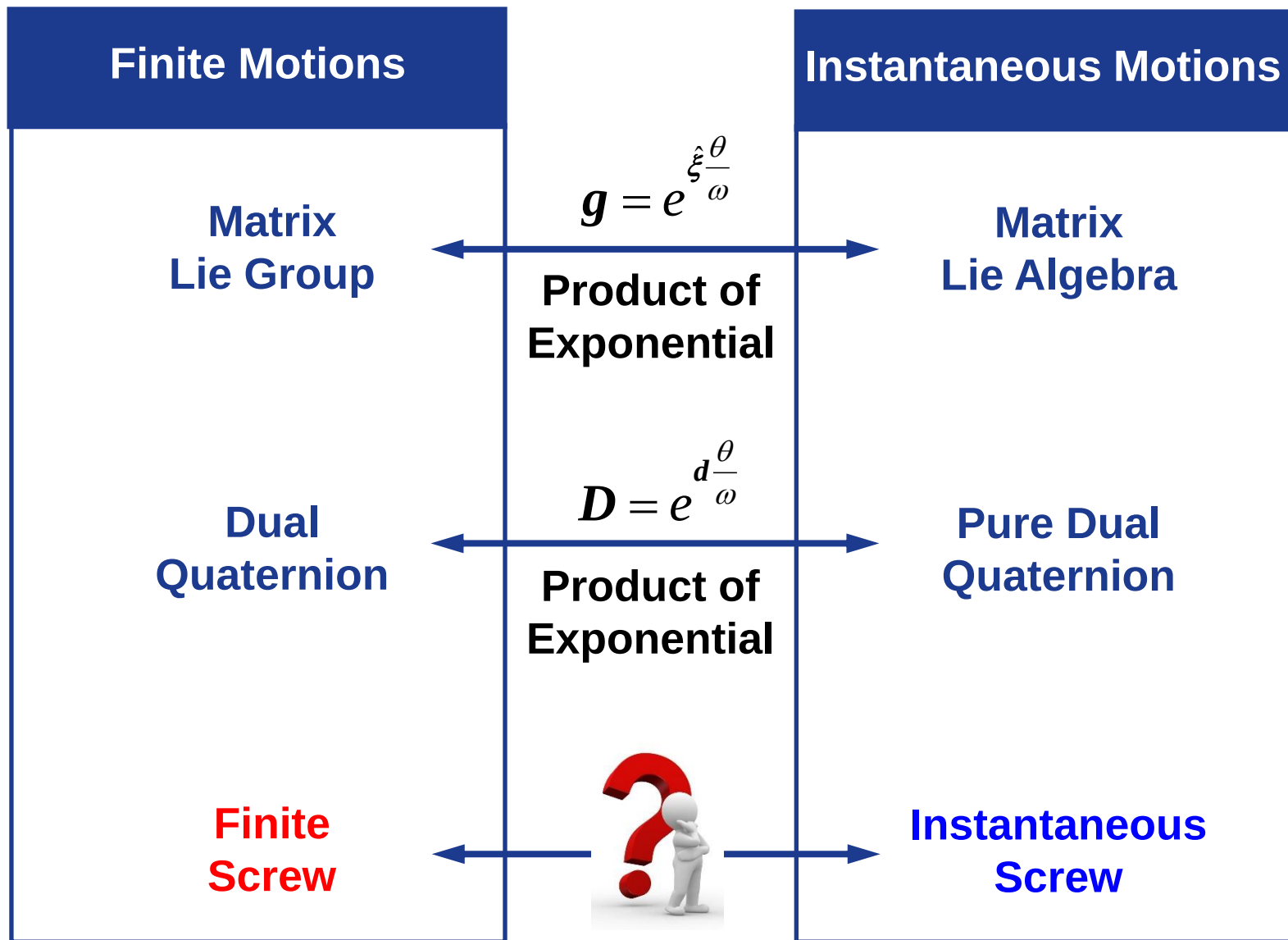
Dual Quaternion

Dual Vector Part Reflects Chasles' Theorem

Rodrigues Formula with Dual Angles

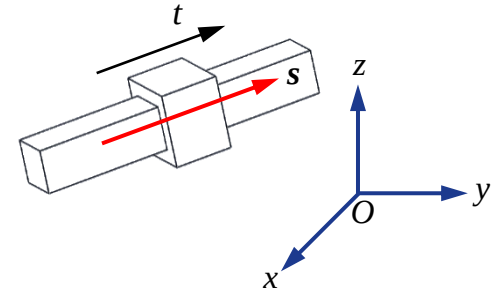
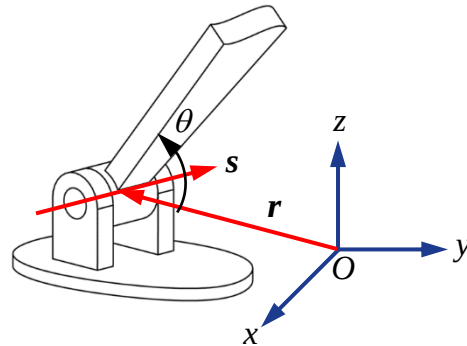
Hard to be Analytically Composited and Computed

2. Finite Screw and Instantaneous Screw



2. Finite Screw and Instantaneous Screw

Differential Mapping between One DoF Screws



Finite Screw

$$\mathbf{S}_f = 2 \tan \frac{\theta}{2} \begin{pmatrix} \mathbf{s} \\ \mathbf{r} \times \mathbf{s} \end{pmatrix}$$

$$\mathbf{S}_f = t \begin{pmatrix} \mathbf{0} \\ \mathbf{s} \end{pmatrix}$$

Solving Differential

$$\theta = 0 \quad t = 0$$

$$\dot{\theta} = \omega \quad \dot{t} = v$$

$$\dot{\mathbf{S}}_f = \frac{\dot{\theta}}{\cos^2 \left(\frac{\theta}{2} \right)} \begin{pmatrix} \mathbf{s} \\ \mathbf{r} \times \mathbf{s} \end{pmatrix}$$

$$\dot{\mathbf{S}}_f = \dot{t} \begin{pmatrix} \mathbf{0} \\ \mathbf{s} \end{pmatrix}$$

Instantaneous Screw

$$\mathbf{S}_t = \omega \begin{pmatrix} \mathbf{s} \\ \mathbf{r} \times \mathbf{s} \end{pmatrix}$$

$$\mathbf{S}_t = v \begin{pmatrix} \mathbf{0} \\ \mathbf{s} \end{pmatrix}$$

2. Finite Screw and Instantaneous Screw

Differential Mapping between Multi-DoFs Screws

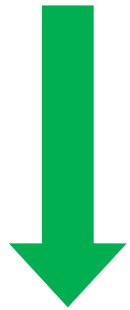
$$\mathbf{S}_f = 2 \tan \frac{\theta}{2} \begin{pmatrix} \mathbf{s}_f \\ \mathbf{r}_f \times \mathbf{s}_f \end{pmatrix} + t \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_f \end{pmatrix} \longrightarrow \dot{\mathbf{S}}_f \Big|_{\substack{\theta=0 \\ t=0}} = \dot{\theta} \begin{pmatrix} \mathbf{s}_t \\ \mathbf{r}_t \times \mathbf{s}_t \end{pmatrix} + \dot{t} \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix} = \mathbf{S}_t$$

Differential Property

$$\mathbf{S}_{f,ab} = \mathbf{S}_{f,a} \Delta \mathbf{S}_{f,b}$$

$$\dot{\mathbf{S}}_{f,ab} = \dot{\mathbf{S}}_{f,a} \Delta \mathbf{S}_{f,b} + \mathbf{S}_{f,a} \Delta \dot{\mathbf{S}}_{f,b}$$

$$\mathbf{S}_{t,ab} = \mathbf{S}_{t,a} + \mathbf{S}_{t,b}$$



$$(\mathbf{S}_{f,a} \Delta \mathbf{S}_{f,b}) \Delta \mathbf{S}_{f,c} = \mathbf{S}_{f,a} \Delta (\mathbf{S}_{f,b} \Delta \mathbf{S}_{f,c})$$

Using Associativity
One DoF to Multi-DoF

$$\mathbf{S}_{f,12\dots n} = \mathbf{S}_{f,1} \Delta \mathbf{S}_{f,2} \Delta \dots \Delta \mathbf{S}_{f,n} \longrightarrow \dot{\mathbf{S}}_{f,12\dots n} \Big|_{\substack{\theta_j=0 \\ t_j=0, j=1,2,\dots,n}} = \mathbf{S}_{t,1} + \mathbf{S}_{t,2} + \dots + \mathbf{S}_{t,n}$$

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



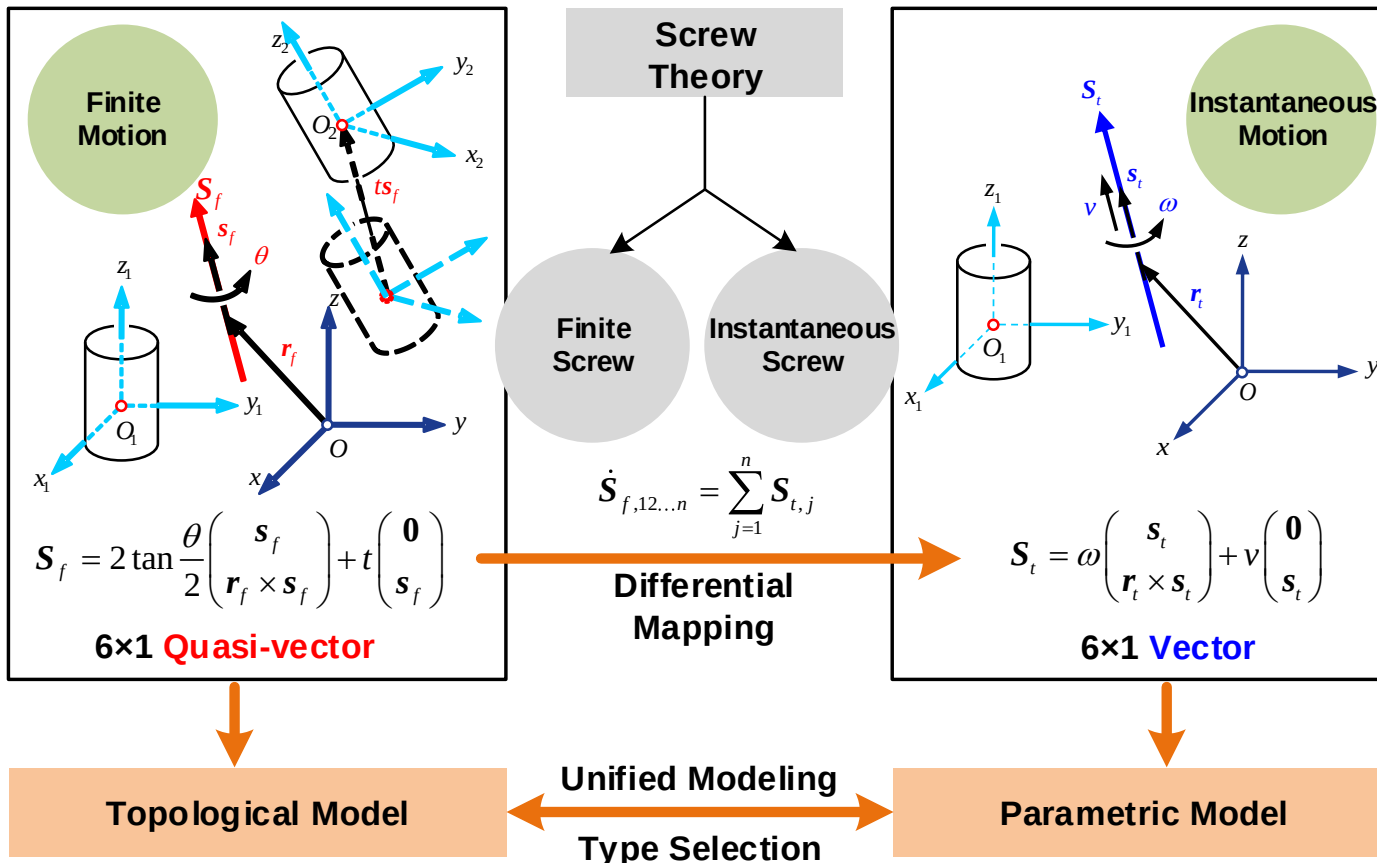
Example: Metrom



Conclusions

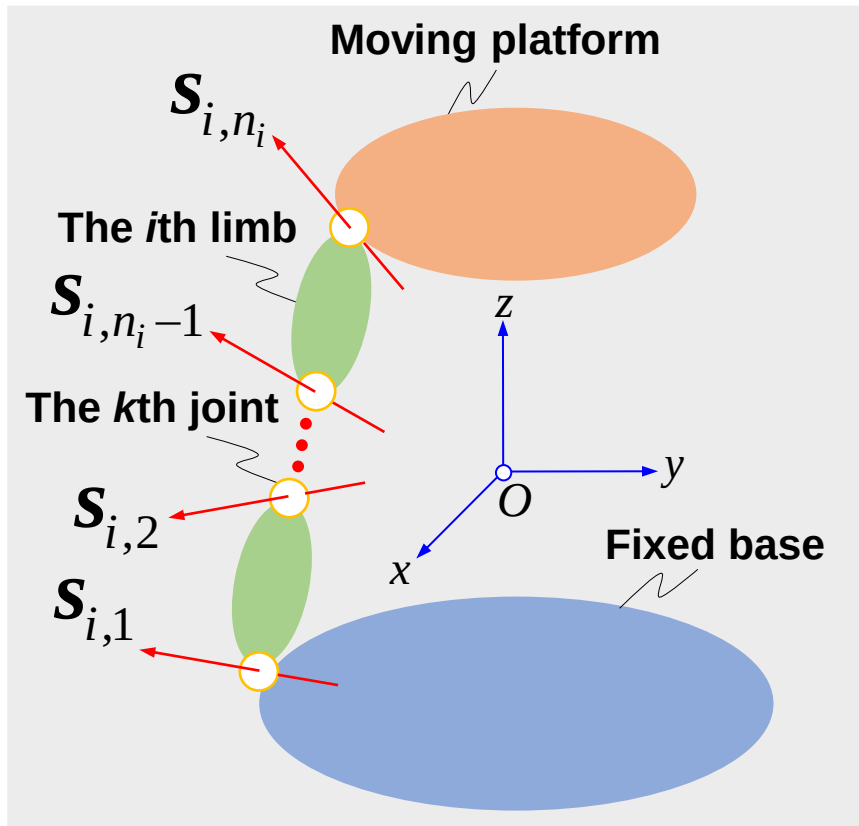
3. Topology and Performance Design

- ❑ The origin of finite screw is proven, and its simplest general form is given.
- ❑ The differential mapping between finite and instantaneous screws is revealed.
- ❑ Compared with existing methods, the simpler screw expressions have more clear physical meanings, are easier to be composited and computed.



3. Topology and Performance Design

A parallel mechanism consists of l limbs that connect the fixed base and the moving platform. Each limb can be regarded as the combination of n_i ($i = 1, 2, \dots, l$) one-DoF joints.



For Parallel Mechanism

$$S_{f,PM} = S_{f,1} \cap S_{f,2} \cap \dots \cap S_{f,l}$$

For Its Limbs

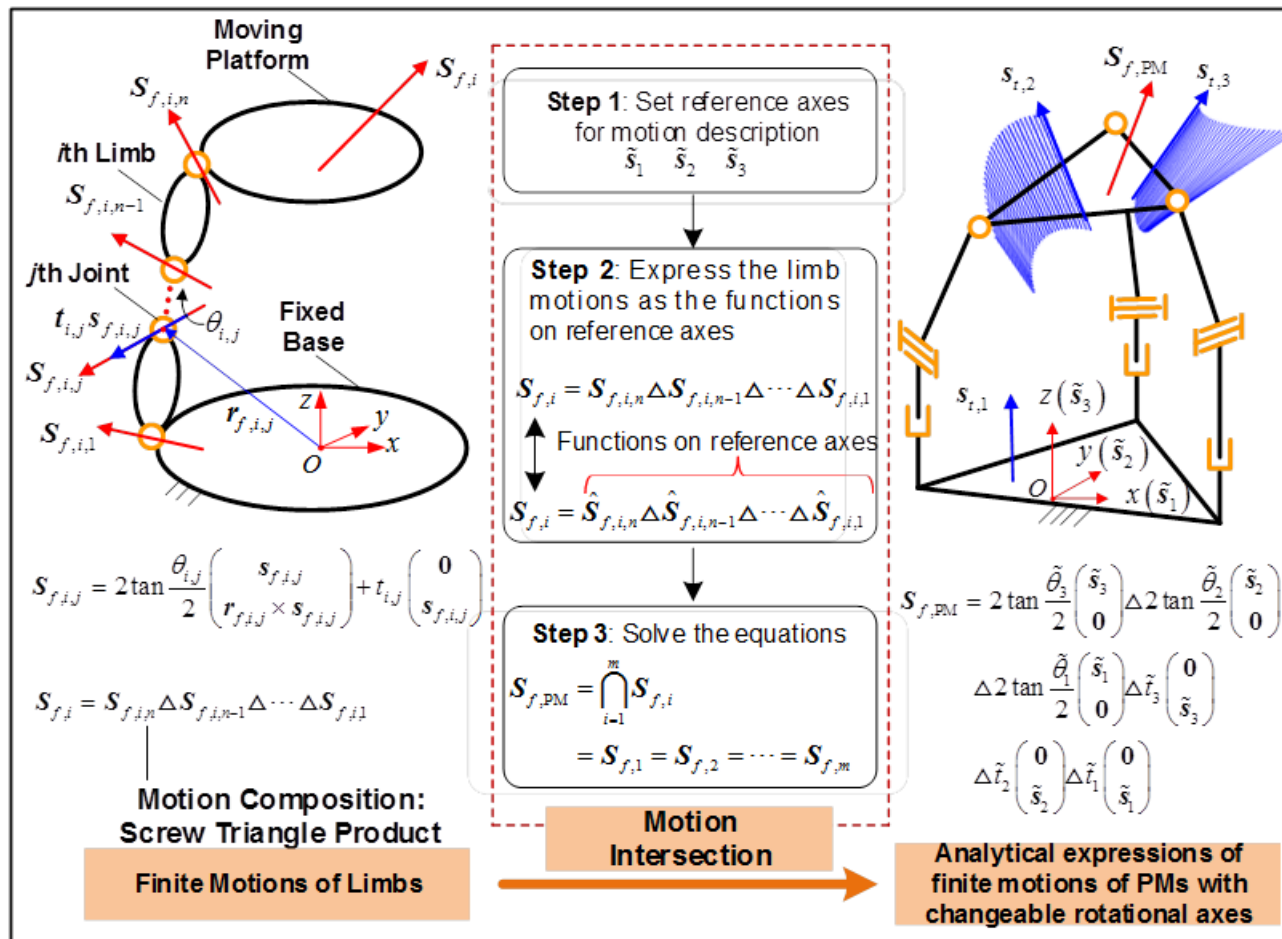
$$S_{f,i} = S_{f,i,n_i} \Delta S_{f,i,n_i-1} \Delta \dots \Delta S_{f,i,1}$$

For the Joints

$$S_{f,i,k} = \begin{cases} 2 \tan \frac{\theta_{i,k}}{2} \begin{pmatrix} s_{i,k} \\ r_{i,k} \times s_{i,k} \end{pmatrix} & \text{R joint} \\ t_{i,k} \begin{pmatrix} 0 \\ s_{i,k} \end{pmatrix} & \text{P joint} \end{cases}$$

3. Topology and Performance Design

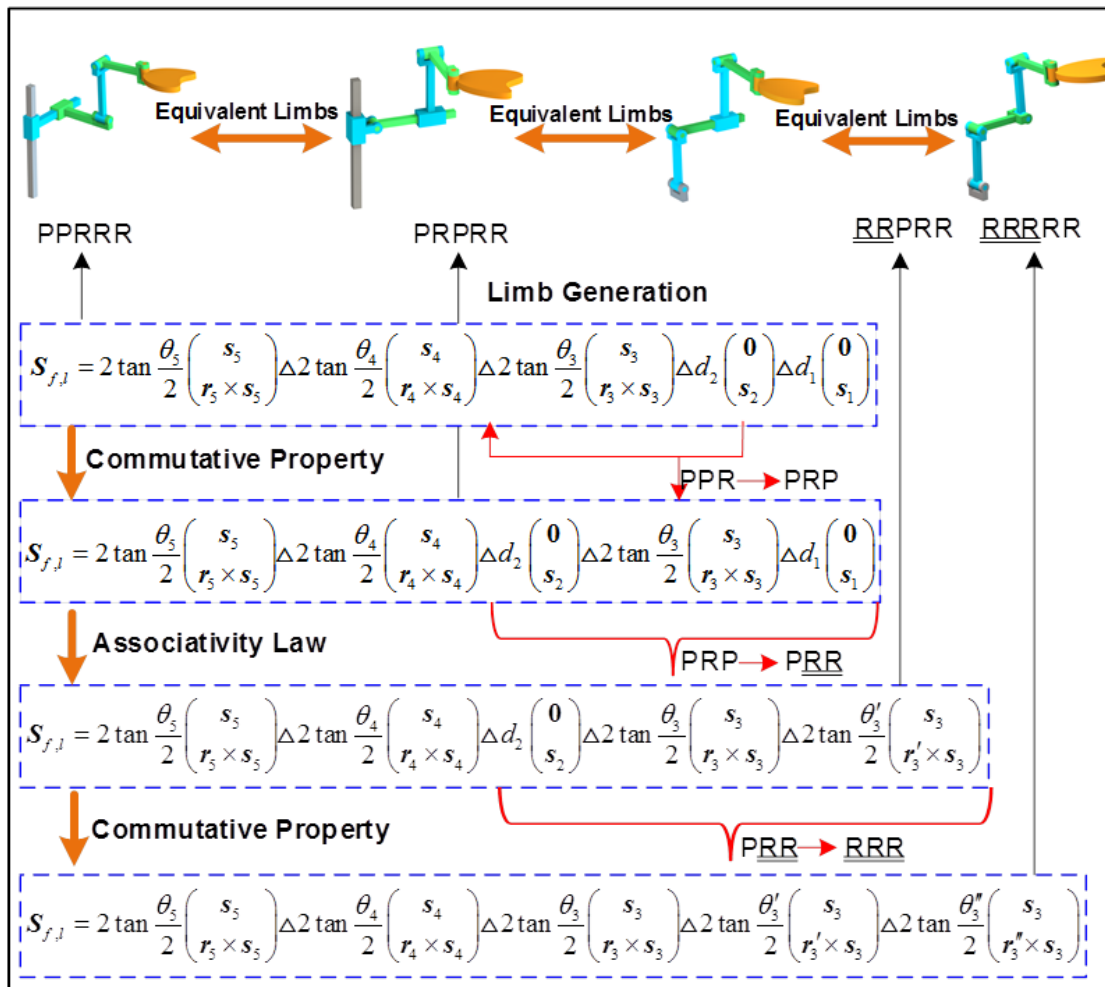
An algebraic topology design (type synthesis) method based on finite screw is proposed, in which composition and intersection of finite motions are computed analytically using screw triangle product.



A general flow for topology design method

3. Topology and Performance Design

The associativity law and commutative properties of screw triangle product are proved, resulting in an algebraic method for limb synthesis purely using analytical derivations.



3. Topology and Performance Design

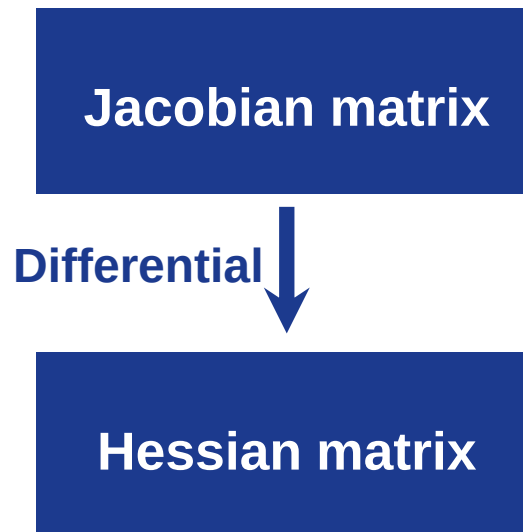
The performance (kinematic, static and dynamic, etc.) models would be obtained directly by differentiating the topology model.

For Parallel Mechanism

$$\begin{aligned}
 \mathbf{S}_{t,PM} &= \dot{\mathbf{S}}_{f,PM} \Big|_{\substack{\theta_{i,k}=0 \quad i=1,2,\dots,l \\ t_{i,k}=0 \quad k=1,2,\dots,n_i}} \\
 &= \dot{\mathbf{S}}_{f,1} \Big|_{\substack{\theta_{1,k}=0 \\ t_{1,k}=0}, k=1,2,\dots,n_1} \cap \dot{\mathbf{S}}_{f,2} \Big|_{\substack{\theta_{2,k}=0 \\ t_{2,k}=0}, k=1,2,\dots,n_2} \cap \dots \cap \dot{\mathbf{S}}_{f,l} \Big|_{\substack{\theta_{l,k}=0 \\ t_{l,k}=0}, k=1,2,\dots,n_l} \\
 &= \mathbf{S}_{t,1} \cap \mathbf{S}_{t,2} \cap \dots \cap \mathbf{S}_{t,l}
 \end{aligned}$$

For Its Limbs

$$\begin{aligned}
 \mathbf{S}_{t,i} &= \dot{\mathbf{S}}_{f,i} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}, k=1,2,\dots,n_i} \\
 &= \sum_{k=1}^{n_i} \dot{\mathbf{S}}_{f,i,k} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}, k=1,2,\dots,n_i} \\
 &= \sum_{k=1}^{n_i} \mathbf{S}_{t,i,k}
 \end{aligned}$$



For the Joints

$$\begin{aligned}
 \mathbf{S}_{t,i,k} &= \dot{\mathbf{S}}_{f,i,k} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}} \\
 &= \begin{cases} \omega_{i,k} \begin{pmatrix} \mathbf{s}_{i,k} \\ \mathbf{r}_{i,k} \times \mathbf{s}_{i,k} \end{pmatrix} & \text{R joint} \\ v_{i,k} \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_{i,k} \end{pmatrix} & \text{P joint} \end{cases}
 \end{aligned}$$

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



Example: Metrom



Conclusions

4. Example: Metrom



$$\mathbf{S}_{f,PKM} = t_2 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_2 \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_1 \end{pmatrix} \Delta 2 \tan \frac{\theta_c}{2} \begin{pmatrix} \mathbf{s}_c \\ \mathbf{r}_o \times \mathbf{s}_c \end{pmatrix} \\ \Delta 2 \tan \frac{\theta_b}{2} \begin{pmatrix} \mathbf{s}_b \\ \mathbf{r}_o \times \mathbf{s}_b \end{pmatrix} \Delta 2 \tan \frac{\theta_a}{2} \begin{pmatrix} \mathbf{s}_a \\ \mathbf{r}_o \times \mathbf{s}_a \end{pmatrix}$$

$$\mathbf{S}_{f,PKM} \subseteq \mathbf{S}_{f,i} \longrightarrow \mathbf{S}_{f,i} = \mathbf{S}_{f,PKM} \Delta t_3 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_3 \end{pmatrix}$$

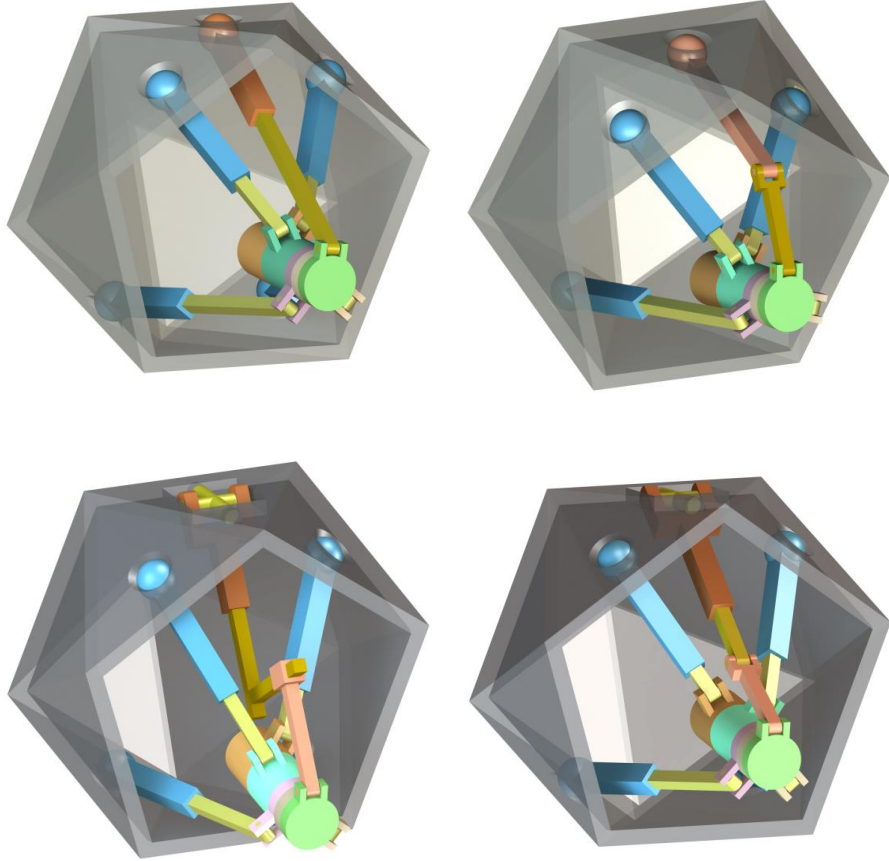
$$\mathbf{S}'_{f,L1} = 2 \tan \frac{\theta_c}{2} \begin{pmatrix} \mathbf{s}_c \\ \mathbf{r}_o \times \mathbf{s}_c \end{pmatrix} \Delta t_2 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_2 \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_1 \end{pmatrix} \\ \Delta 2 \tan \frac{\theta_b}{2} \begin{pmatrix} \mathbf{s}_b \\ \mathbf{r}_o \times \mathbf{s}_b \end{pmatrix} \Delta 2 \tan \frac{\theta_a}{2} \begin{pmatrix} \mathbf{s}_a \\ \mathbf{r}_o \times \mathbf{s}_a \end{pmatrix}$$

Table 1. 5-DOF limb structures for 2T3R PKM

Standard limbs	Derivative limbs
SP ₁ P ₂	UP ₁ P ₂ R _c , UP ₁ R _c P ₂
	SP ₁ R _c , SR _c R _c
	UP ₁ R _c R _c , SR _c P ₁

Metrom is with 5-DoF machining capacity
Big workspace, Flexibility
Large components with complex surface

4. Example: Metrom



Many topology structures
will be synthesized

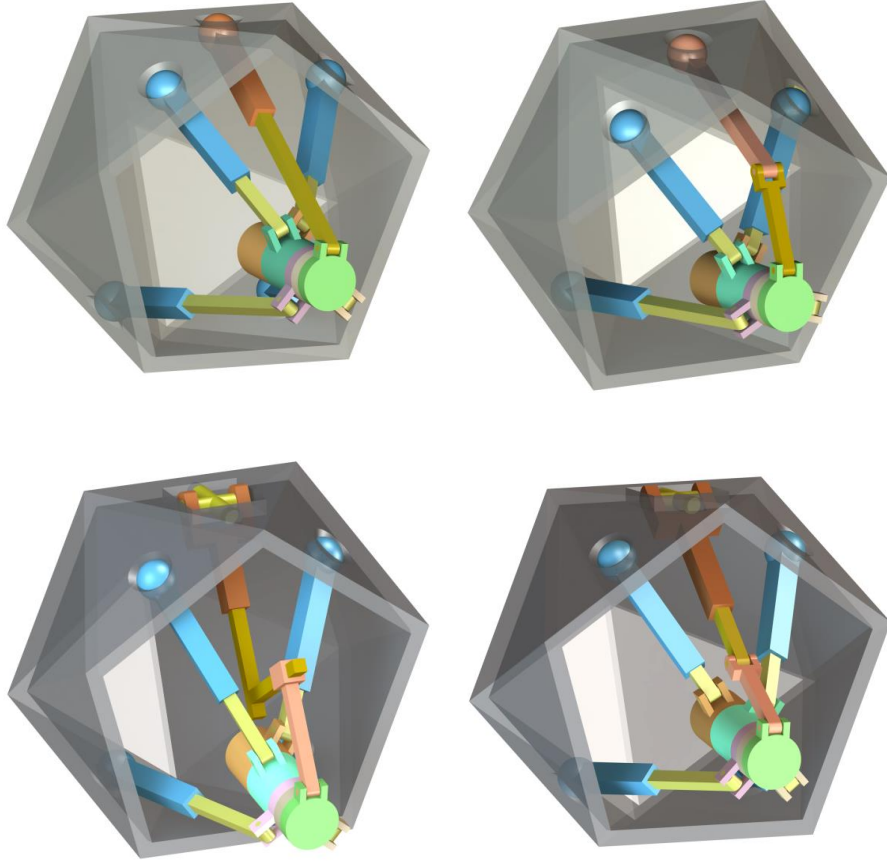
$$\mathbf{S}_{f, \text{METROM}} = t_2 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_2 \end{pmatrix} \Delta t_1 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_1 \end{pmatrix} \Delta 2 \tan \frac{\theta_c}{2} \begin{pmatrix} \mathbf{s}_c \\ \mathbf{r}_o \times \mathbf{s}_c \end{pmatrix} \\ \Delta 2 \tan \frac{\theta_b}{2} \begin{pmatrix} \mathbf{s}_b \\ \mathbf{r}_o \times \mathbf{s}_b \end{pmatrix} \Delta 2 \tan \frac{\theta_a}{2} \begin{pmatrix} \mathbf{s}_a \\ \mathbf{r}_o \times \mathbf{s}_a \end{pmatrix}$$

$$\mathbf{S}_{t, \text{METROM}} = \dot{\mathbf{S}}_{f, \text{METROM}} \bigg|_{\substack{\theta_m=0 \\ t_n=0, m=a,b,c, n=1,2}} \\ = \dot{t}_2 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_2 \end{pmatrix} + \dot{t}_1 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_1 \end{pmatrix} + \dot{\theta}_c \begin{pmatrix} \mathbf{s}_c \\ \mathbf{r}_o \times \mathbf{s}_c \end{pmatrix} + \dot{\theta}_b \begin{pmatrix} \mathbf{s}_b \\ \mathbf{r}_o \times \mathbf{s}_b \end{pmatrix} + \dot{\theta}_a \begin{pmatrix} \mathbf{s}_a \\ \mathbf{r}_o \times \mathbf{s}_a \end{pmatrix} \\ = \mathbf{v}_2 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_2 \end{pmatrix} + \mathbf{v}_1 \begin{pmatrix} \mathbf{0} \\ \mathbf{s}_1 \end{pmatrix} + \omega_c \begin{pmatrix} \mathbf{s}_c \\ \mathbf{r}_o \times \mathbf{s}_c \end{pmatrix} + \omega_b \begin{pmatrix} \mathbf{s}_b \\ \mathbf{r}_o \times \mathbf{s}_b \end{pmatrix} + \omega_a \begin{pmatrix} \mathbf{s}_a \\ \mathbf{r}_o \times \mathbf{s}_a \end{pmatrix}$$

$$\mathbf{S}_{t, \text{METROM}} = \begin{bmatrix} \mathbf{s}_a & \mathbf{s}_b & \mathbf{s}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{r}_o \times \mathbf{s}_a & \mathbf{r}_o \times \mathbf{s}_b & \mathbf{r}_o \times \mathbf{s}_c & \mathbf{s}_1 & \mathbf{s}_2 \end{bmatrix} \begin{bmatrix} \omega_a \\ \omega_b \\ \omega_c \\ \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$

Jacobian matrix

4. Example: Metrom



Many topology structures
will be synthesized

Jacobian matrix

$$\mathbf{S}_{t, \text{METROM}} = \begin{bmatrix} \mathbf{s}_a & \mathbf{s}_b & \mathbf{s}_c & \mathbf{0} & \mathbf{0} \\ \mathbf{r}_O \times \mathbf{s}_a & \mathbf{r}_O \times \mathbf{s}_b & \mathbf{r}_O \times \mathbf{s}_c & \mathbf{s}_1 & \mathbf{s}_2 \end{bmatrix} \begin{bmatrix} \omega_a \\ \omega_b \\ \omega_c \\ v_1 \\ v_2 \end{bmatrix}$$



Hessian matrix

$$\begin{aligned} \mathbf{S}_{a, \text{METROM}} &= \mathbf{J}\dot{\mathbf{q}} + \dot{\mathbf{J}}\mathbf{q} \\ &= \mathbf{J}\dot{\mathbf{q}} + \begin{bmatrix} 0 & ((\mathbf{s}_a \times \mathbf{s}_c)\omega_a + (\mathbf{s}_b \times \mathbf{s}_c)\omega_b) & 0 & 0 \\ 0 & \begin{pmatrix} \mathbf{s}_b^T \mathbf{r}_O \mathbf{s}_a \\ -\mathbf{s}_a^T \mathbf{r}_O \mathbf{s}_b \end{pmatrix} \omega_a & ((\mathbf{s}_a \times \mathbf{s}_1)\omega_a + (\mathbf{s}_b \times \mathbf{s}_1)\omega_b) & ((\mathbf{s}_a \times \mathbf{s}_2)\omega_a + (\mathbf{s}_b \times \mathbf{s}_2)\omega_b) \\ 0 & ((\mathbf{s}_c^T \mathbf{r}_O \mathbf{s}_a - \mathbf{s}_a^T \mathbf{r}_O \mathbf{s}_c)\omega_a + (\mathbf{s}_c^T \mathbf{r}_O \mathbf{s}_b - \mathbf{s}_b^T \mathbf{r}_O \mathbf{s}_c)\omega_b) & +(\mathbf{s}_c \times \mathbf{s}_1)\omega_c & +(\mathbf{s}_c \times \mathbf{s}_2)\omega_c \end{bmatrix} \mathbf{q} \end{aligned}$$

Outline



Background



Finite Screw and Instantaneous Screw



Topology and Performance Design



Example: Metrom



Conclusions

5. Conclusions

- ✓ The composition, associativity and intersection computational properties of finite screws are investigated and proved.
- ✓ The differential mapping between finite screw and instantaneous screw are revealed.
- ✓ The finite screw based topological model of PMs for topology design is formulated, leading to an algebraic type synthesis method, which is applicable for PMs with changeable rotational axes.
- ✓ The instantaneous screw based parametric model of PMs for performance design is directly derived from the topological model, which is ready for velocity, force, stiffness, accuracy and dynamic analysis.
- ✓ The strongly related topological and parametric models result in an unified method for integrated design of PMs under the simple and concise framework of screw theory.

Thank you!

Tao SUN, Shuofei YANG, Tian HUANG, Jian S. DAI

Tianjin University



天津大学
Tianjin University