

Additional file 4

Leakage-free covariate adjustment

Let v be the target variable or feature which we want to adjust by covariates $C_1, C_2, \dots, C_r$. Denote by v_i and $C_{i1}, C_{i2}, \dots, C_{ir}$ the observed values of v and the C s for subject i .

Suppose we are given a cross-validation split of the subjects into a training and a testing set.

Continuous v

If v is continuous, we use the **training set** to fit a linear regression model:

$$v = \beta_0 + \beta_1 C_1 + \beta_2 C_2 + \dots + \beta_r C_r.$$

For each subject i (**whether in the training or testing set**), we subtract the model-fitted values from v_i :

$$v_i^{\text{adj}} = v_i - (\hat{\beta}_0 + \hat{\beta}_1 C_{i1} + \hat{\beta}_2 C_{i2} + \dots + \hat{\beta}_r C_{ir}).$$

Binary v

If v is binary with values are 0 and 1, we use the **training set** to fit a logistic regression model:

$$\text{logit}(\pi) = \beta_0 + \beta_1 C_1 + \beta_2 C_2 + \dots + \beta_r C_r$$

where $\pi = \text{Prob}(v = 1)$.

For each subject i (**whether in the training or testing set**), we subtract the model-fitted values from the observed outcomes:

$$v_i^{\text{adj}} = v_i - \hat{\pi}_i$$

where

$$\hat{\pi}_i = \frac{\exp(\hat{\beta}_0 + \hat{\beta}_1 C_{i1} + \hat{\beta}_2 C_{i2} + \dots + \hat{\beta}_r C_{ir})}{1 + \exp(\hat{\beta}_0 + \hat{\beta}_1 C_{i1} + \hat{\beta}_2 C_{i2} + \dots + \hat{\beta}_r C_{ir})}.$$

Multiclass ordinal v

This is a simple extension of the binary case. If v is multiclass ordinal with values $0, 1, \dots, K$, we use the **training set** to fit a multinomial logistic regression model, deriving values $\hat{\pi}_k$ for $k = 1, \dots, K$.

For each subject i (**whether in the training or testing set**), we subtract the model-fitted values from the observed outcomes:

$$v_i^{\text{adj}} = v_i - \sum_{k=1}^K \hat{\pi}_{ki} k.$$