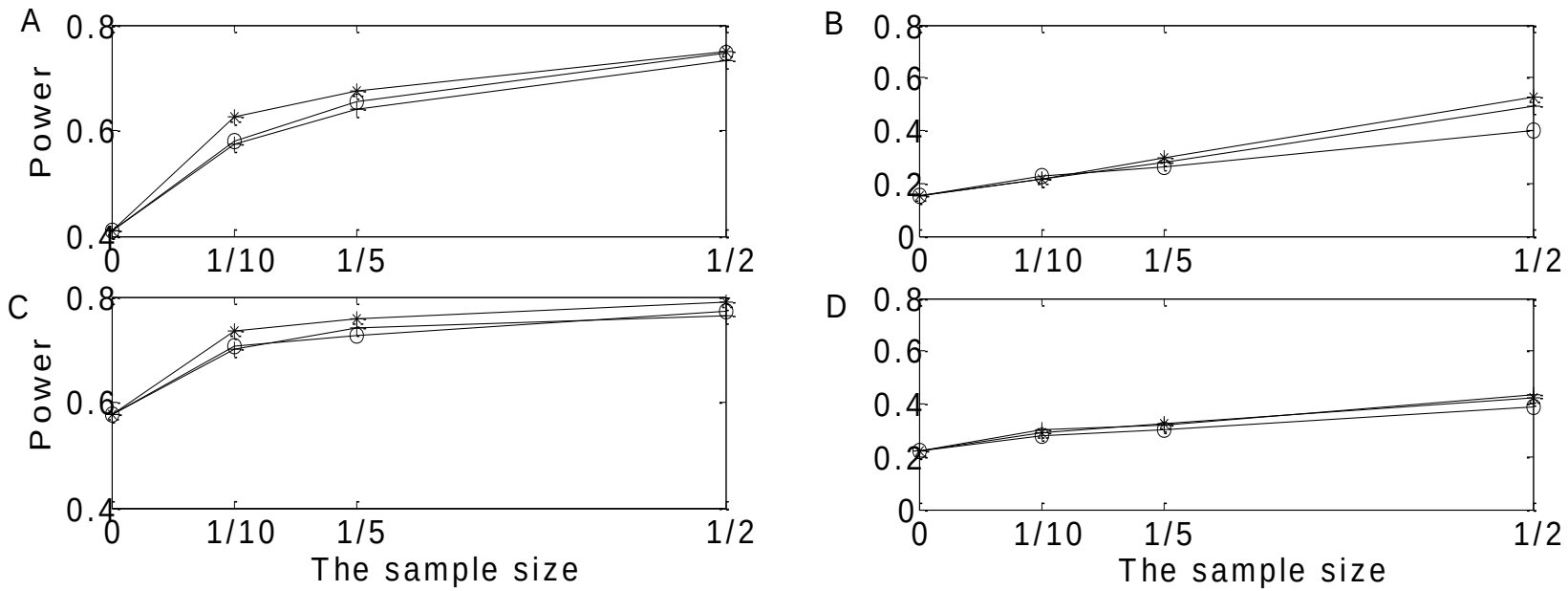

Fig.S1 Empirical power against the sample size at the 0.05 significance level in population stratification when there are 20% non-causal variants.

Note: A and **B** are for $\tilde{Z}_C$, and **C** and **D** are for TDT_{BRV} when causal variants have different effects with the same direction and causal variants have opposite effects, respectively. The sample size $N=N_0$, $N_0+1/10N_0$, $N_0+1/5N_0$, $N_0+1/2N_0$ with $N_0=500$ denoted by 0, 1/10, 1/5, and 1/2 respectively. $\Omega_{0+\mathrm{I}}$ (○), $\Omega_{0+\mathrm{II}}$ (*), $\Omega_{0+\mathrm{I}+\mathrm{II}}$ (+).

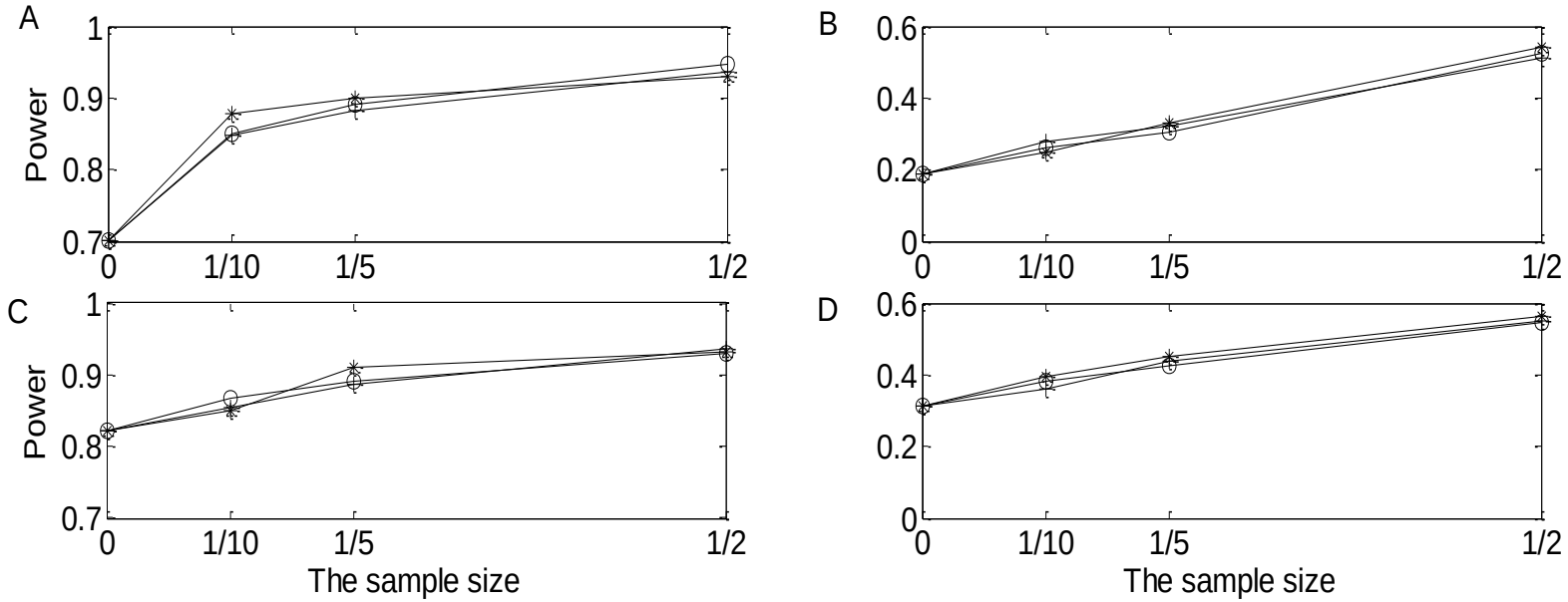

Fig.S2 Empirical power against the sample size at the 0.05 significance level in population stratification when there are 40% non-causal variants.

Note: A and **B** are for $\tilde{Z}_C$, and **C** and **D** are for TDT_{BRV} when causal variants have different effects with the same direction and causal variants have opposite effects, respectively. The sample size $N=N_0$, $N_0+1/10N_0$, $N_0+1/5N_0$, $N_0+1/2N_0$ with $N_0=500$ denoted by 0, 1/10, 1/5, and 1/2 respectively. $\Omega_{0+\mathrm{I}}$ (○), $\Omega_{0+\mathrm{II}}$ (*), $\Omega_{0+\mathrm{I}+\mathrm{II}}$ (+).

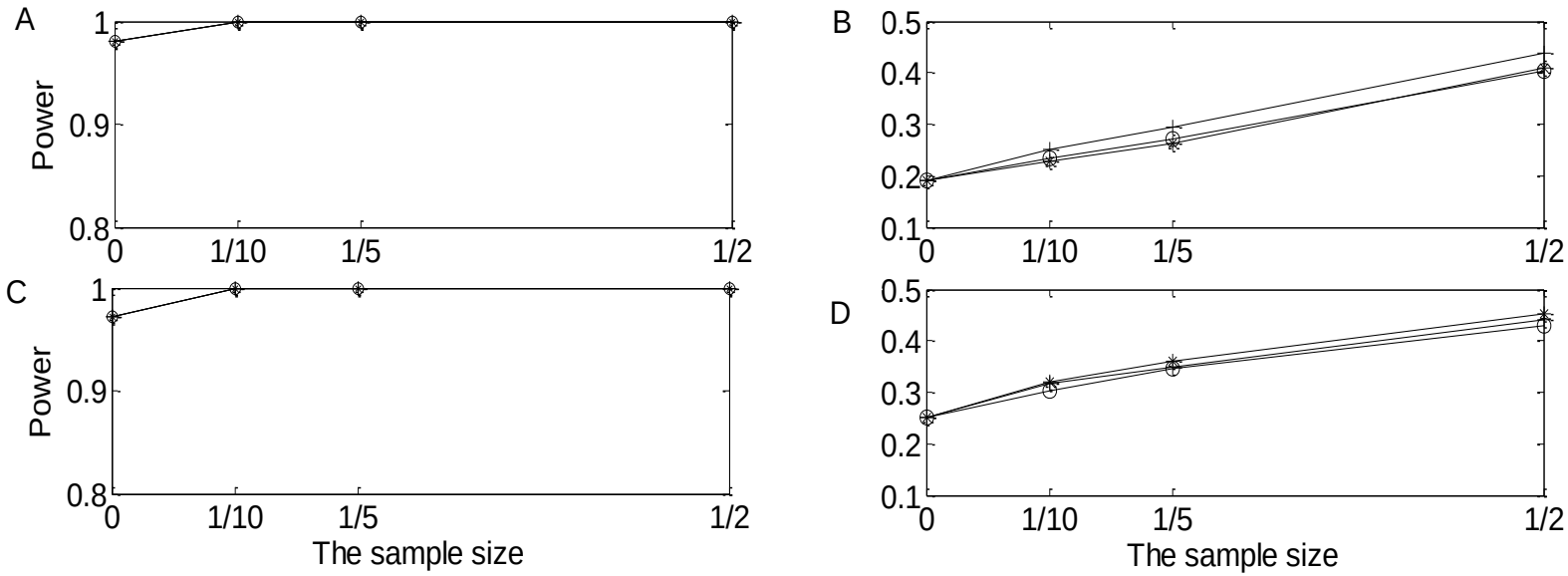

Fig.S3 Empirical power against the sample size at the 0.05 significance level in population stratification when there are 60% non-causal variants.

Note: A and **B** are for $\tilde{Z}_C$, and **C** and **D** are for TDT_{BRV} when causal variants have different effects with the same direction and causal variants have opposite effects, respectively. The sample size $N=N_0$, $N_0+1/10N_0$, $N_0+1/5N_0$, $N_0+1/2N_0$ with $N_0=500$ denoted by 0, 1/10, 1/5, and 1/2 respectively. $\Omega_{0+\mathrm{I}}$ (○), $\Omega_{0+\mathrm{II}}$ (*), $\Omega_{0+\mathrm{I}+\mathrm{II}}$ (+).

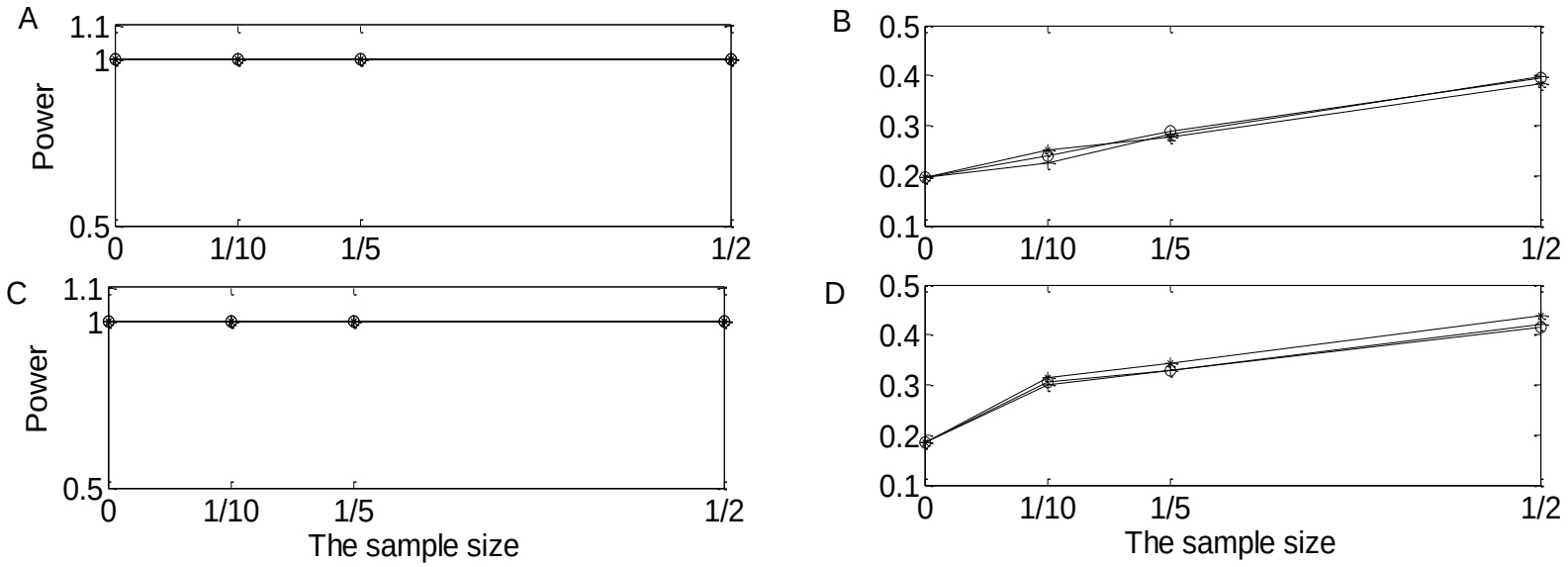

Fig.S4 Empirical power against the sample size at the 0.05 significance level in population stratification when there are 80% non-causal variants.

Note: A and **B** are for $\tilde{Z}_C$, and **C** and **D** are for TDT_{BRV} when causal variants have different effects with the same direction and causal variants have opposite effects, respectively. The sample size $N=N_0$, $N_0+1/10N_0$, $N_0+1/5N_0$, $N_0+1/2N_0$ with $N_0=500$ denoted by 0, 1/10, 1/5, and 1/2 respectively. Ω_{0+I} (○), Ω_{0+II} (*), Ω_{0+I+II} (+).