Modified ICC calculation from generalized linear mixed-effects models (GLMMs) with Poisson error structure and log-link.

GLMM structure

$Y_{ij} \sim Poisson\ (\mu_{ij})$
$\mu_{ij} = \exp(\beta_0 + \alpha_i + e_{ij})$
$\alpha_i \sim Gaussian\ (0, \sigma_\alpha^2)$
$e_{ij} \sim Gaussian\ (0, \sigma_e^2)$

where Y_{ij} is the observed count for the *i*th individual at the *j*th occasion, μ_{ij} is the underlying (latent) mean for the *i*th individual at the *j*th occasion, β_0 is the intercept on the link scale, α_i is the individual (random) effect term with a between-individual variance σ_α^2 and e_{ij} is the additive over-dispersion (residual) term on the log-normal scale with a variance of σ_e^2.

Residual variance

$$\sigma_\varepsilon^2 = \omega + \ln({}^{1}/_{\exp(\beta_0)} + 1)$$

where corresponding symbols are same as above with ω being the dispersion parameter for the model.

Modified ICC on latent scale

$$ICC_{\text{lnormA}} = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_e^2 + \ln({}^{1}/_{\exp(\beta_0)+1})}$$

where ICC_{lnormA} is the log-normal scale ICC for count data with additive over-dispersion.

Modified ICC on original scale

$$ICC_{countA} = \frac{E[Yij]_A \cdot (\exp(\sigma_\alpha^2) - 1)}{E[Yij]_A \cdot (\exp(\sigma_\alpha^2 + \sigma_e^2) - 1) + 1}$$

where ICC_{countA} is the count-scale ICC estimated from a Poisson model with additive over-dispersion, $E[Y_{ij}]_{\text{A}}$ is the expectation for Y_{ij}.

$$E[Y_{ij}]_{\text{A}} = \exp(\beta_0 + \frac{\sigma_\alpha^2 + \sigma_e^2}{2})$$

Adapted from work by Nakagawa et al [1]

Reference

1. Nakagawa S, Schielzeth H. Repeatability for Gaussian and non-Gaussian data: a practical guide for biologists. *Biological Reviews.* 2010;85(4):935-956.