Additional Files

MCCC Gradient ascendant step-by-step solution

This section detail the Gradient solution using the Complex Correntropy as a cost function. Let J be a cost function as

$$J = E[G_\sigma^c(e)] = E[G_\sigma^c(d-y)] \tag{39}$$

Using the Gaussian kernel and computing the expected value over a window with N samples, one could write :

$$J_n = \frac{1}{2\pi\sigma^2}\frac{1}{N}\sum_{i=n-N+1}^{n}\exp\left(-\frac{(d_i-y_i)(d_i-y_i)^*}{2\sigma^2}\right) = \frac{1}{2\pi\sigma^2}\frac{1}{N}\sum_{i=n-N+1}^{n}\exp\left(-\frac{e_ie_i^*}{2\sigma^2}\right) \tag{40}$$

Expanding the exponential argument, $e_ie_i^*$:

$$\begin{aligned} e_ie_i^* &= (d_i-y_i)(d_i-y_i)^* \\ e_ie_i^* &= (d_i-y_i)(d_i^*-y_i^*) \\ e_ie_i^* &= (d_id_i^* - d_iy_i^* - y_id_i^* + y_iy_i^*) \end{aligned} \tag{41}$$

Knowing that $y_i = w^Hx_i$

$$\begin{aligned} y_i &= w^Hx_i \\ y_i^* &= (w^Hx_i)^* \\ y_i^* &= (w^H)^*(x_i)^* \end{aligned} \tag{42}$$

But, $w^H = (w^*)^T = (w^T)^*$, then:

$$\begin{aligned} y_i^* &= (w^H)^*(x_i)^* \\ y_i^* &= ((w^T)^*)^*(x_i)^* \\ y_i^* &= w^Tx_i^* \end{aligned} \tag{43}$$

Then, one could rewrite 41 to achieve:

$$\begin{aligned} e_ie_i^* &= (d_id_i^* - d_iy_i^* - y_id_i^* + y_iy_i^*) \\ e_ie_i^* &= (d_id_i^*) - (d_iw^Tx_i^*) - (w^Hx_id_i^*) + (w^Hx_iw^Tx_i^*) \end{aligned} \tag{44}$$

In order to use the ascend gradient, one could do:

$$W_{n+1} = W_n + \mu \bigtriangledown J_n \tag{45}$$

Then, using Wirtinger calculus to obtain the gradient:

$$\bigtriangledown J_n = \frac{\partial J_n}{\partial w^*} = \frac{1}{2\pi\sigma^2}\frac{1}{N}\sum_{i=n-N+1}^{n}\exp\left(-\frac{(d_i-y_i)(d_i-y_i)^*}{2\sigma^2}\right)\frac{(-1)}{2\sigma^2}\frac{\partial(e_ie_i^*)}{\partial w^*} \tag{46}$$

Then, obtaining the derivation of the exponential argument:

$$\frac{\partial(e_ie_i^*)}{\partial w^*} = \frac{\partial(d_id_i * -d_iy_i^* - y_id_i^* + y_iy_i^*)}{\partial w^*} \tag{47}$$

which could be rewrite as:

$$\frac{\partial(e_ie_i^*)}{\partial w^*} = \frac{\partial(d_id_i^*)}{\partial w^*} - \frac{\partial(d_iw^Tx_i^*)}{\partial w^*} - \frac{\partial(w^Hx_id_i^*)}{\partial w^*} + \frac{\partial(w^Hx_iw^Tx_i^*)}{\partial w^*} \tag{48}$$

$$\frac{\partial(e_i e_i^*)}{\partial w^*} = \frac{\partial(d_i d_i^*)}{\partial w^*} - \frac{\partial(d_i w^T x_i^*)}{\partial w^*} - \frac{\partial((w^*)^T x_i d_i^*)}{\partial w^*} + \frac{\partial((w^*)^T x_i\, w^T x_i^*)}{\partial w^*} \tag{49}$$

Resulting in

$$\begin{aligned}
\frac{\partial(e_i e_i^*)}{\partial w^*} &= 0 - 0 - x_i d_i^* + x_i\, w^T x_i^* \\
&= -x_i d_i^* + x_i\, w^T x_i^* \\
&= -(d_i^* - w^T x_i^*) x_i \\
&= -(d_i^* - (w^T x_i^*)^{**}) x_i \\
&= -(d_i^* - (w^H x_i)^*) x_i \\
&= -(d_i - w^H x_i)^* x_i \\
\frac{\partial(e_i e_i^*)}{\partial w^*} &= -(d_i - w^H x_i)^* x_i = -(d_i - y_i)^* x_i = -e_i^* x_i
\end{aligned} \tag{50}$$

always using the denominator layout. Then, Eq. 46 could be written as

$$\bigtriangledown J_n = \frac{\partial J_n}{\partial w^*} = + \frac{1}{4\pi\sigma^4} \frac{1}{N} \sum_{i=n-N+1}^{n} \exp\left(-\frac{e_i e_i^*}{2\sigma^2}\right) (e_i^* x_i) \tag{51}$$

Which implies in a update rule:

$$W_{n+1} = W_n + \frac{\mu}{N 4\pi\sigma^3} \sum_{i=n-N+1}^{n} \exp\left(-\frac{e_i e_i^*}{2\sigma^2}\right) e_i^* X_i \tag{52}$$

where $e_i = d_i - W_n^H X_i$. Using the stochastic gradient, one could obtain

$$W_{n+1} = W_n + \frac{\mu}{N 4\pi\sigma^3} \exp\left(-\frac{e_n e_n^*}{2\sigma^2}\right) e_n^* X_n \tag{53}$$

finishing the demonstration.