Supplementary Resource Document for Bego et al. (2024) "Single-Paper Meta-Analyses of the Effects of Spaced Retrieval Practice in Nine Introductory STEM Courses: Is the Glass Half Full or Half Empty?"

Assessment Items for Calculus I for Engineering Students (from Pearson's MyMathLab®)

Note: Pearson's online textbooks consist of questions with algorithmic variants; each time the question appears, it is slightly different, with different coefficients or attributes but focused on the same learning objective. Therefore, in this implementation, we selected only one question for each learning objective, and asked it multiple times, as described in the methodology.

LO1: Shift graphs of functions and write equations for the shifted graphs.

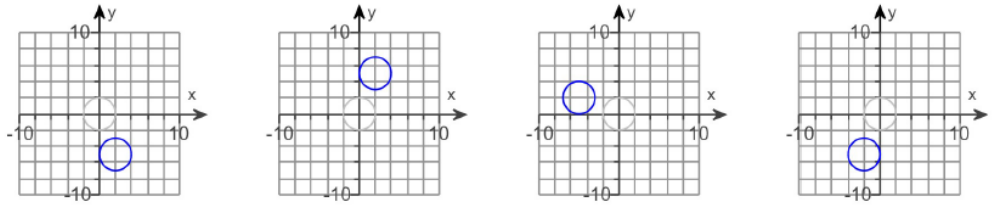
The number of units and the directions the graph of the equation is to be shifted are given. Give an equation for the shifted graph. Then sketch the original and shifted graphs together.

$x^2 + y^2 = 4$; down 5, right 2

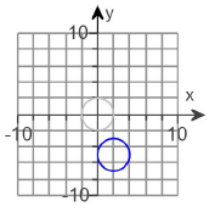
The equation of the shifted graph is $\square = 4$.

Which of the following shows the original graph in gray and the shifted graph in blue?

A. B. C. D.



Answers $(x-2)^2 + (y+5)^2$



A.

ID: 1.2.28

LO2: Find the domains and ranges of functions.

Find the domain and range of the function.

$$g(x) = \sqrt{x^2 - 7x}$$

The domain of the function is ☐.
(Type your answer in interval notation.)

The range of the function is ☐.
(Type your answer in interval notation.)

Answers $(-\infty, 0] \cup [7, \infty)$

$[0, \infty)$

ID: 1.1.4

LO3: Determine the domain of the function.

Find the domain of the function.

$$f(x) = \sqrt{\frac{5x}{x+2} - 4}$$

The domain is ☐.
(Type your answer in interval notation. Use integers or fractions for any numbers in the expression.)

Answer: $(-\infty, -2) \cup [8, \infty)$

ID: 2.7.63

LO4: Use long division to divide polynomials.

Divide using long division. State the quotient, q(x), and the remainder, r(x).

$$\frac{3x^3 + 7x^2 + 9x - 14}{x + 2}$$

$$\frac{3x^3 + 7x^2 + 9x - 14}{x + 2} = \square + \frac{\square}{x + 2}$$

(Simplify your answers. Do not factor.)

Answers $3x^2 + x + 7$

-28

ID: 2.4.9

LO5: Construct functions from formulas.

For $f(x) = x - 2$ and $g(x) = 4x^2 - 3$, find the following functions.

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(0)$; **d.** $(g \circ f)(0)$

a. $(f \circ g)(x) =$ ☐ (Simplify your answer.)

b. $(g \circ f)(x) =$ ☐ (Simplify your answer.)

c. $(f \circ g)(0) =$ ☐ (Simplify your answer.)

d. $(g \circ f)(0) =$ ☐ (Simplify your answer.)

Answers $4x^2 - 5$

$4x^2 - 16x + 13$

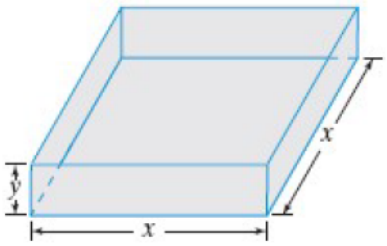
-5

13

ID: Instructor-created question

LO6: Solve applications involving modeling with functions.

The figure to the right shows an open box with a square base. The box is to have a volume of 6 cubic feet. Express the amount of material needed to construct the box, A, as a function of the length of a side of its square base, x.



A(x) = [ ] (Simplify your answer.)

Answer: $x^2 + \frac{24}{x}$

ID: 1.10.31

LO7: Find limits of polynomial and rational functions.

Find the following limit.

$$\lim_{u \to 1} \frac{u^3 - 1}{u^4 - 1}$$

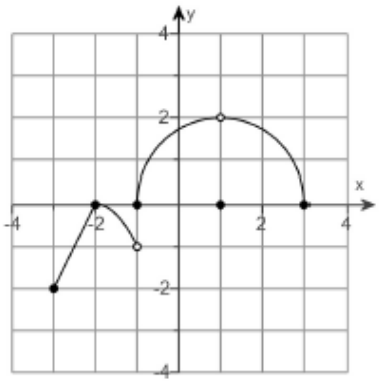
$\lim_{u \to 1} \frac{u^3 - 1}{u^4 - 1}$ = [ ] (Simplify your answer.)

Answer: $\frac{3}{4}$

ID: Instructor-created question

LO8: Find the limit of functions defined by graphs.

Which of the statements a through i about the function $y = f(x)$ graphed here are true, and which are false?



a. The statement $\lim_{x \to 1} f(x)$ does not exist is

(1) _____

b. The statement $\lim_{x \to 1} f(x) = 1$ is (2) _____

c. The statement $\lim_{x \to -1} f(x)$ does not exist is

(3) _____

d. The statement $\lim_{x \to c} f(x)$ exists at every point c in

$(-3, -1)$ is (4) _____

e. The statement $\lim_{x \to c} f(x)$ exists at every point c in $(-1, 3)$ is

(5) _____

(1) ○ true. ○ false. (2) ○ false. ○ true. (3) ○ true. ○ false. (4) ○ false. ○ true. (5) ○ true. ○ false.

Answers (1) false.

(2) false.

(3) true.

(4) true.

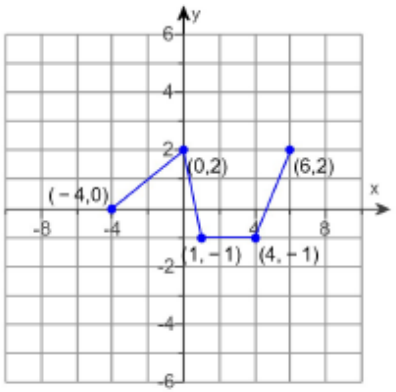
(5) true.

ID: Instructor-created question

LO9: Match a graphed function with the graph of its derivative.

(a) The graph in the figure given below is made of line segments joined end to end. At which points of the interval $[-4,6]$ is f' not defined? Give reasons for your answer.

(b) Graph the derivative of f.

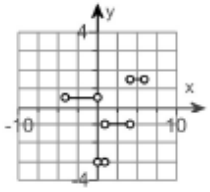


(a) Choose the correct answer below.

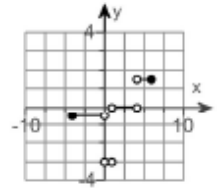
- **A.** f' is not defined at $x = 0$, $x = 1$, and $x = 4$ because at these points the graph has corners.
- **B.** f' is not defined at $x = 0$, $x = 1$, and $x = 6$ because at these points the graph has cusps.
- **C.** f' is not defined at $x = -4$, $x = 1$, and $x = 4$ because at these points the graph has corners.
- **D.** f' is not defined at $x = -4$, $x = 1$, and $x = 4$ because at these points the graph has cusps.

(b) Choose the correct graph of the derivative of f below.

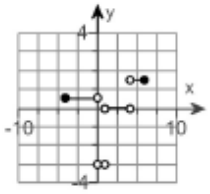
A.



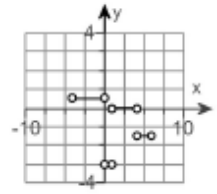
B.



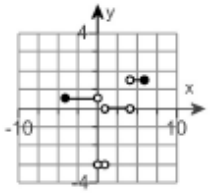
C.



D.



Answers A. f' is not defined at $x = 0$, $x = 1$, and $x = 4$ because at these points the graph has corners.



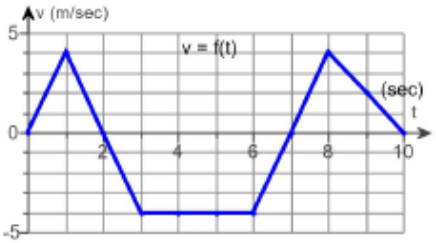
C.

ID: 3.2.31

LO10: Analyze the motion of an object given a graph of its acceleration or velocity.

The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)$ (m / sec) of a body moving along a coordinate line.

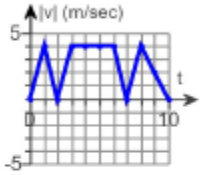
a. When does the body reverse direction?
b. When is it moving at a constant speed?
c. Graph the body's speed for $0 \le t \le 10$.
d. Graph the acceleration, where defined.



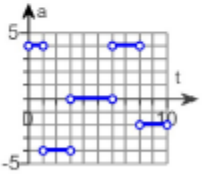
Answers 2,7

3

6



C.



B.

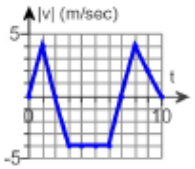
ID: 3.4.15

The body reverses direction at t = [ ]. (Use a comma to separate answers as needed.)

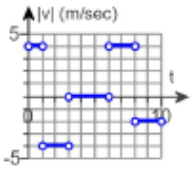
The body moves at a constant speed at [ ] ≤ t ≤ [ ].

Choose the correct graph of the body's speed for $0 \le t \le 10$.

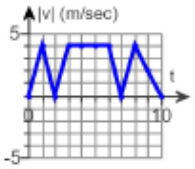
○ A.



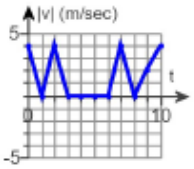
○ B.



○ C.

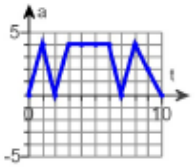


○ D.

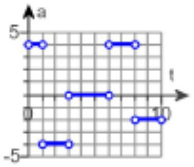


Choose the correct graph of the acceleration, where defined.

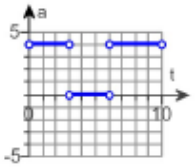
○ A.



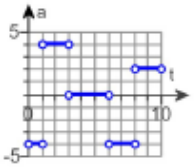
○ B.



○ C.



○ D.



LO11: Find slopes, tangents, and normal lines of a function. CALCULATE

The curve $y = ax^2 + bx + c$ passes through the point (1,7) and is tangent to the line $y = 4x$ at the origin. Find a, b, and c.

a = [ ], b = [ ], c = [ ]

Answers 3

4

0

ID: 3.3.45

LO12: Use the Chain Rule to evaluate derivatives at given values.

Assume that $f'(2) = -6$, $g'(3) = \frac{5}{8}$, $g(3) = 2$, and $y = f(g(x))$. What is y' at $x = 3$?

At $x = 3$, $y' =$ [ ].
(Simplify your answer.)

Answer: $-\frac{15}{4}$

ID: 3.6.73

LO13: Find derivatives using implicit differentiation.

Use implicit differentiation to find $\frac{dy}{dx}$ using the following equation.

$$x^5 + y^5 = 40xy$$

Choose the correct answer below.

- **A.** $\frac{dy}{dx} = \frac{8x - y^4}{x^4 - 8y}$
- **B.** $\frac{dy}{dx} = \frac{y^4 - 8x}{8y + x^4}$
- **C.** $\frac{dy}{dx} = \frac{x^4 - 8y}{y^4 - 8x}$
- **D.** $\frac{dy}{dx} = \frac{8y - x^4}{y^4 - 8x}$

Answer: D. $\frac{dy}{dx} = \frac{8y - x^4}{y^4 - 8x}$

ID: 3.7.21

LO14: Find derivatives using the chain rule.

Calculate the derivative of the following function.

$y = f(g(7x^2))$, where f and g are differentiable for all real numbers

What is $\frac{d}{dx} f(g(7x^2))$?

- A. $f'(g'(14x))$
- B. $f'(g(7x^2))g'(14x)$
- C. $f'(g(7x^2))g'(7x^2)14x$
- D. $f'(g(7x^2))g(7^2)g'(7x^2)14x$

Answer: C. $f'(g(7x^2))g'(7x^2)14x$

ID: 3.6.53

LO15: Find limits related to derivatives.

The following limit represents f′(a) for some function f and some value of a.

$$\lim_{h \to 0} \frac{\sqrt{81+h} - \sqrt{81}}{h}$$

a. Find a function f and a number a.
b. Determine the value of the limit by finding f′(a).

a. f(x) = ____
(Type an expression using x as the variable. Type an exact answer, using radicals as needed.)

a = ____
(Type an integer or a decimal.)

b. f′(a) = ____
(Type an exact answer, using radicals as needed.)

Answers $\sqrt{x}$

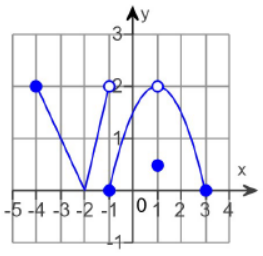
81

$\frac{1}{18}$

ID: 3.2.59

LO16: Determine at which points functions are differentiable.

The figure to the right shows the graph of a function over the closed interval D: $-4 \le x \le 3$.
At what domain points does the function appear to be differentiable?



The function appears to be differentiable on _____.
(Use interval notation.)

Answer: $[-4,-2) \cup (-2,-1) \cup (-1,1) \cup (1,3]$

ID: Instructor-created question

LO17: Graph and identify characteristics of trigonometric functions.

Graph the function. What is the period of the function?

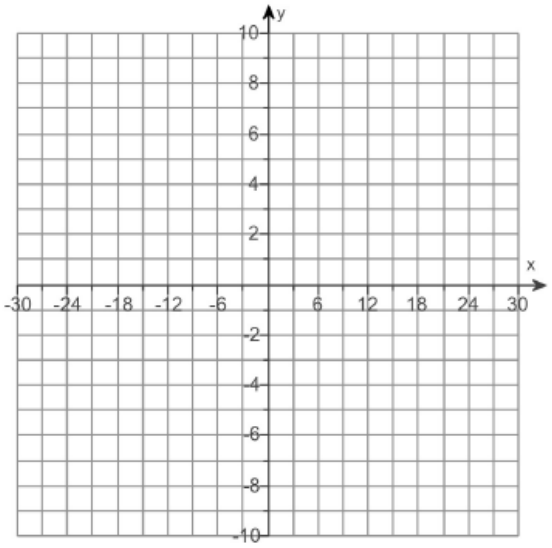
$$-\sin\frac{\pi x}{11}$$

The period of $-\sin\frac{\pi x}{11}$ is [ ].

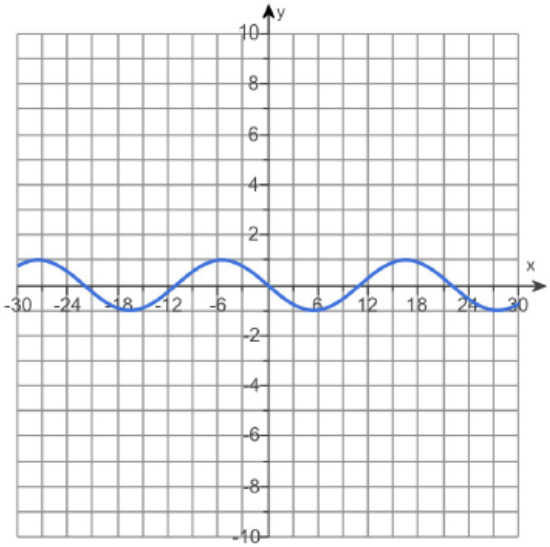
(Type an exact answer.)

Use the graphing tool to graph the equation.

(Type the word pi to insert the symbol π as needed.)



Answers 22



ID: 1.3.17

LO18: Identify the transformations of sine curves and graph the functions.

For $f(x) = A \sin\left(\frac{2\pi}{B}(x - C)\right) + D$, identify A, B, C, and D for $y = 4 \sin\left(x + \frac{\pi}{2}\right) - 2$ and sketch its graph.

Identify A, B, C, and D.

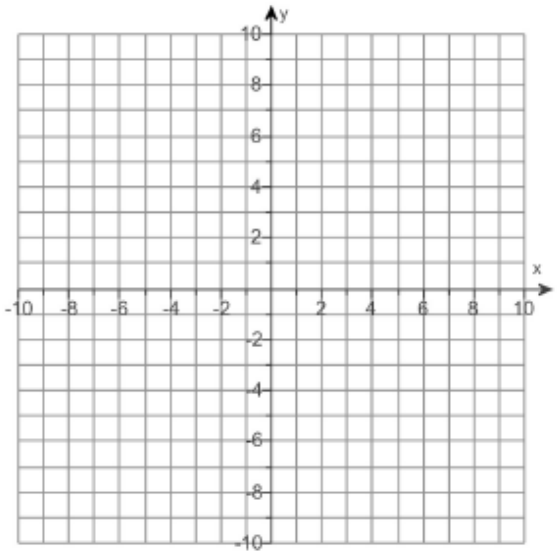
A = B =

C = D =

(Type exact answers, using π as needed.)

Use the graphing tool to graph the function.

(Type the word pi to insert the symbol π as needed.)

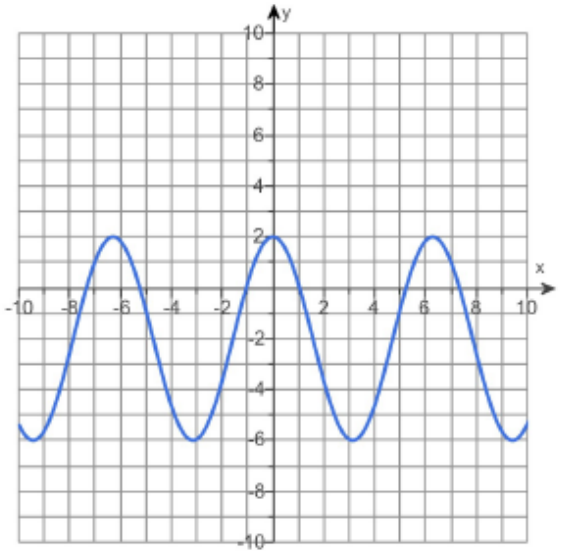


Answers 4

2π

$-\frac{\pi}{2}$

-2



ID: 1.3.67

LO19: Find the limits of trigonometric functions using the limit of sin(theta)/theta. CALC

Use the relation $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ to determine the limit shown below.

$$\lim_{\theta \to 0} \frac{\tan \theta}{\theta^2 \cot 4\theta}$$

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $\lim_{\theta \to 0} \frac{\tan \theta}{\theta^2 \cot 4\theta} =$ [ ] (Type an integer or a simplified fraction.)
- **B.** The limit does not exist.

Answer: A. $\lim_{\theta \to 0} \frac{\tan \theta}{\theta^2 \cot 4\theta} =$ **4** (Type an integer or a simplified fraction.)

ID: 2.4.43

LO20: Find derivatives of trigonometric functions.

Find $\frac{dy}{dx}$.

$$y = 2 \cos x \cot x$$

$\frac{dy}{dx} =$ [ ]

Answer: $-2 \cos x \csc^2 x - 2 \cos x$

ID: 3.5.7

LO21: Use the Chain Rule to find first derivatives.

Find $\frac{dy}{dt}$.

$y = \sin(\cos(3t - 7))$

$\frac{dy}{dt} =$ [ ]

Answer: $-3\cos(\cos(3t - 7))\sin(3t - 7)$

ID: 3.6.49

LO22: Use implicit differentiation to find first derivatives.

Find $\frac{dr}{d\theta}$.

$\cos 4r + \cot 6\theta = 5r$

$\frac{dr}{d\theta} =$ [ ]

Answer: $-\frac{6\csc^2(6\theta)}{4\sin(4r) + 5}$

ID: 3.7.18

LO23: Solve applications involving linearization and differentials. CALC

The volume of a solid can be expressed as $V = 3x^3$. The volume is to be calculated with an error of no more than 2% of the true value. Find approximately the greatest error that can be tolerated in the measurement of x, expressed as a percentage of x.

The greatest tolerated error in the measurement of x is [ ]%.
(Type an integer or a simplified fraction.)

Answer: $\frac{2}{3}$

ID: 3.9.47

LO24: Evaluate trigonometric functions.

The value of **tan** x is given. Find **sin** x and **cos** x if x lies in the specified interval.

$$\tan x = \frac{1}{7},\ x \in \left[\pi, \frac{3\pi}{2}\right]$$

Find **sin** x.

sin x = ☐

(Type an exact answer, using radicals as needed.)

Find **cos** x.

cos x = ☐

(Type an exact answer, using radicals as needed.)

Answers $-\frac{1}{5\sqrt{2}}$

$-\frac{7}{5\sqrt{2}}$

ID: Instructor-created question