Additional file 1: Reducing calculation time of the spherical harmonic expansion

In order to reduce the calculation of the toroidal-poloidal power, for almost all the Legendre polynomials, P_l^m, we do not use the definition of Legendre polynomials

$$P_l^m(\cos\theta) = \sin^m\theta\left(\frac{dP_l(\cos\theta)}{d\cos\theta}\right) \tag{1}$$

where l, m, and θ are degree, order, and colatitude respectively, and the P_l is Legendre function

$$P_l(\cos\theta) = \sum_{s=0}^{\leq l/2}(-1)^s\frac{(2l-2s)!}{2^l s!(l-s)!(l-2s)!}\cos^{l-2s}\theta, \tag{2}$$

because the calculation takes a long time due to the summation and the factorials at high degree. Instead, we utilize recursive formulae of Legendre polynomials

$$(l-m+1)P_{l+1}^m - (2l+1)\cos\theta P_l^m + (l+m)P_{l-1}^m = 0 \tag{3}$$

$$(l-m)\cos\theta P_l^m - (l+m)P_l^m + \sin\theta P_l^{m+1} = 0 \tag{4}$$

$$P_l^{m+2} - 2(m+1)\frac{\cos\theta}{\sin\theta}P_l^{m+1} + (l-m)(l+m+1)P_l^m = 0 \tag{5}$$

and parity of Legendre polynomials

$$P_l^m(\cos(\pi-\theta)) = P_l^m(-\cos\theta) = (-1)^{l+m}P_l^m(\cos\theta) \tag{6}$$

$$\frac{dP_l^m(\cos(\pi-\theta))}{d(\pi-\theta)} = -\frac{dP_l^m(\cos(\pi-\theta))}{d\theta} = (-1)^{l+m+1}\frac{dP_l^m(\cos\theta)}{d\theta}, \tag{7}$$

which means that calculation for the Northern Hemisphere ($0 \leq \theta \leq \pi/2$) can be used to that for the Southern Hemisphere ($\pi/2 \leq \theta \leq \pi$), reducing the calculation time to about half.