RESEARCH

Supplementary Material: Investigation of regional variation in core flow models using spherical Slepian functions

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Abstract

By assuming that changes in the magnetic field in the Earth's outer core are advection-dominated on short timescales, models of the core surface flow can be deduced from secular variation. Such models are known to be under-determined and thus require other assumptions to produce feasible flows. There are regions where poor knowledge of the core flow dynamics gives rise to further uncertainty, such as within the tangent cylinder, and assumptions about the nature of the flow may lead to ambiguous patches, such as if it is assumed to be strongly tangentially geostrophic. We use spherical Slepian functions to spatially and spectrally separate core flow models, confining the flow to either inside or outside these regions of interest. In each region we examine the properties of the flow and analyze its contribution to the overall model. We use three forms of flow model: (a) synthetic models from randomly generated coefficients with blue, red and white energy spectra, (b) a snapshot of a numerical geodynamo simulation and (c) a model inverted from satellite magnetic field measurements. We find that the Slepian decomposition generates unwanted spatial leakage which partially obscures flow in the region of interest, particularly along the boundaries. Possible reasons for this include the use of spherical Slepian functions to decompose a scalar quantity that is then differentiated to give the vector function of interest, and the spectral frequency content of the models. These results will guide subsequent investigation of flow within localized regions, including applying vector Slepian decomposition methods.

Keywords: Spherical Slepian functions, Outer Core Flow, Geostrophic flows, Tangent Cylinder

1 Supplementary Material

To support the article, we wish to include some further images to demonstrate the application of the technique. Firstly we present a figure showing that the absolute difference between the input and summed regions is $\leq 5 \times 10^{-13}$ to confirm that a true recreation can be achieved. An example for the numerical dynamo data is shown in Figure 1.

Next, we present decompositions of a flow within and outside the tangent cylinder for a snapshot of the numerical dynamo as well as for flow from inverted satellite data. This demonstrates the results for an alternative region to the tangentially geostrophic ambiguous patch shown in the main text. Discussion of the main findings and considerations for the tangent cylinder can be found in Section 6. Finally, we present the same tangent cylinder decomposition for the inverted satellite flow plotted as vector, stream function and potential function (as in Figure 11 in the main text).

The datasets supporting the conclusions of this article are available by contacting either Julien Aubert or Kathy Whaler for the numerical dynamo or flow derived from satellite data, respectively. The code used to generate these images is available on request with supporting functions from Simons, Dahlen and Wieczorek at GitHub, doi: 10.1137/S0036144504445765, or http://geoweb.princeton.edu/people/simons/software.html.

Availability of data and material

The code and data used to generate these findings is available on request.

Competing interests

The authors declare that they have no competing interests.

Funding

HFR is funded through the E3 DTP at the University of Edinburgh by NERC grant NE/L002558/1, and a NERC CASE studentship with a British Geological Survey BUFI grant (BGS contract GA/17S/009).

Author's contributions

HR adapted and wrote new code for the experiments. CDB and KAW helped with the analysis and discussion of the results. All authors contributed to the writing of the manuscript.

Acknowledgements

We thank Julien Aubert for providing the Coupled Earth numerical geodynamo model snapshot, Frederik Simons for making his and co-authors' Slepian code available (doi: 10.1137/S0036144504445765, http://geoweb.princeton.edu/people/simons/software.html), and Jarno Saarimäki, for his work on previous scripts focusing on the crustal field. We also wish to thank Lara Kalnins and Ashley Smith for their assistance with the project. We thank the Hagay Amit and two anonymous reviewers for their feedback and suggestions for improvement on earlier versions of the paper.

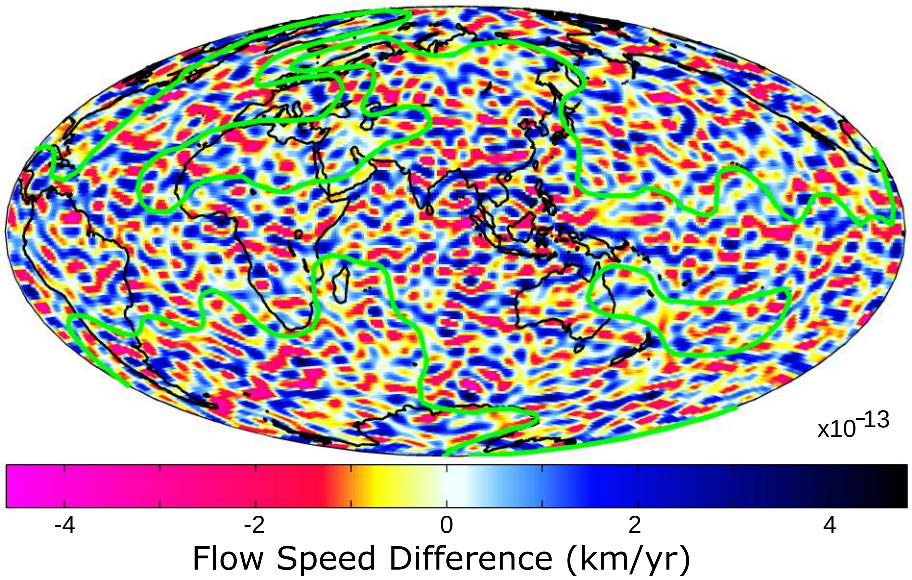

Figure 1 Absolute difference between numerical dynamo ϕ-component flow from input and summed decomposition coefficients. By visually comparing maps and spectra of the input and sum based on the decompositions of the numerical dynamo, it appeared that a perfect recreation was achieved. To confirm this, the absolute values of one map was subtracted from the absolute values of the other to give difference values of $\leq 5 \times 10^{-13}$. This is much smaller than the input values and a perfect reconstruction is achieved except for errors due to computational rounding.

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References

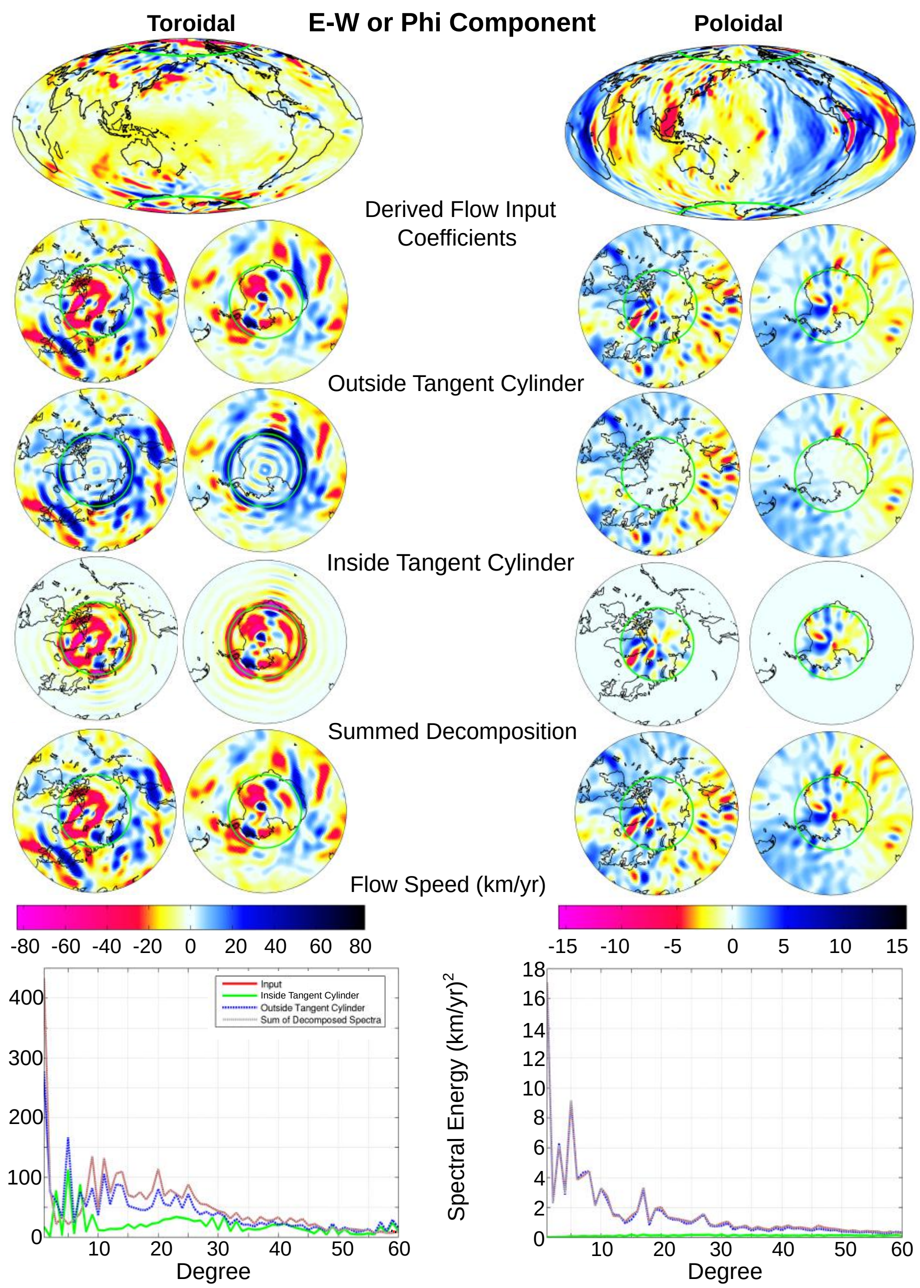

Figure 2 **ϕ-component of the tangent cylinder decomposition of the toroidal flow scalar for numerical dynamo data.** Optimum decomposition of degree 20 derived flow model using $K = 18$ to represent the flow within the ambiguous patch. The Shannon number is much lower ($K = 14$) due to the smaller region of interest. The images show that a good recreation of the input can be achieved but the signal is obscured by the large boundary signals generated during the decomposition. This is consistent with findings using the tangentially geostrophic ambiguous patch.

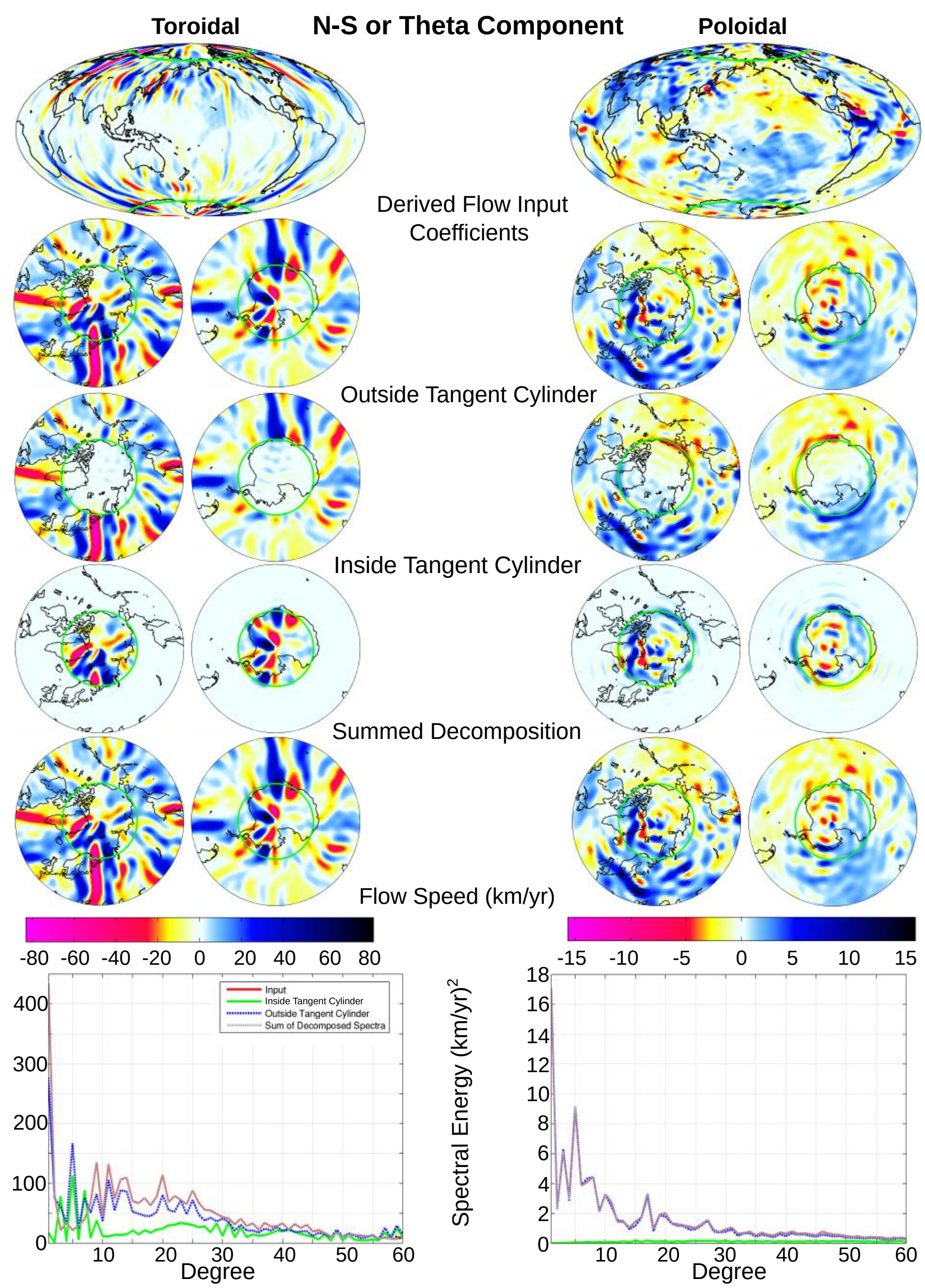

Figure 3 **θ-component of the tangent cylinder decomposition of the toroidal flow scalar for numerical dynamo data.** As for Figure 2 for the θ-component

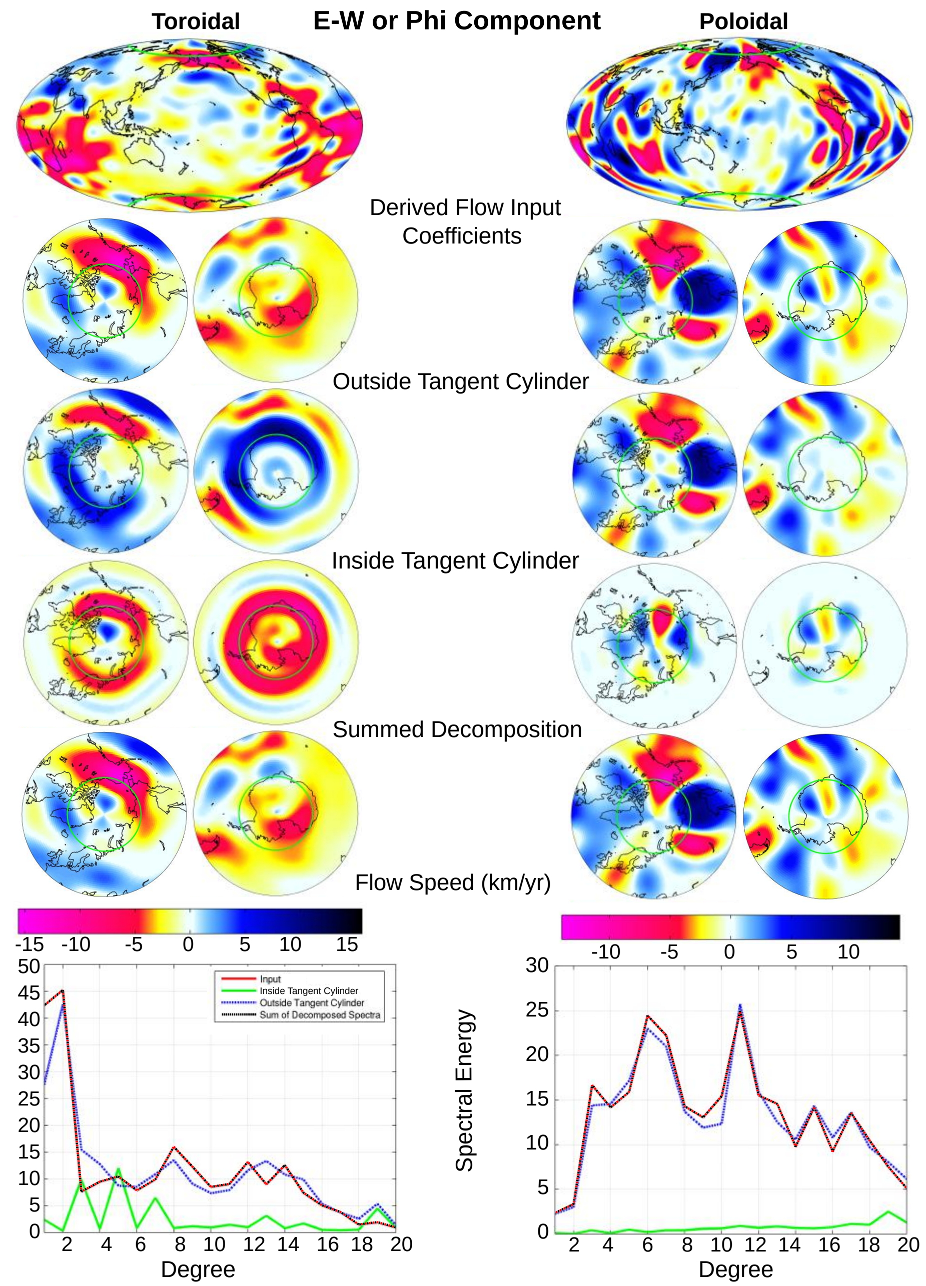

Figure 4 **ϕ-component of the tangent cylinder decomposition of the toroidal flow scalar derived from satellite data.** Optimum decomposition of degree 20 derived flow model using $K = 18$ to represent the flow within the ambiguous patch. The Shannon number is much lower $(K = 14)$ due to the smaller region of interest. The images show that a good recreation of the input can be achieved but the signal is obscured by the large boundary signals generated during the decomposition. This is consistent with findings using the tangentially geostrophic ambiguous patch.

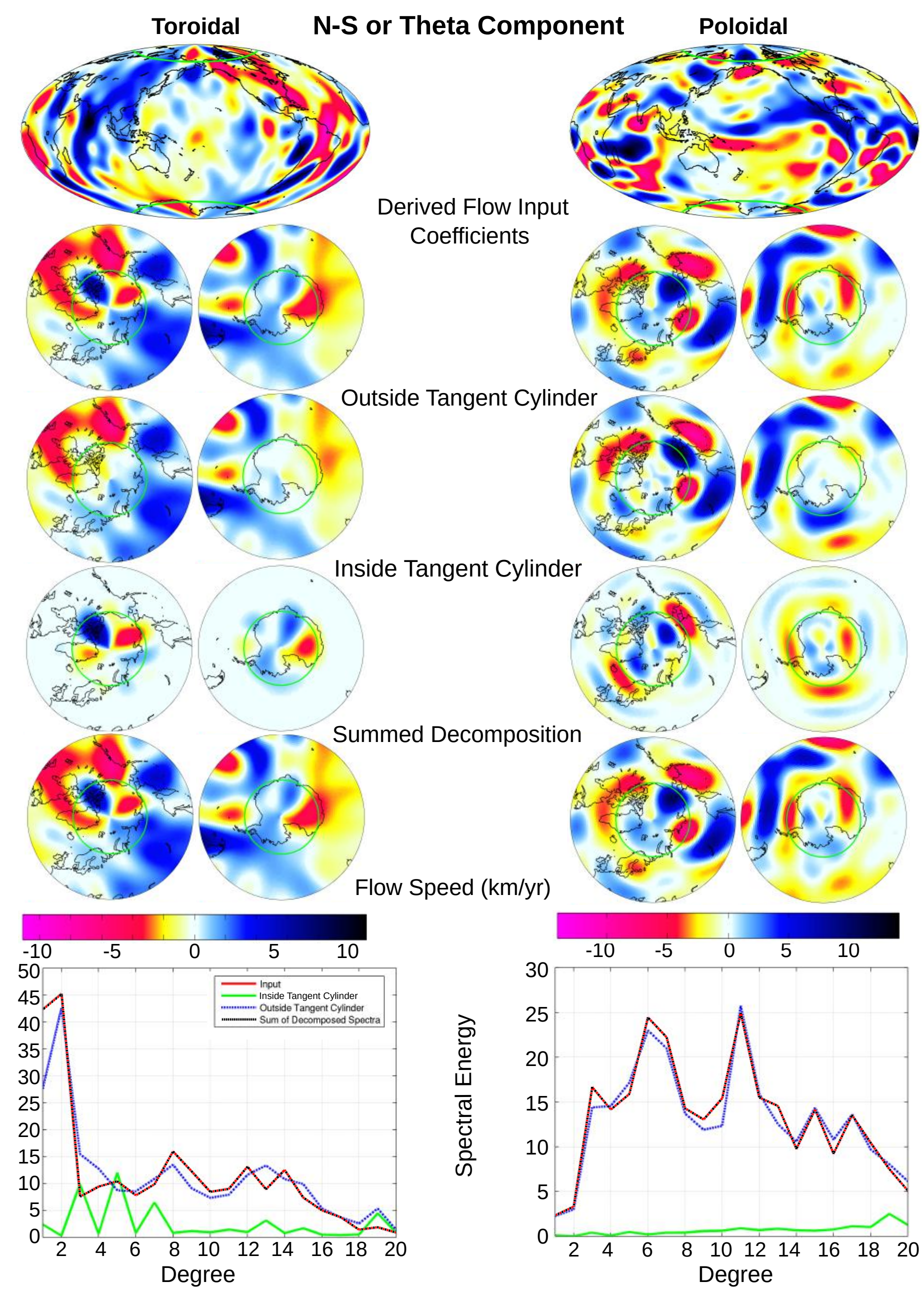

Figure 5 θ-component of the tangent cylinder decomposition of the toroidal flow scalar derived from satellite data. As for Figure 4 for the θ-component

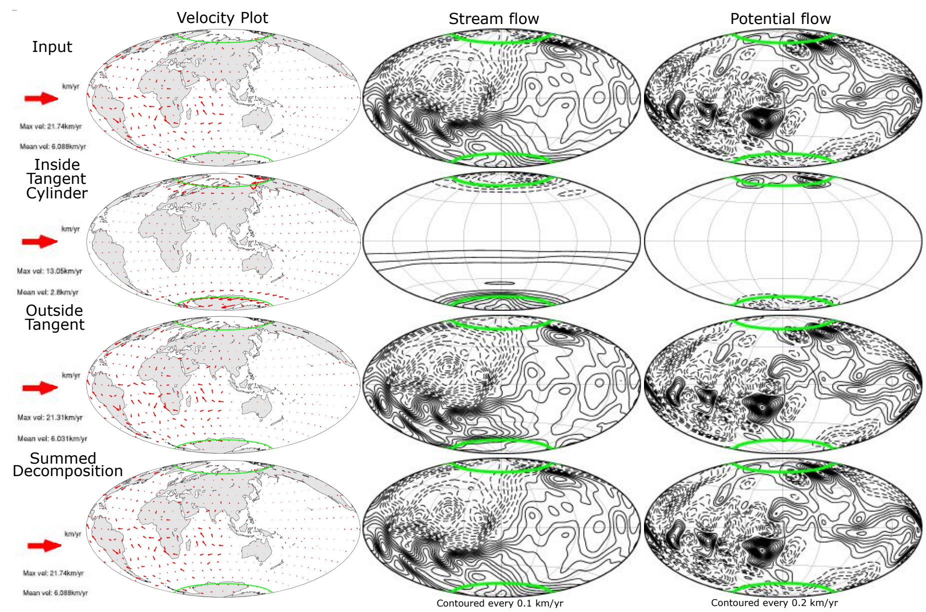

Figure 6 Decomposition of the tangent cylinder for flow inverted from satellite data plotted as vector flow, stream flow and potential flow. A decomposition of the toroidal and poloidal components of the inverted flow from satellite data, seen in Figures 4 and 1, as velocity (column one), stream flow (column two) and potential flow (third column). The input, top row, is split into inside and outside the tangentially geostrophic ambiguous patches, rows two and three, before being summed together in the final row. As with previous decompositions, the input and the summed decomposition are identical and flow is restricted to the region of interest with the decomposed maps. These plotting styles show just as much leakage at the region edge but it is less obvious at first glance.