Permutation entropy of a time series

In this text, we describe how permutation entropy is computed for the following time series:

$$x(t) = [4, 3, 2, 4, 6, 1, 4, 2, 5, 6, 2, 3, 1, 5],$$

plotted in Figure 1a.

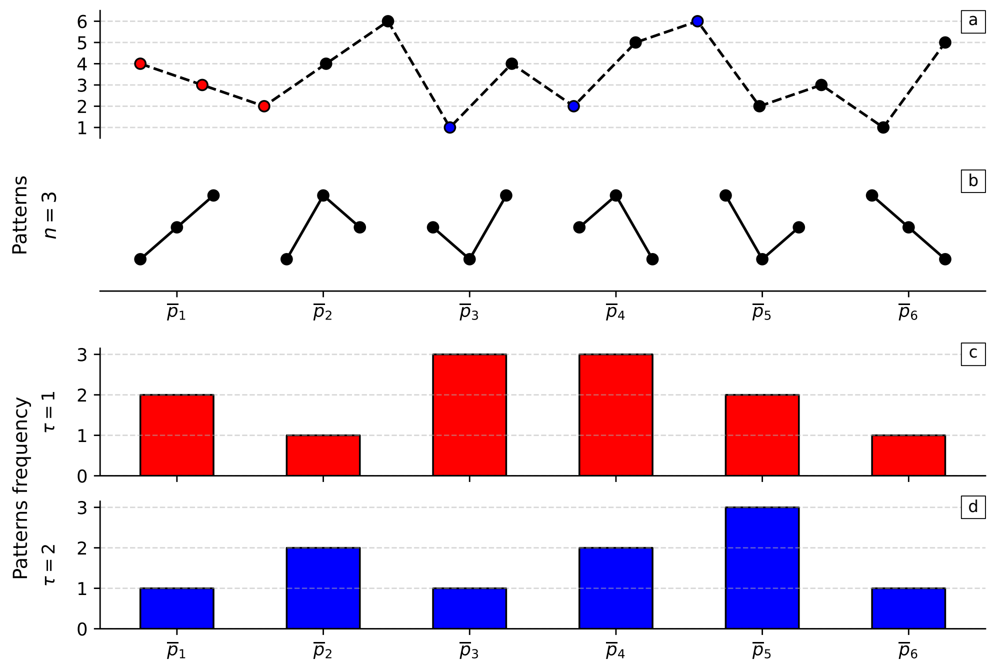

Figure 1: Example of how to compute permutation entropy for the time series given in (a). (b) All the possible pattern for $n = 3$. (c) Distribution of patterns occurrence for sequences of $\tau = 1$, red dots in (a). (d) Distribution of patterns occurrence for sequences of $\tau = 2$, blue dots in (a).

In symbolic analysis, each sample of a time series is a symbol, and the combination of symbols is a pattern (see Daw et al., 2003, for a review). Permutation entropy is a measure based on the occurrence frequency of patterns in symbolic data. All possible patterns are defined by setting the length of the pattern n, being $n!$. For example, Figure 1b shows the 6 possible patterns ($\overline{p}_k$, for $k = 1, ..., n!$) for $n = 3$. Counting how many patterns compose the time series depends on how we construct each sequence of symbols (a sequence is just a combination of n symbols extracted from the symbolic data and used to recognize a pattern). Therefore, this process depends on the second parameter, τ, which can be defined as the time delay between equidistant symbols used to build each sequence.

Figure 1c shows the distribution of patterns occurrence for sequences of $\tau = 1$ (red dots in Figure 1a), whereas Figure 1c shows the same distribution using $\tau = 2$ (blue dots in Figure 1a). Note that in the determination of the patterns, the values are not important; the only thing that matters is if the next symbol is greater or smaller than the previous one. Permutation entropy is then defined as the Shannon entropy of the relative frequency of patterns (Riedl et al., 2013), i.e.,

$$h_{\tau:1} = -\sum_{k=1}^{6} p_k \log_2 p_k = -2\left(\frac{1}{12}\right)\log_2\frac{1}{12} - 2\left(\frac{2}{12}\right)\log_2\frac{2}{12} - 2\left(\frac{3}{12}\right)\log_2\frac{3}{12} = 2.45$$

$$h_{\tau:2} = -\sum_{k=1}^{6} p_k \log_2 p_k = -3\left(\frac{1}{10}\right)\log_2\frac{1}{10} - 2\left(\frac{2}{10}\right)\log_2\frac{2}{10} - 1\left(\frac{3}{10}\right)\log_2\frac{3}{10} = 2.44$$

References

Daw, C. S., Finney, C. E., and Tracy, E. R. (2003). A review of symbolic analysis of experimental data. *Review of Scientific Instruments*, 74(2):915–930.

Riedl, M., Müller, A., and Wessel, N. (2013). Practical considerations of permutation entropy. *The European Physical Journal Special Topics*, 222(2):249–262.