Additional file 2: FDTD simulation

We performed a finite-difference time-domain (FDTD) simulation of infrasound propagation to investigate the acoustic resonance behavior in an actual volcanic environment. Here, the details of the calculation are described. We used a GPU-based 3-D FDTD code (Kim et al. 2015) for the calculation. The resonant frequencies calculated under the following conditions were used to estimate the depth.

- calculation space: 2,000 m $\times$ 2,000 m $\times$ 2,000 m (applied perfectly matched layer (PML) boundary)
- spatial grid: 2 m
- calculation duration: 10 s
- time step: 0.001 s
- source time function: the Blackman-Harris function as the mass flow rate (Fig. S2-1)
- 1-D sound velocity profile (Fig. S2-2)
  - continuous profile (Yokoo et al. 2019)
  - different profiles inside and outside the vent: (outside) same as in the air (inside) 300, 400, 500, 600, 700, 800 m/s
- elevation data (Fig. S2-3)
  - a flat plane
  - DEM data of Aso volcano with a 1-m spacing (obtained in March 2015)
- frustum pipe
  - pipe radius: (open end) 25 m (closed end) 35, 46, 58, 71 m
  - pipe length: 25, 50, 75,100, 125, 150, 175, 200, 225, 250, 275, 300, 400, 500, 600, 800, 1000 m

Assuming a continuous profile of the sound velocity (Yokoo et al. 2019), the computed resonant frequencies are identical to the theoretical values (Figure 3a; Figure S2-4a). On the other hand, when the sound velocity inside the pipe is higher than the outside, the obtained results of the FDTD simulation show lower resonant frequencies than those expected from the theoretical formulas (Figure 3b; Figure S2-4d) Although not yet strictly confirmed, the cause of such discrepancies may be the density profile which is given for the calculation. If the sound velocity inside the pipe is high due to high temperatures, the density inside the pipe is lower than the outside considering the assumption of the ideal gas. In this way, when the inside of the pipe is in a higher velocity and lower density environment than the outside, the calculated resonance frequency does not match the theoretical value (Figure S2-4d). However, if the density profile is continuous and only the velocity profile has a difference between inside and outside the pipe, the calculated resonant frequency is the same as the theoretical resonant frequency (Figure S2-4b). Conversely, when the velocity profile is continuous and only the density profile provides a difference, the calculated frequency does not the theoretical frequency (Figure S2-4c). Therefore, the discrepancy between the theoretical and computed frequencies might be attributed to the large change in the density value at the exit of the pipe. These results do not depend on the shape of the pipe. In the case of a cylindrical pipe, similar results are obtained (Figure S2-5).

References

Kim K, Fee D, Yokoo A, and Lees J. M (2015). Acoustic source inversion to estimate volume flux from volcanic explosions. Geophysical Research Letters 42(13):5243–5249. doi:10.1002/2015GL064466.

Kinsler L, Frey A, Coppens A, and Sanders J (1999). Fundamentals of acoustics. Wiley New York 4th edition.

Yokoo A, Ishii K, Ohkura T, and Kim K (2019). Monochromatic infrasound waves observed during the 2014–2015 eruption of Aso volcano, Japan. Earth, Planets and Space 71(1):1–14. doi:10.1186/s40623-019-0993-y.

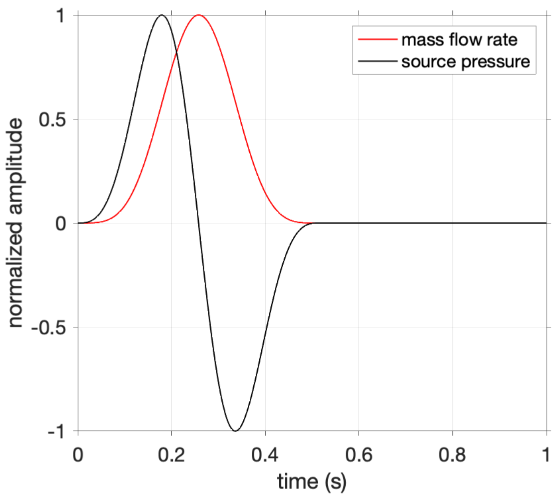

Figure S2- 1: Source time function. Normalized waveforms of the mass flow rate (Blackman-Harris function; red line) and source pressure (black line).

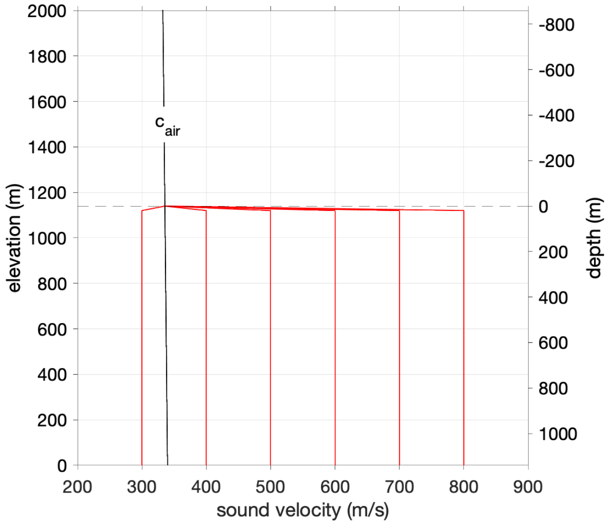

Figure S2- 2: 1-D profiles of the sound velocity. The black line shows the sound velocity in the air assuming a temperature lapse rate of 0.006 K/m (Yokoo et al. 2019). In the profiles in which the velocities inside and outside the vent are different, the velocity inside the vent is homogeneous at most depths and transitions linearly from 20 m to the crater floor (0 m; dashed line). For the simulations, 1-D profiles of the density were calculated on the assumption of the ideal gas.

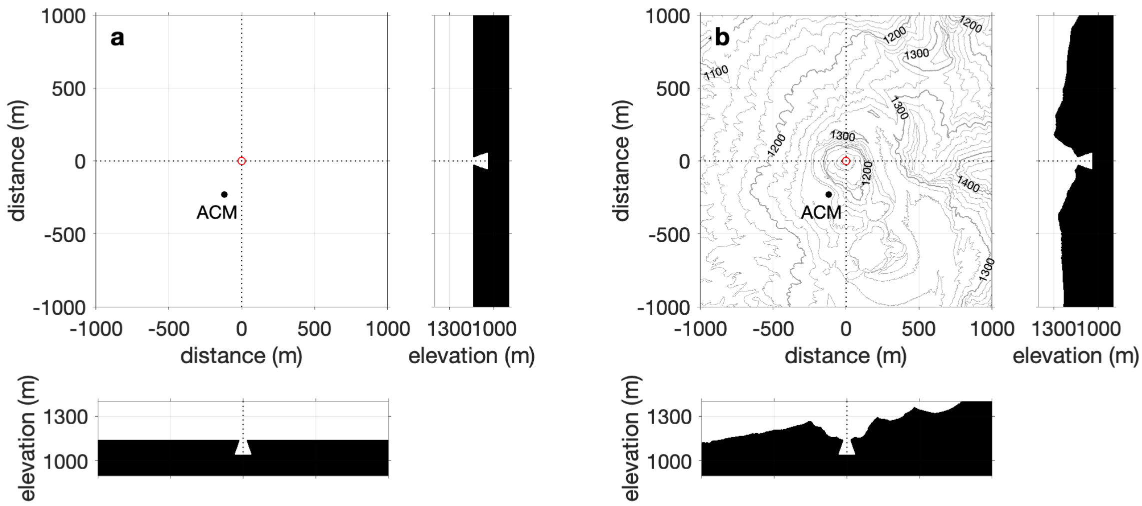

Figure S2- 3: **a** Elevation data of a flat plane and **b** DEM data of Aso volcano. A frustum pipe is connected to each data. A source is set at the bottom of the frustum pipe. The length of the pipes in these figures is 100 m.

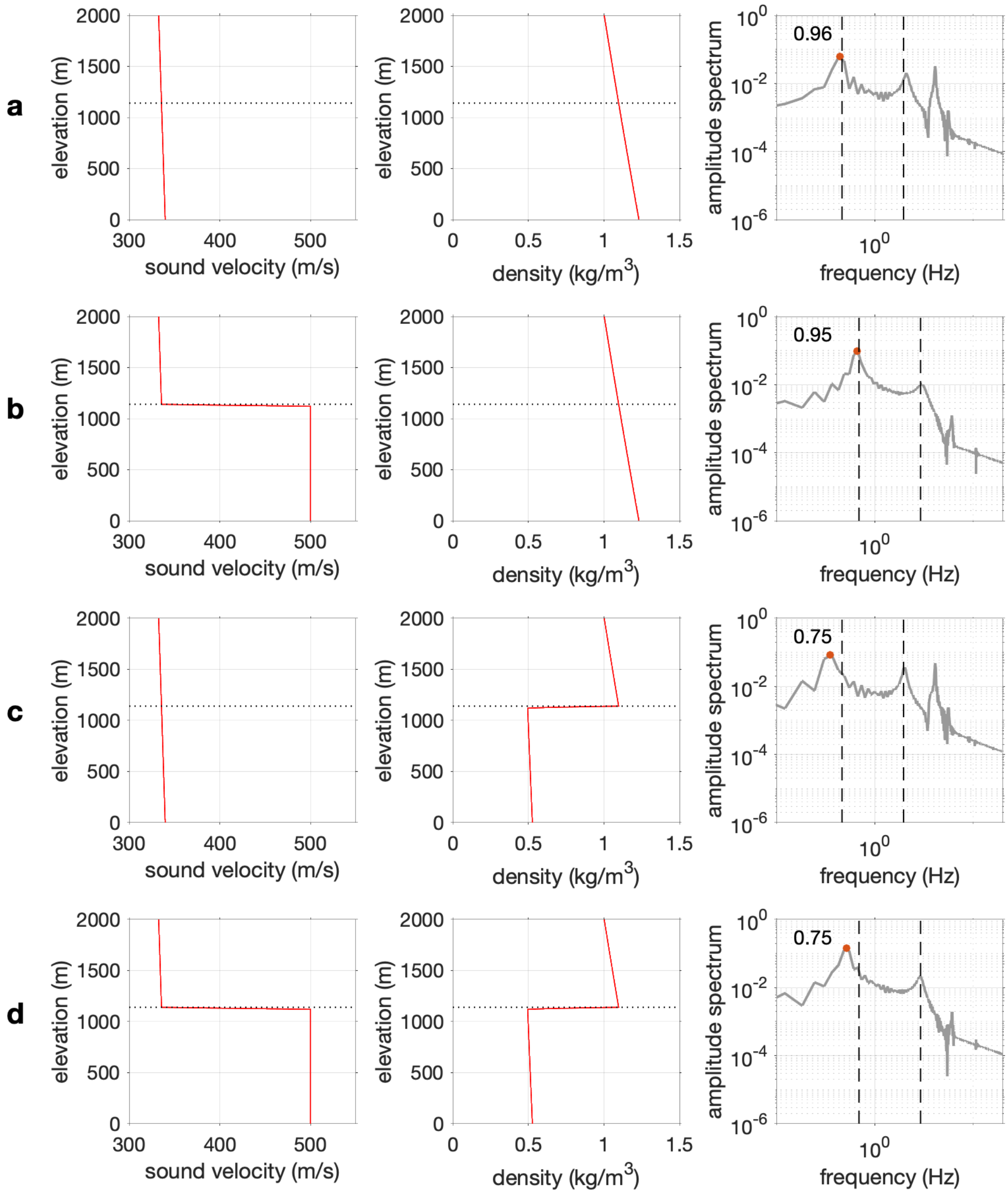

Figure S2- 4: The variation in the amplitude spectrum with the density and sound velocity profiles. The leftmost column shows the sound velocity profiles, the middle column shows the density profiles, and the rightmost column shows the frequency spectra obtained by FDTD simulation. The pipe length and the radii of the frustum pipe (a_o and a_c) are 100 m, 25 m, and 58 m, respectively. Here, the frustum pipe is connected to a flat plane (Figure S2-3a). Black dotted lines show the elevation of the ground surface. Black broken lines indicate the resonant frequencies (f_0 and f_1) calculated from Eq. (1). When the density is continuous inside and outside the pipe, the calculated resonant frequency agrees with the theoretical value (**a** and **c**). However, when the density is different between inside and outside the pipe, the calculated frequency is different from the theoretical value (**b** and **d**). The numbers shown in the rightest panels are the frequency ratio of the fundamental resonant frequencies f_0 (simulation result/theoretical value).

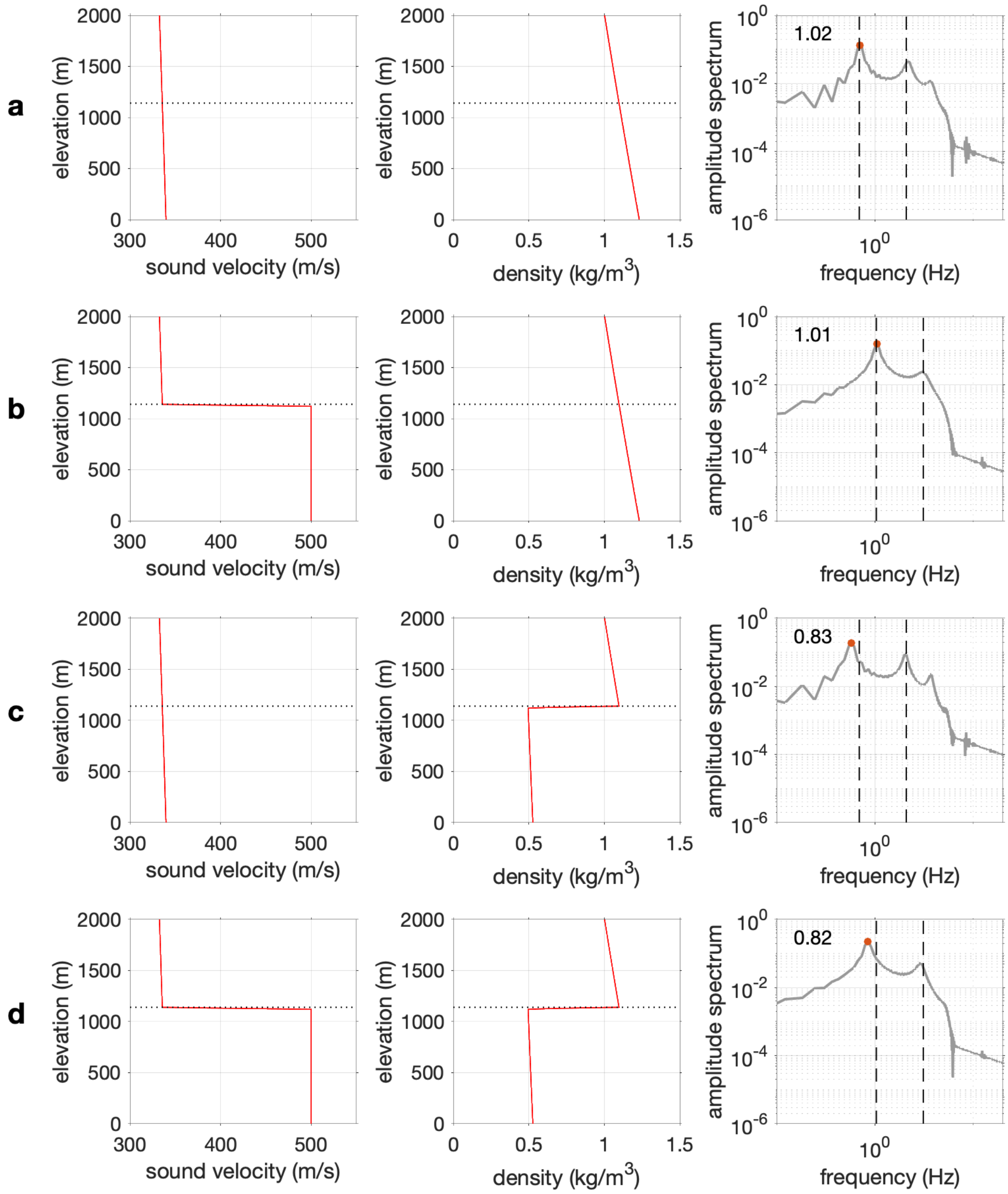

Figure S2- 5: In the case of a cylindrical pipe in a flat plane (a_o = a_c =25 m, pipe length 100 m). All the diagrams are described as the same as those of Figure S2-4. As the case of the conical frustum pipe (Figure S2-4), when the density inside the pipe is different than that outside the pipe, the calculated frequency is different from the theoretical value (**b** and **d**). The theoretical frequencies are based on Kinsler et al. (1999).