

## Additional file 3: How to calculate PDF data

### 1. How to calculate PDF data in Figure 2b

The histogram of Figure 2b shows the distribution of the frequency ratio ( $f_1/f_0$ ). This distribution is calculated from the average amplitude spectrum of infrasound signals (black line in Figure 2a). Figure S3-1 shows the average amplitude spectrum (both axes are linear scale). The spectra in the range that satisfies half or more of the maximum value of the peaks ( $f_0$  and  $f_1$ ) are extracted (solid red line). Then, each of these functions is normalized so that their sum is one. We consider these functions as the probability distributions of  $f_0$  and  $f_1$ . Using these probability distributions, we calculated the distribution of the PDF of  $f_1/f_0$ . By reflecting the shape of the average spectrum, the uncertainty of the peak frequency is reflected in the uncertainty of the pipe shape.

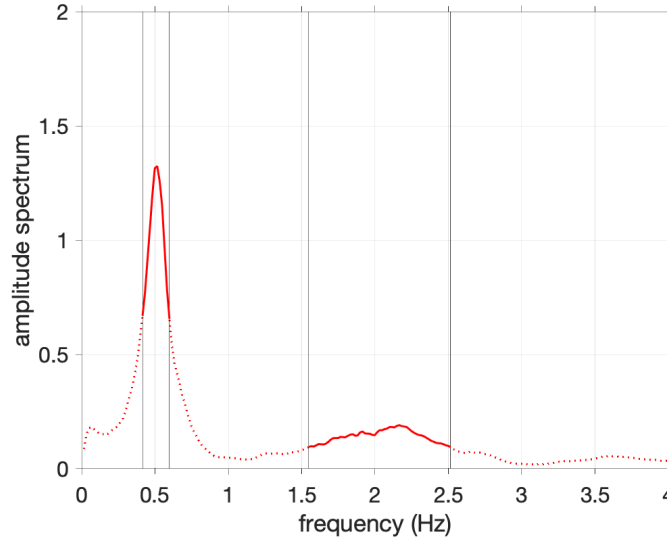


Figure S3- 1: Average amplitude spectrum of the infrasound signal. Solid red lines indicate the ranges that satisfy half or more of the maximum value of the peaks.

### 2. How to calculate PDF data in Figure 5

In Figure 5, the 2-D PDF in a coordinate space of depth and sound velocity is shown. The distribution of the PDF is calculated based on that of the PDF of observed parameters ( $f_0$  and  $\Delta t$ ; Figure 4a). Here, how the PDF in Figure 5 is transformed from that in Figure 4a is explained.

There are two steps:

#### (i) coordinate transformation

When a pair of observed parameters ( $f_{0i}$  and  $\Delta t_i$ ) is focused, how often the pair is observed is represented as  $\text{PDF}_i$  in Figure 4a. As mentioned in the Method section, the pair of the observed parameters ( $f_{0i}$  and  $\Delta t_i$ ) provides the depth of the magma-air interface ( $d_i$ ) and the sound velocity inside the vent ( $c_i$ ). Therefore,  $\text{PDF}_i$  also indicates how often the depth and sound velocity ( $d_i$  and  $c_i$ ) were realized. In other words,  $\text{PDF}_i$  can be distributed in depth-velocity coordinates.

(ii) accumulation of  $\text{PDF}_i$

Next, all the pairs of  $f_{0i}$  and  $\Delta t_i$  included in the area shown in Figure 4a are considered. When the coordinate transformation procedure is performed for all pairs of  $f_{0i}$  and  $\Delta t_i$ , all the  $\text{PDF}_i$ s ( $i=1,2,\dots, n$ ) are plotted in depth-velocity space. The cumulative result of these  $\text{PDF}_i$ s is the distribution of PDFs shown in Figure 5.

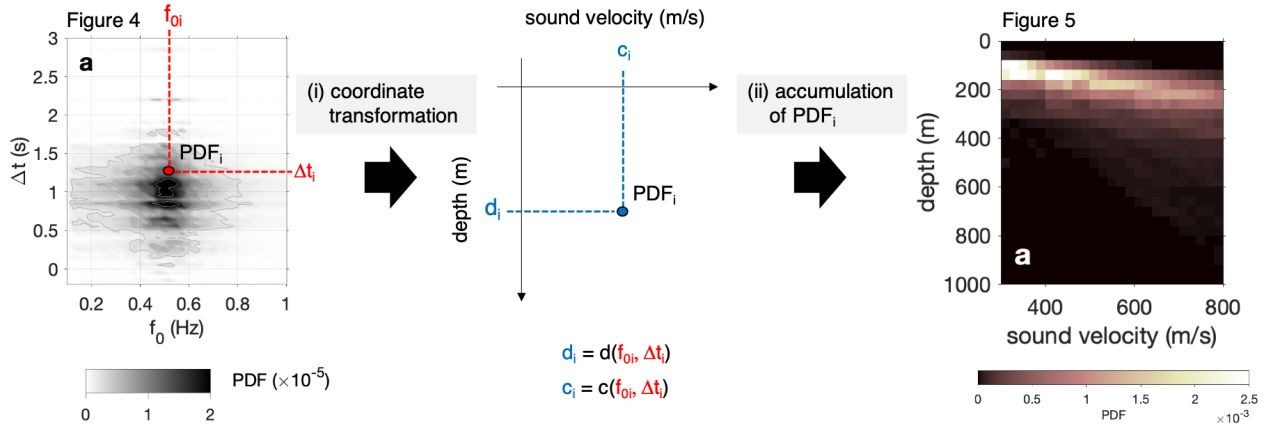


Figure S3- 2: How to calculate the PDF in Figure 5.