Additional file to: On temporal variations of coastal tidal ocean-dynamo signals

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1 Short Info

Here we provide additional but more specifically more technical information. This includes a mathematical explanation of the amplitude modulation as well as tables that provide additional details on the used methods and figures used to support our argumentation.

1.1 Amplitude Modulation

In the following, we describe the amplitude modulation model we used. A sinusoidal signal, in our specific case the M_2 tidal constituent (or higher order of it), is identified as the carrier wave $c(t)$:

$$c(t) = A \sin\left(\omega_{M_2} t\right). \tag{1}$$

The carrier wave $c(t)$ is modulated by a the message signal $m(t)$ with the modulation frequency ω_m.

$$m(t) = m \cos\left(\omega_m t\right). \tag{2}$$

The modulated signal can thus be described as:

$$y(t) = [1 + m(t)]\, c(t) = [1 + m \cos\left(\omega_m t\right)]\, A \sin\left(\omega_{M_2} t\right), \tag{3}$$

where m is the modulation index which indicates the modulation of the carrier wave amplitude in percent. Using the trigonometric identities we can describe the modulated signal $y(t)$ as:

$$y(t) = A \sin\left(\omega_{M_2} t\right) + \frac{1}{2} Am \left[\sin\left(\left(\omega_{M_2} + \omega_m\right) t\right) + \sin\left(\left(\omega_{M_2} - \omega_m\right) t\right)\right]. \tag{4}$$

The modulated signal has consequently three components, the unchanged carrier signal $c(t)$ and two sidebands. Please note, ω_m is usually much lower than the carrier frequency. The sideband frequency are below $(f = \omega_{M_2} - \omega_m)$ and above $(f = \omega_{M_2} + \omega_m)$the carrier frequency, with the frequency difference of the message signal. Since the mentioned effects acting on the M_2 signal occur seasonally, we chose the seasonal frequencies for the message signal (c.f. table 1.3).

1.2 Descriptions and Details of the Presented Fit Functions

Table S1. Names, Identifiers and short description of harmonic models

Identifier	Name (hsum_)	# of Tidal Frequencies	Description
1	M_2	1	Harmonic sum comprising only the oscillation of the M_2, comparable to Maus and Kuvshinov (2004)
2	M_2_overtides	8	Harmonic sum comprising all tidal harmonics of the lunar tide ranging from M_1 to M_8
3	M_2_modulation	15	Harmonic sum modelling the amplitude variation of the carrier frequency of the M_2 long periodic modulation frequencies
4	M_2_overtides_modulation	120	Harmonic sum modelling the amplitude variation of the a non-sinusoidal carrier signal comprising the M_2 tide and higher harmonics (see hsum_M_2_overtides) modulated on long period
5	Schnepf_2014	15	Harmonic sum used in the analysis of coastal TODS of Schnepf et al. (2014)
6	Schnepf_2014_short	12	Harmonic sum similar to the Schnepf_2014 model except that it does not include low frequency models. It is closely related to the model of Schnepf et al. (2018) but unlike their model our model assures consistency and comparability by including the O_1 and P_1 components like in the publication of Schnepf et al. (2014)
7	NOAA	37	Harmonic sum comprising the 37 tidal constituents of the NOAA model
8	NOAA_short	32	hsum_NOAA without low frequency components

The focus of models 1-4 is to analyse the TODS of the M_2 tide, its deviation from the sinusoidal signal and its amplitude modulation. For the models 5-8, the focus is analysing the influence of incorporating additional oscillations on the extracted TODS amplitude of the M_2 tide

1.3 Tidal Component used in this study

Table S2. Long periodic tidal constituents

Species	Darwin Symbol	Period (hr)	Speed ($°/hr$)	Model used
Nodal Modulation	Om_1	163161.711385	0.0022064	5
Solar annual	S_a	8766.15265	0.0410686	3*, 4*, 5, 7
Solar semiannual	S_{sa}	4383.076325	0.0821373	3*, 4*, 5, 7
Solar terannual	S_{ta}	2921.98542416	0.1232039	3*, 4*
Lunar synodic monthly	MS_m	763.486512056	0.4715211	3*, 4*
Lunar monthly	M_m	661.3111655	0.5443747	3*, 4*, 7
Lunisolar synodic fortnightly	MS_f	354.3670666	1.0158958	3*, 4*, 7
Lunisolar fortnightly	M_f	327.8599387	1.0980331	3*, 4*, 7

* in these models the frequencies are included only indirectly as sidebands to the either the M_2 tide alone (model 3) or the M_2 tide and its overtides (model 4).

Table S3. Diurnal tidal constituents

Species	Darwin Symbol	Period (hr)	Speed ($°/hr$)	Model used
Larger elliptic diurnal	$2Q_1$	28.00621204	12.854286	7, 8
Larger lunar elliptic diurnal	Q_1	26.868350	13.39866	7, 8
Larger lunar evectional diurnal	ϱ	26.72305326	13.471514	7, 8
Lunar diurnal	O_1	25.81933871	13.9430356	5, 6, 7, 8
Smaller lunar elliptic diurnal	M_1	24.84120241	14.4920521	2, 7, 8
Solar diurnal	P_1	24.06588766	14.9589314	5, 6, 7, 8
Solar diurnal	S_1	24	15	5, 6, 7, 8
Lunar diurnal	K_1	23.93447213	15.0410686	5, 6, 7, 8
Smaller lunar elliptic diurnal	J_1	23.09848146	15.58544	7, 8
Lunar diurnal	OO_1	22.30608083	16.139101	7, 8

Table S4. Semidiurnal tidal constituents

Species	Darwin Symbol	Period (hr)	Speed ($°/hr$)	Model used
Lunar elliptical semidiurnal second-order	2"N_2	12.90537297	27.8953548	7, 8
Variational	μ_2	12.8717576	27.968208	7, 8
Larger lunar elliptic semidiurnal	N_2	12.65834751	28.4397295	5, 6, 7, 8
Larger lunar evectional	ν_2	12.62600509	28.5125831	7, 8
Principal lunar semidiurnal	M_2	12.4206012	28.9841042	all models
Smaller lunar evectional	λ_2	12.22177348	29.4556253	7, 8
Smaller lunar elliptic semidiurnal	L_2	12.19162085	29.5284789	7, 8
Larger solar elliptic	T_2	12.01644934	29.9589333	7, 8
Principal solar semidiurnal	S_2	12	30	5, 6, 7, 8
Smaller solar elliptic	R_2	11.98359564	30.0410667	7, 8
Lunisolar semidiurnal	K_2	11.96723606	30.0821373	5, 6, 7, 8
Shallow water semidiurnal	$2SM_2$	11.60695157	31.0158958	7, 8

Table S5. Short periodic tidal constituents

Species	Darwin Symbol	Period (hr)	Speed ($°/hr$)	Model used
Shallow water terdiurnal	2"MK_3	8.38630265	42.927139	7, 8
Lunar terdiurnal	M_3	8.280400802	43.4761563	2, 4, 7, 8
Shallow water terdiurnal	MK_3	8.177140247	44.025172	7, 8
Shallow water overtides of principal solar	S_3	8	45	5, 6
Shallow water quarter diurnal	MN_4	6.269173724	57.423833	7, 8
Shallow water overtides of principal lunar	M_4	6.210300601	57.9682084	2, 4, 7, 8
Shallow water quarter diurnal	MS_4	6.103339275	58.984104	7, 8
Shallow water overtides of principal solar	S_4	6	60	5, 6, 7, 8
Shallow water overtides of principal lunar	M_5	4.96824048818	72.4602605	2, 4
Shallow water overtides of principal solar	S_5	4.8	75	5, 6
Shallow water overtides of principal lunar	M_6	4.140200401	86.9523127	2, 4, 7, 8
Shallow water overtides of principal solar	S_6	4	90	5, 6, 7, 8
Shallow water overtides of principal lunar	M_7	3.54874320584	101.4443647	2, 4
Shallow water eighth diurnal	M_8	3.105150301	115.9364166	2, 4, 7, 8

2 Histograms of Residual Data

The following histograms depict the residual data obtained from the presented data processing algorithm using smoothing spline subtraction.

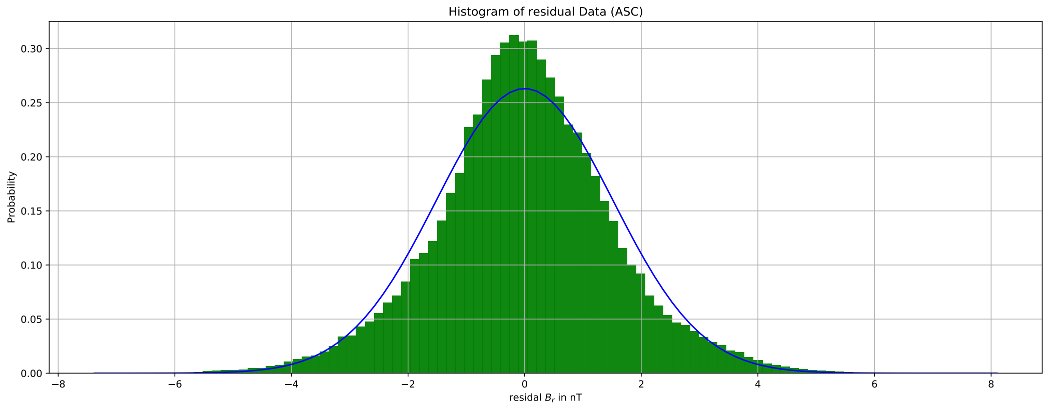

Figure S1. Histogram and gaussian fit to the residual data obtained after processing the data obtained from Ascencion Island (ASC). The residual data is almost gaussian distributed and has a narrow spread from less than 20 nT.

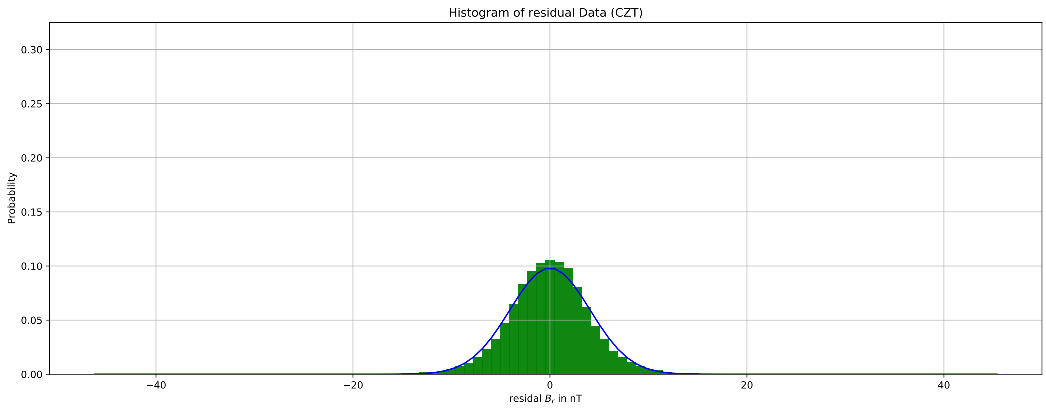

Figure S2. Histogram and gaussian fit to the residual data obtained after processing the data obtained from Crozet Archipelago (CZT). The residual data is almost gaussian distributed and has a spread or more than 80 nT.

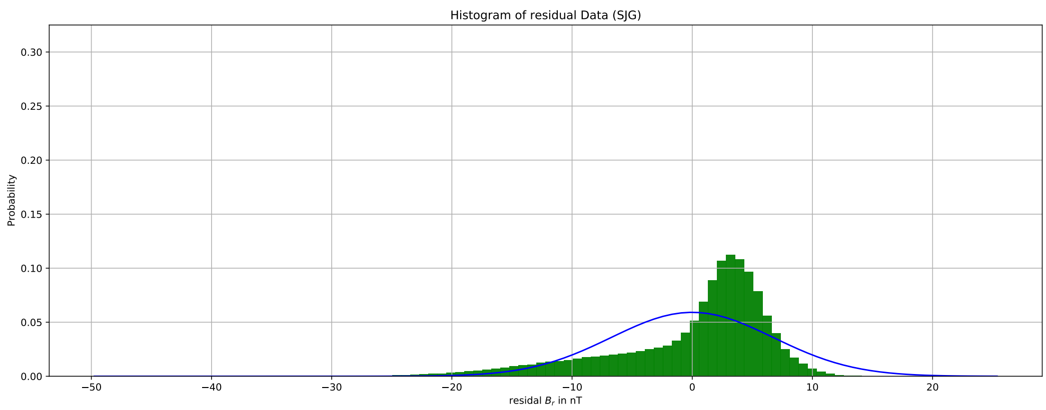

Figure S3. Histogram and gaussian fit to the residual data obtained after processing the data obtained from San Juan (SJG). The residual data is non-gaussian distributed and has a spread or more than 80 nT.

3 Root Mean Square values of fitted functions

3.1 Least Squares fit (forward)

Table S6. Root Mean Square (rms) values for: Least Squares forward (station = ASC)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	1.27565	1.26233	1.24718	1.24498	1.31514	1.36552	1.38708	1.42608	1.44464	1.45247
M2_overtides	1.26463	1.25860	1.24481	1.24288	1.31292	1.36322	1.38496	1.42403	1.44270	1.45106
M2_modulation	0.95953	0.94600	0.94288	0.95081	1.06026	1.11040	1.13701	1.15438	1.16293	1.16834
M2_overtides_modulation	0.75783	0.81606	0.82281	0.83800	0.96279	1.01370	1.04814	1.07705	1.09087	1.10837
Schnepf_01	0.93160	0.92834	0.92107	0.92802	1.04497	1.10083	1.13336	1.14946	1.15689	1.16566
Schnepf_02	0.94689	0.93834	0.92718	0.93381	1.04859	1.10461	1.13751	1.15321	1.16053	1.16880
NOAA	0.74724	0.74591	0.73321	0.72648	0.92523	1.00352	1.04545	1.06150	1.06678	1.07777
NOAA_short	0.78466	0.77732	0.76125	0.75872	0.94212	1.01546	1.05515	1.07277	1.07909	1.08912

Table S7. Root Mean Square (rms) values for: Least Squares forward (station = CZT)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	3.42758	3.36271	3.28992	3.33998	3.53061	3.65739	3.74355	3.84795	3.90769	3.95176
M2_overtides	3.42289	3.36060	3.28865	3.33939	3.52961	3.65652	3.74289	3.84749	3.90724	3.95135
M2_modulation	3.14189	3.09700	3.04601	3.08067	3.34924	3.49152	3.57162	3.68517	3.74867	3.78673
M2_overtides_modulation	2.98966	2.99475	2.97308	3.03659	3.31365	3.46087	3.55090	3.67207	3.73818	3.77775
Schnepf_01	2.97308	2.91589	2.86976	2.88502	3.22607	3.42258	3.48862	3.62941	3.70556	3.75289
Schnepf_02	2.97460	2.91659	2.87038	2.88563	3.22646	3.42284	3.48887	3.62961	3.70566	3.75299
NOAA	2.99920	2.94776	2.89964	2.92657	3.24821	3.43052	3.49955	3.63371	3.70369	3.74940
NOAA_short	3.00256	2.95066	2.90166	2.92789	3.24919	3.43098	3.50008	3.63418	3.70397	3.74973

Table S8. Root Mean Square (rms) values for: Least Squares forward (station = SJG)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	6.38117	6.11447	5.97537	5.89907	5.95257	6.21765	6.34400	6.46053	6.61318	6.71727
M2_overtides	6.36291	6.09981	5.96664	5.89669	5.95153	6.21606	6.34339	6.45947	6.61247	6.71578
M2_modulation	5.78963	5.58611	5.45850	5.38908	5.66756	6.00562	6.16602	6.25850	6.38208	6.50293
M2_overtides_modulation	5.43155	5.39400	5.30318	5.29476	5.59013	5.92036	6.08038	6.19149	6.32150	6.45644
Schnepf_01	3.65208	3.72228	3.65562	3.62483	4.29061	4.68073	4.83550	4.85176	4.89372	5.02178
Schnepf_02	3.66917	3.73031	3.66093	3.63007	4.29404	4.68349	4.83820	4.85427	4.89610	5.02376
NOAA	3.67085	3.74696	3.69965	3.67602	4.29122	4.64153	4.76985	4.80400	4.85554	4.96344
NOAA_short	3.69084	3.75556	3.70419	3.67882	4.29405	4.64377	4.77179	4.80569	4.85768	4.96551

3.2 Least Squares Fit (backward)

Table S9. Root Mean Square (rms) values for: Least Squares backward (station = ASC)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	1.47090	1.58825	1.62875	1.60271	1.58965	1.54669	1.51109	1.48691	1.46788	1.45247
M2_overtides	1.40865	1.56544	1.61846	1.59563	1.58493	1.54312	1.50895	1.48505	1.46610	1.45106
M2_modulation	1.03954	1.19831	1.20458	1.24741	1.24469	1.21108	1.20515	1.19215	1.18057	1.16834
M2_overtides_modulation	0.67566	1.02715	1.08337	1.16044	1.16675	1.13332	1.14534	1.13280	1.11942	1.10837
Schnepf_01	1.09102	1.22326	1.20076	1.25283	1.25818	1.23608	1.21351	1.19316	1.18012	1.16566
Schnepf_02	1.10615	1.22711	1.20860	1.25698	1.26324	1.24127	1.21746	1.19673	1.18318	1.16880
NOAA	0.80863	1.07482	1.07443	1.15703	1.16133	1.12618	1.12137	1.10632	1.09227	1.07777
NOAA_short	0.83077	1.09195	1.09813	1.17201	1.17413	1.13708	1.13184	1.11710	1.10312	1.08912

Table S10. Root Mean Square (rms) values for: Least Squares backward (station = CZT)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	4.36355	4.33521	4.35326	4.40842	4.33601	4.24761	4.14584	4.05470	3.99036	3.95176
M2_overtides	4.35711	4.33236	4.35182	4.40718	4.33471	4.24690	4.14496	4.05411	3.98997	3.95135
M2_modulation	3.93158	3.94664	3.99014	4.17168	4.07742	3.97589	3.93158	3.87380	3.82176	3.78673
M2_overtides_modulation	3.71841	3.84813	3.91667	4.11859	4.04410	3.95370	3.91362	3.85955	3.81023	3.77775
Schnepf_01	4.06287	4.03047	4.07854	4.18881	4.13007	4.03809	3.94535	3.86457	3.79768	3.75289
Schnepf_02	4.06367	4.03099	4.07913	4.18894	4.13026	4.03827	3.94543	3.86464	3.79776	3.75299
NOAA	3.93853	3.93326	3.98094	4.15001	4.07550	3.97431	3.91174	3.84218	3.78744	3.74940
NOAA_short	3.94271	3.93482	3.98236	4.15057	4.07627	3.97483	3.91204	3.84246	3.78775	3.74973

Table S11. Root Mean Square (rms) values for: Least Squares backward (station = SJG)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	7.92396	7.96341	7.69606	7.55999	7.53728	7.26564	7.00169	6.85276	6.75032	6.71727
M2_overtides	7.82773	7.94384	7.68436	7.54945	7.53040	7.26166	6.99860	6.84945	6.74791	6.71578
M2_modulation	7.47010	7.62838	7.32767	7.32633	7.33740	7.09553	6.83281	6.66788	6.55185	6.50293
M2_overtides_modulation	6.32313	7.12440	7.12768	7.21822	7.25193	7.01569	6.77486	6.61010	6.50150	6.45644
Schnepf_01	5.25956	5.69868	5.22038	5.50359	5.59224	5.48017	5.31681	5.19032	5.10019	5.02178
Schnepf_02	5.28275	5.70692	5.22595	5.50861	5.59744	5.48423	5.31980	5.19278	5.10226	5.02376
NOAA	4.79789	5.55682	5.16307	5.39839	5.44433	5.30296	5.19719	5.10260	5.02554	4.96344
NOAA_short	4.81049	5.56706	5.16740	5.40258	5.44781	5.30656	5.19940	5.10473	5.02722	4.96551

3.3 Iterative Robuts Least Squares

Table S12. Root Mean Square (rms) values for: IRLS forward (station = ASC)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	1.27689	1.26380	1.24838	1.24608	1.31556	1.36575	1.38724	1.42631	1.44495	1.45280
M1_overtides	1.26649	1.26064	1.24661	1.24437	1.31439	1.36474	1.38653	1.42571	1.44415	1.45244
M2_modulation	0.96116	0.94734	0.94397	0.95193	1.06103	1.11154	1.13814	1.15549	1.16403	1.16945
M2_overtides_modulation	0.76721	0.82126	0.82585	0.84041	0.96646	1.01788	1.05214	1.08080	1.09429	1.11158
Schnepf_01	0.93334	0.92954	0.92210	0.92912	1.04608	1.10220	1.13475	1.15078	1.15812	1.16696
Schnepf_02	0.94822	0.93918	0.92800	0.93475	1.04939	1.10561	1.13863	1.15406	1.16131	1.16964
NOAA	0.74977	0.74726	0.73408	0.72730	0.93103	1.01140	1.05211	1.06788	1.07250	1.08306
NOAA_short	0.78727	0.77872	0.76219	0.75947	0.94609	1.02067	1.05946	1.07690	1.08283	1.09254

Table S13. Root Mean Square (rms) values for: forward (station = CZT)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	3.43030	3.36530	3.29161	3.34197	3.53387	3.66162	3.74715	3.85161	3.91107	3.95495
M2_overtides	3.42701	3.36399	3.29094	3.34181	3.53315	3.66104	3.74669	3.85130	3.91077	3.95467
M2_modulation	3.15863	3.11057	3.05572	3.09210	3.36234	3.50477	3.58438	3.69729	3.75965	3.79675
M2_overtides_modulation	3.02413	3.01705	2.99029	3.05277	3.33121	3.47862	3.56714	3.68776	3.75251	3.79078
Schnepf_01	2.98774	2.92594	2.87871	2.89463	3.24131	3.44073	3.50486	3.64644	3.72174	3.76870
Schnepf_02	2.98814	2.92561	2.87794	2.89396	3.24060	3.44015	3.50433	3.64562	3.72075	3.76765
NOAA	3.01858	2.96350	2.91224	2.94112	3.26565	3.45101	3.51852	3.65302	3.72278	3.76708
NOAA_short	3.02111	2.96518	2.91361	2.94192	3.26642	3.45095	3.51865	3.65282	3.72244	3.76670

Table S14. Root Mean Square (rms) values for: forward (station = SJG)

Name (hsum)	year=1	year=2	year=3	year=4	year=5	year=6	year=7	year=8	year=9	year = 10
Kuvshinov_01	6.38121	6.11565	5.97647	5.90009	5.95293	6.21780	6.34414	6.46075	6.61348	6.71744
M2_overtides	6.36941	6.10396	5.96986	5.89881	5.95257	6.21716	6.34462	6.46098	6.61395	6.71704
M2_modulation	5.80954	5.59621	5.46921	5.39993	5.67207	6.01087	6.17189	6.26337	6.38603	6.50731
M2_overtides_modulation	5.56846	5.50235	5.38607	5.35634	5.62355	5.95148	6.10931	6.21930	6.35279	6.48359
Schnepf_01	3.67023	3.74311	3.67961	3.64505	4.33635	4.77032	4.94974	4.95324	4.98385	5.12264
Schnepf_02	3.68140	3.74308	3.67222	3.64048	4.33479	4.75657	4.91849	4.92678	4.95845	5.09176
NOAA	3.71224	3.78154	3.73711	3.71118	4.33722	4.70610	4.82785	4.86186	4.91015	5.01669
NOAA_short	3.72493	3.78264	3.73139	3.70406	4.33176	4.69964	4.81935	4.85474	4.90407	5.01035

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