

Universal orbital angular momentum spectrum analyzer for beams:

Supplementary Materials

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1. Phase term ϕ determination

The accurate determination of phase ϕ is essential for the accurate OAM spectrum analyzing. The method we use here is actually a helical wavefront sensing scheme [1]. When a reference Gaussian beam E_R is combined coaxially with the tested OAM beam E , the intensity profile of the interference field $I_{\cos}$ reads:

$$I_{\cos}=|E+E_R|^2=|E|^2+|E_R|^2+2|E||E_R|\cos(\phi-\phi_R) \quad (S1)$$

with ϕ_R the phase of reference beam. If an additional phase delay $\pi/2$ is introduced for the reference Gaussian beam E_R , the interference field $I_{\sin}$ reads:

$$I_{\cos}=|E+E_R\exp(i\pi/2)|^2=|E|^2+|E_R|^2+2|E||E_R|\sin(\phi-\phi_R) \quad (S2)$$

Considering that a Gaussian beam can be approximated as a plane wave, thus ϕ_R is a constant and can be ignored. From Eqs. (S1) and (S2), one can obtain the phase term as:

$$\phi=\arctan\left(\frac{I_{\sin}-I_{\cos}}{I_{\cos}-I_{\cos}}$$

Therefore if the intensity patterns of the tested OAM beam I , the reference beam I_R , the two interference fields $I_{\cos}$ and $I_{\sin}$ are captured, one can determine the phase term ϕ , and thus to analyze the OAM spectrum.

2. RMSE evaluation for OAM spectrum

We introduce the value RMSE of OAM spectrum to evaluate the difference between the simulatedly or experimentally analyzed OAM spectra and theoretical one. From Eq. (5) the intensity proportion for various OAM channels can be expressed as a sequence $\{R_l\}$. The RMSE of OAM spectrum is defined as:

$$\text{RMSE}=\sqrt{\frac{\sum_{l=a}^b(A_l-T_l)^2}{b-a+1}} \quad (S4)$$

where $\{R_l\}=\{A_l\}$ denotes the simulatedly or experimentally analyzed OAM spectra and $\{R_l\}=\{T_l\}$ denotes the theoretical OAM spectrum. Parameters a and b are the lower and upper limits of l . The smaller the RMSE value, the more accurate the analyzed OAM spectrum.

3. Holograms to tailor multi-ring OAM beams

Multi-ring OAM beams have multi-concentric-circles-shaped intensity profile, and have two eigen values as the radial index p determines the intensity fluctuation along radial direction and

topological charge l determines the helical phase along azimuthal angle. Therefore when generating multi-ring OAM beams one must introduce phase difference along both radial and azimuthal directions. Here we employ the method proposed by Y. Ohtake et al. [2] to achieve this goal, where a π phase shift is brought in each concentric circle. The phase distribution of the tailoring hologram under polar coordinates reads $\varphi_{MR} = -l\varphi + \pi\Theta[-L_p^{|l|}(2r^2/\omega^2)]$, with $\Theta(\zeta)$ the step function. When a Gaussian beam passing through such hologram encoded SLM, the multi-ring OAM beam with selective radial index p and topological charge l will present at diffraction fields. The correlation coefficients between the observed and the theoretical two-dimensional intensity profiles to universally can reach more than 0.946 [2].

4. Influence of CCD's resolution on the measurement accuracy of OAM spectrum

Obviously the resolution of the CCD camera employed in the system will affect the measurement accuracy of OAM spectrum. Such phenomenon can be vividly understood as such resolution determines the dispersion degree of the measured complex amplitude along azimuth. Here we study the influence of the resolution of the CCD camera on the measured OAM spectra by simulation, where takes two-fold multiplexed OAM beams ($l_1=2$, $l_2=6$, intensity proportion 1:2) as example. The simulated results are displayed in Fig. S1.

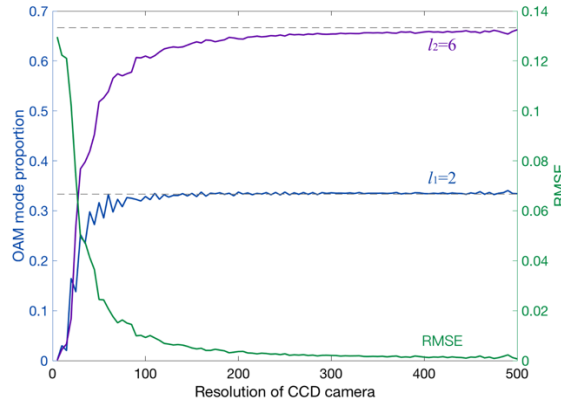


Figure S1 | Diagram to show the influence of the resolution of the CCD camera on the measured OAM spectra.

With the increasing of the CCD's resolution, the OAM mode proportions of the present OAM channels are getting closer to the theoretical value, and the RMSE decreases significantly.

Here the image surface of the CCD camera is a square, and the abscissa of Fig. S1 is the number of pixels along one direction. We evaluate the measured OAM mode proportion and the RMSE performance separately. From Fig. S1 one can acquire that the resolution of CCD camera has great effects on the accuracy of OAM spectra diagnostic. As the resolution increases, all of the three curves all tend to flatten out, where the OAM mode proportions are very closed to the theoretical value (gray dashed lines). When the resolution reaches 300, the RMSE value is small as 2.1×10^{-3} .

References

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