

Supplementary Material for

Large-scale phase retrieval

Xuyang Chang, Liheng Bian,* and Jun Zhang

¹*School of Information and Electronics & Advanced Research Institute of Multidisciplinary Science, Beijing Institute of Technology, Beijing 100081, China*

**bian@bit.edu.cn*

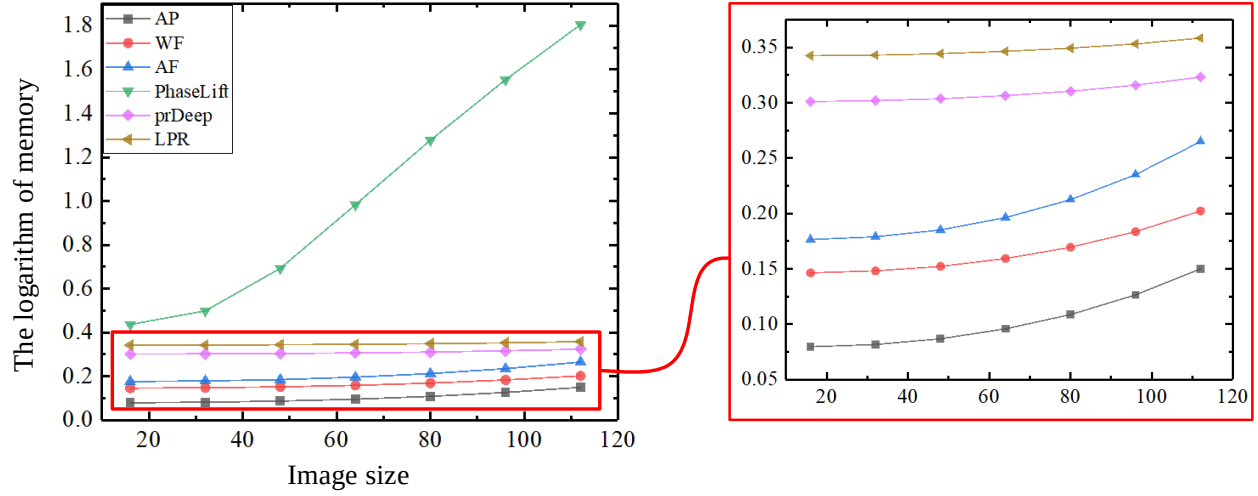


Figure S1: **The relationship between memory requirement and image size under the CDI modality.** The horizontal axis represents the image size in one dimension, and the vertical axis is the logarithm of required memory (GB). These algorithms ran on another computer with 64GB memory. We can see that except for the PhaseLift technique, the required memory of the other algorithms increase linearly as the image size enlarges.

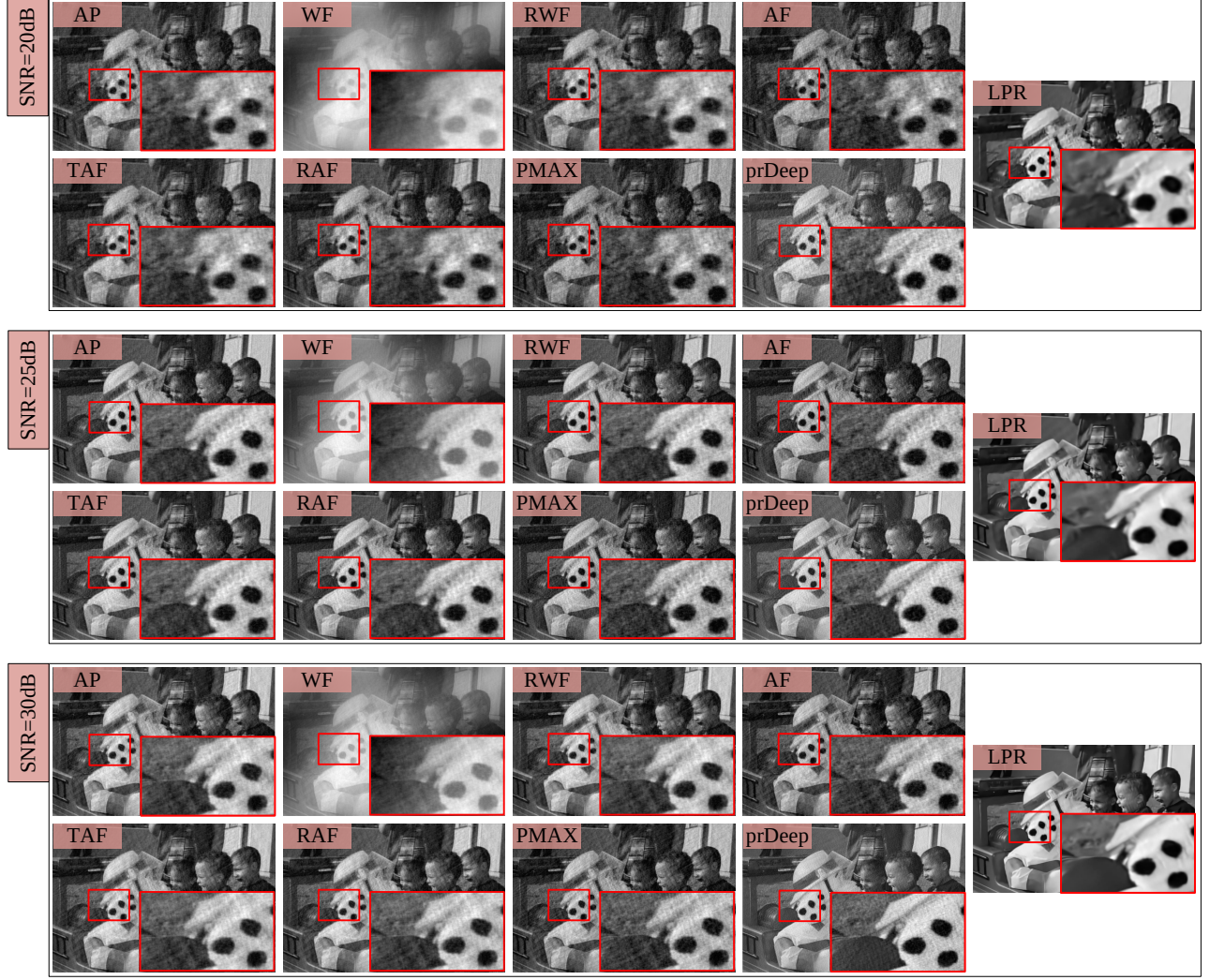


Figure S2: **Visual comparison under the CDI modality.** The spatial resolution of ground truth is 1356×2040 , and zeros are padded around the image matrix to generate a 2712×4080 image to fulfill the oversampling requirement. From the results, we can see that most of the conventional methods (except for prDeep) obtain little improvement or even degradation compared to AP. These algorithms can not handle measurement noise well and produce low fidelity. In contrast, the reported *LPR* technique is effective to attenuate noise and artifacts, producing smooth background and high fidelity.

Table S1: **Quantitative comparison under the CDI modality (onion cell).**

Algorithm	SNR=20dB			SNR=25dB			SNR=30dB		
	PSNR	SSIM	TIME	PSNR	SSIM	TIME	PSNR	SSIM	TIME
AP	18.69	0.50	782.51	20.71	0.60	791.49	23.47	0.68	803.90
WF	19.42	0.47	+29.66	20.96	0.57	+28.48	23.75	0.66	+30.83
RWF	18.95	0.46	+24.74	20.94	0.56	+26.20	23.78	0.66	+29.07
AF	15.97	0.31	+25.93	18.42	0.44	+27.80	21.61	0.57	+27.05
TAF	18.65	0.48	+26.72	20.94	0.57	+22.18	23.81	0.65	+27.99
RAF	16.41	0.31	+23.08	18.36	0.44	+21.59	21.24	0.57	+23.40
PLIFT	✗-memory limitation			✗-memory limitation			✗-memory limitation		
PLAMP	✗-memory limitation			✗-memory limitation			✗-memory limitation		
PMAX	19.14	0.47	+40.52	21.01	0.57	+37.91	24.01	0.67	+36.08
CD	✗-no convergence			✗-no convergence			✗-no convergence		
KAC	✗-no convergence			✗-no convergence			✗-no convergence		
prDeep	20.20	0.52	+45.82	21.01	0.57	+40.65	24.92	0.70	+36.08
LPR	23.02	0.70	+27.60	25.56	0.73	+27.19	28.02	0.78	+28.99

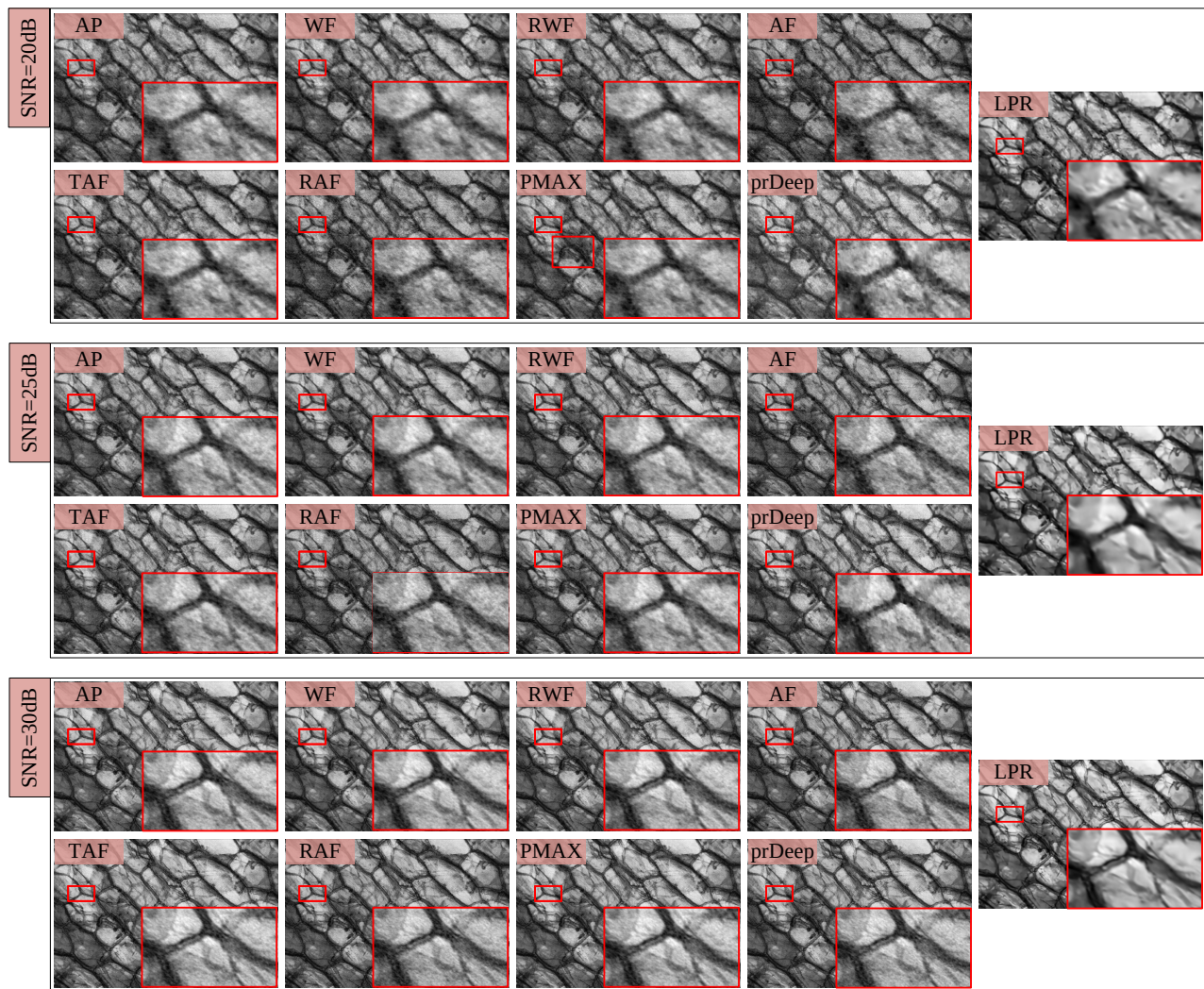


Figure S3: **Visual comparison under the CDI modality (onion cell).**

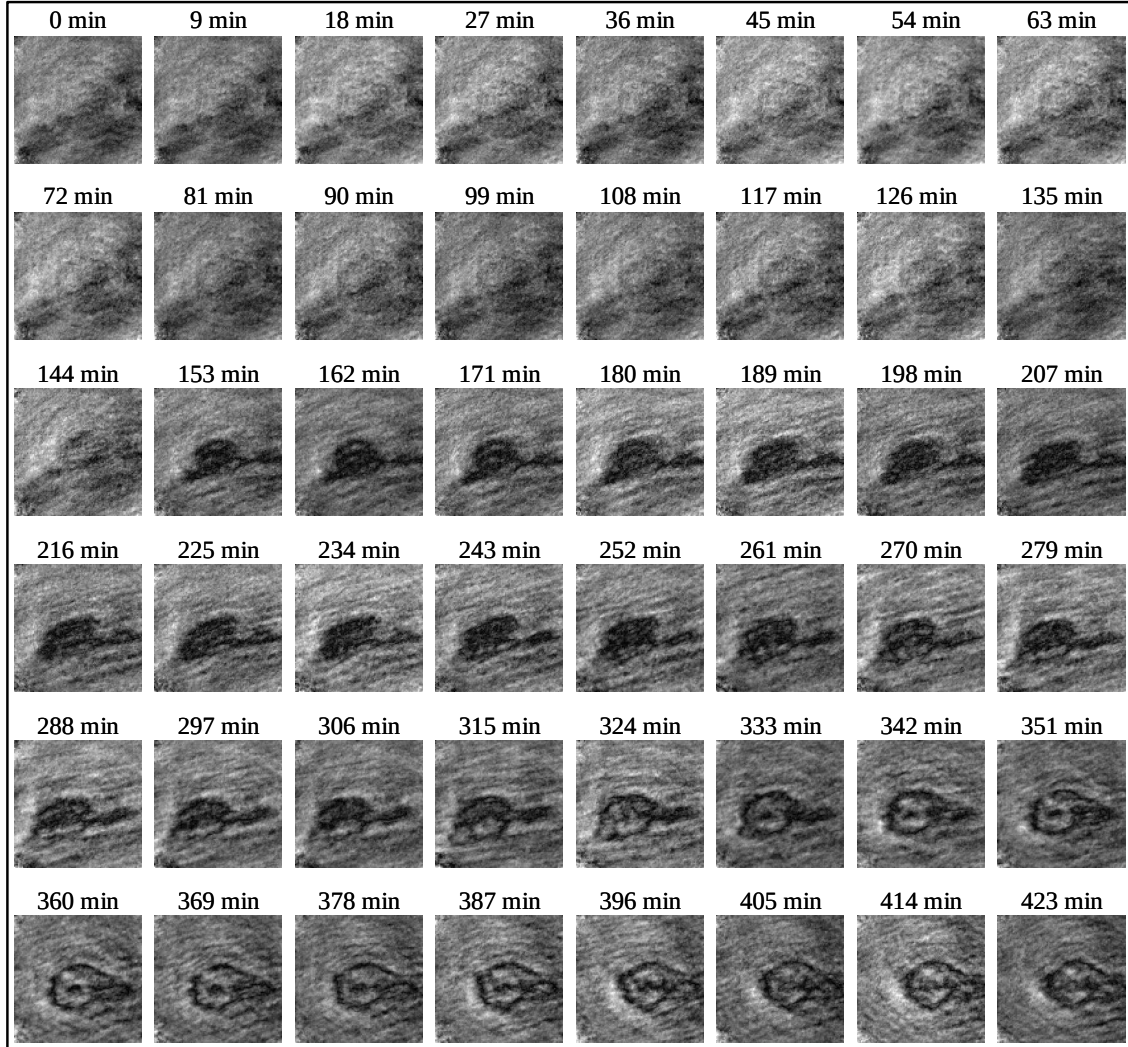


Figure S4: **Experiment amplitude results of AP under the CDI modality.** The results describe that a 25 μ m cancer cell gradually attached to a 100 μ m large cell within two hours. In about 6 hours, they merged and formed a high-density area. The amplitude reconstruction of AP maintains serious noise. We can hardly observe cell's contour and edge in most of monments.

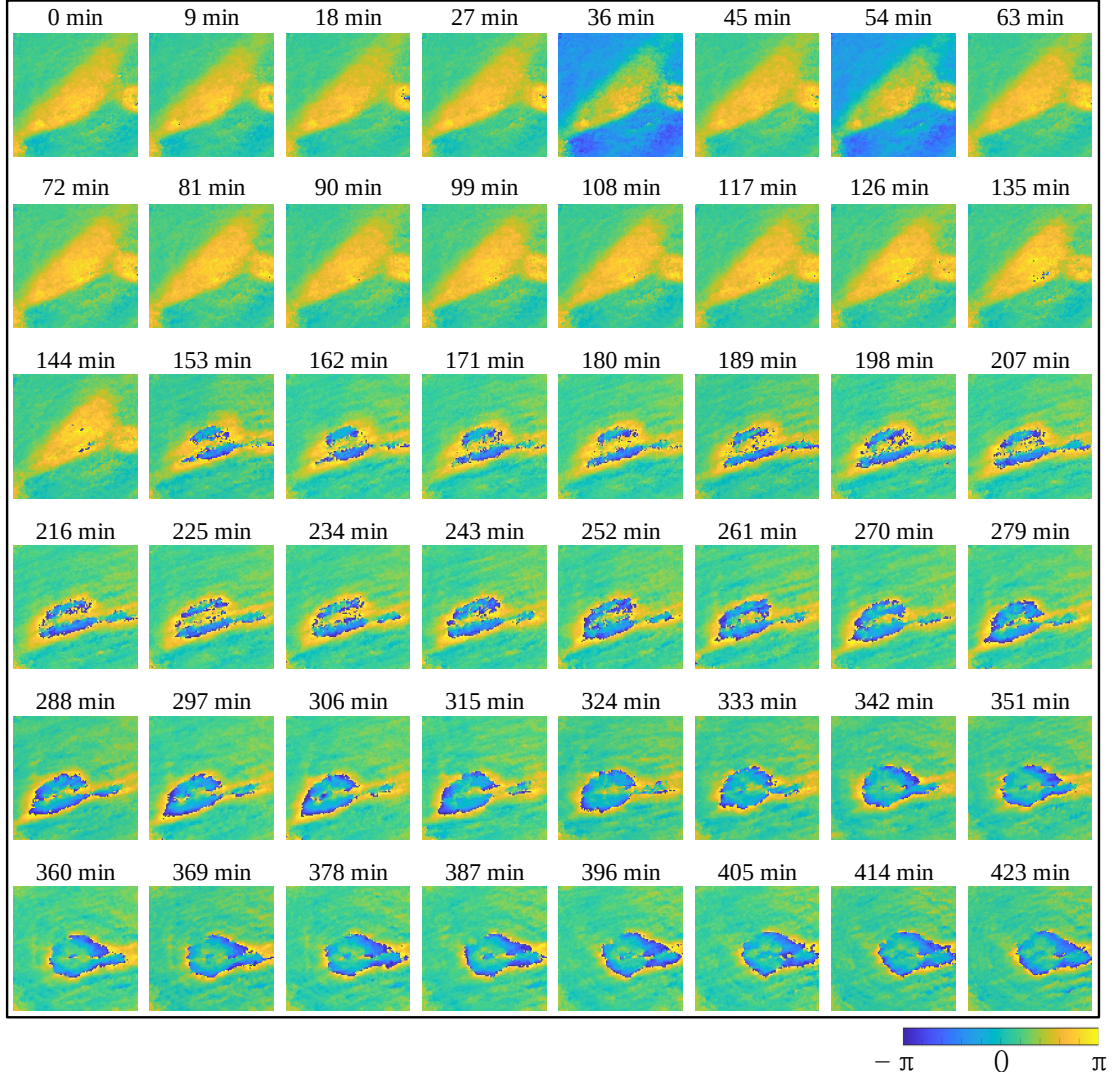


Figure S5: **Experiment phase results of AP under the CDI modality.** The phase reconstruction of AP produces more details compared to its amplitude. However, it still suffers from measurement noise.

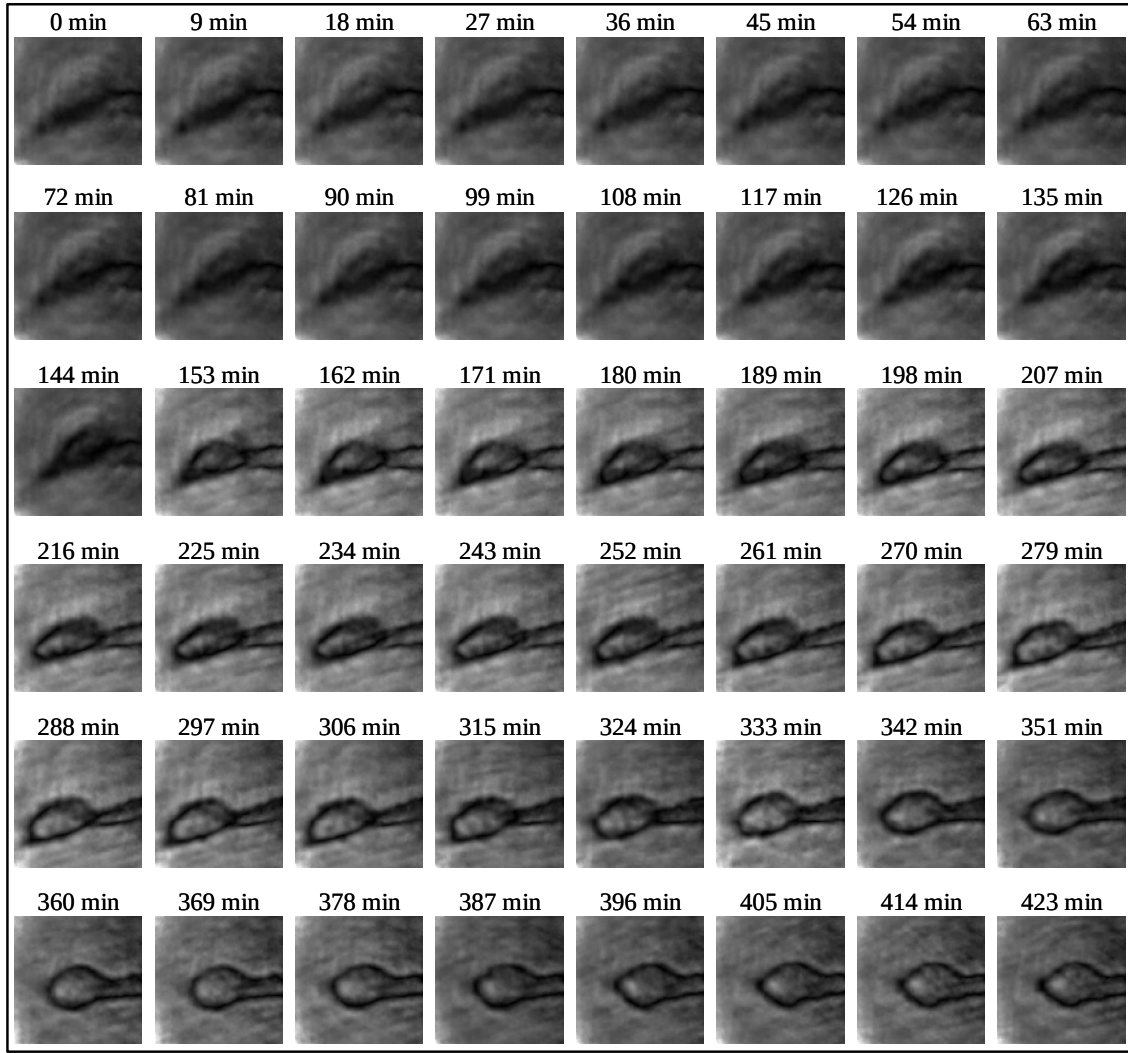


Figure S6: **Experiment amplitude results of *LPR* under the CDI modality.** The amplitude reconstruction of *LPR* obtains more details about the cell's contour (especially at the moments of 0 min to 144 min)

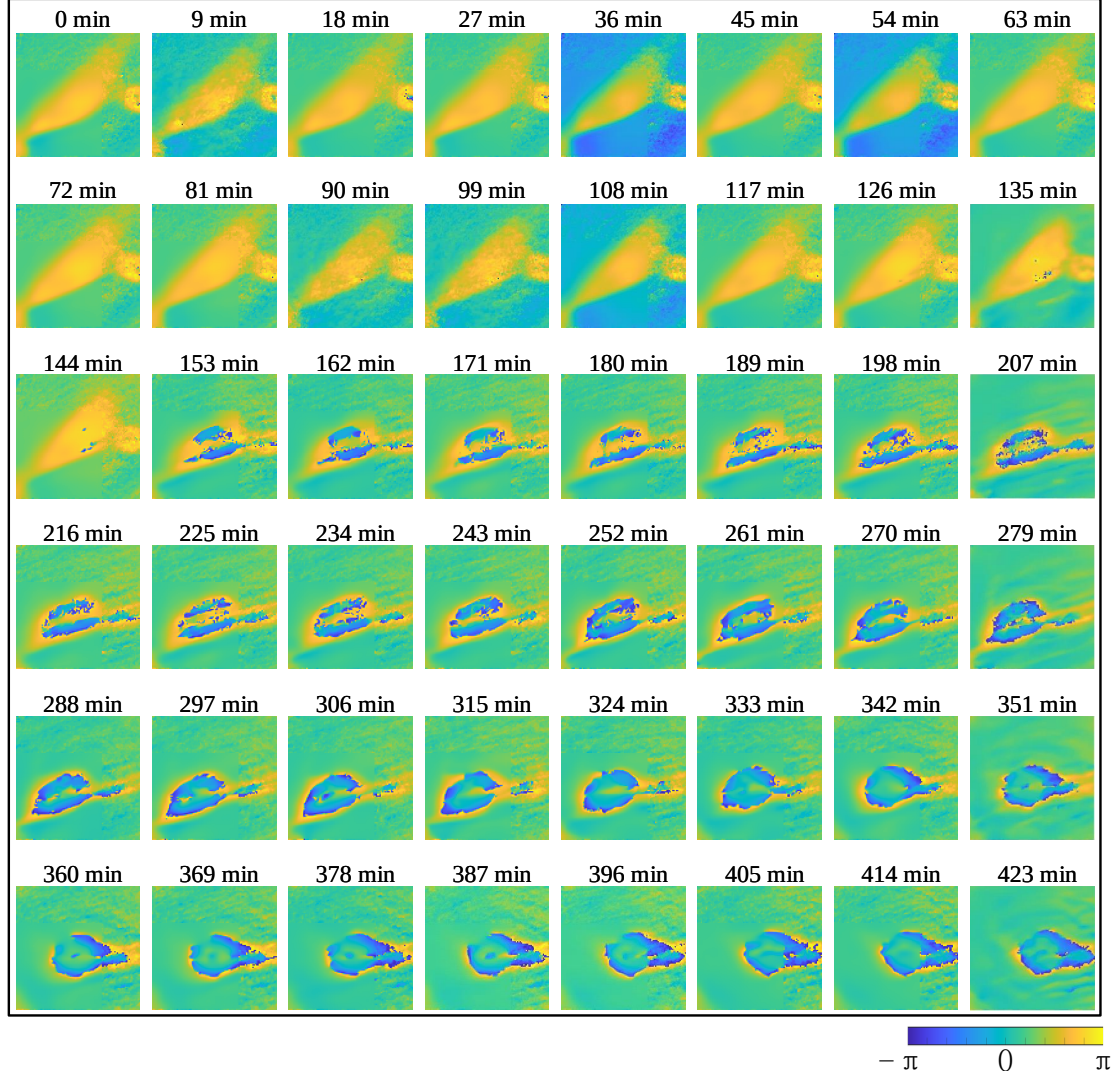


Figure S7: **Experiment phase results of *LPR* under the CDI modality.** The reported *LPR* technique can effectively attenuate measurement noise, with smooth background and high fidelity compared to AP.

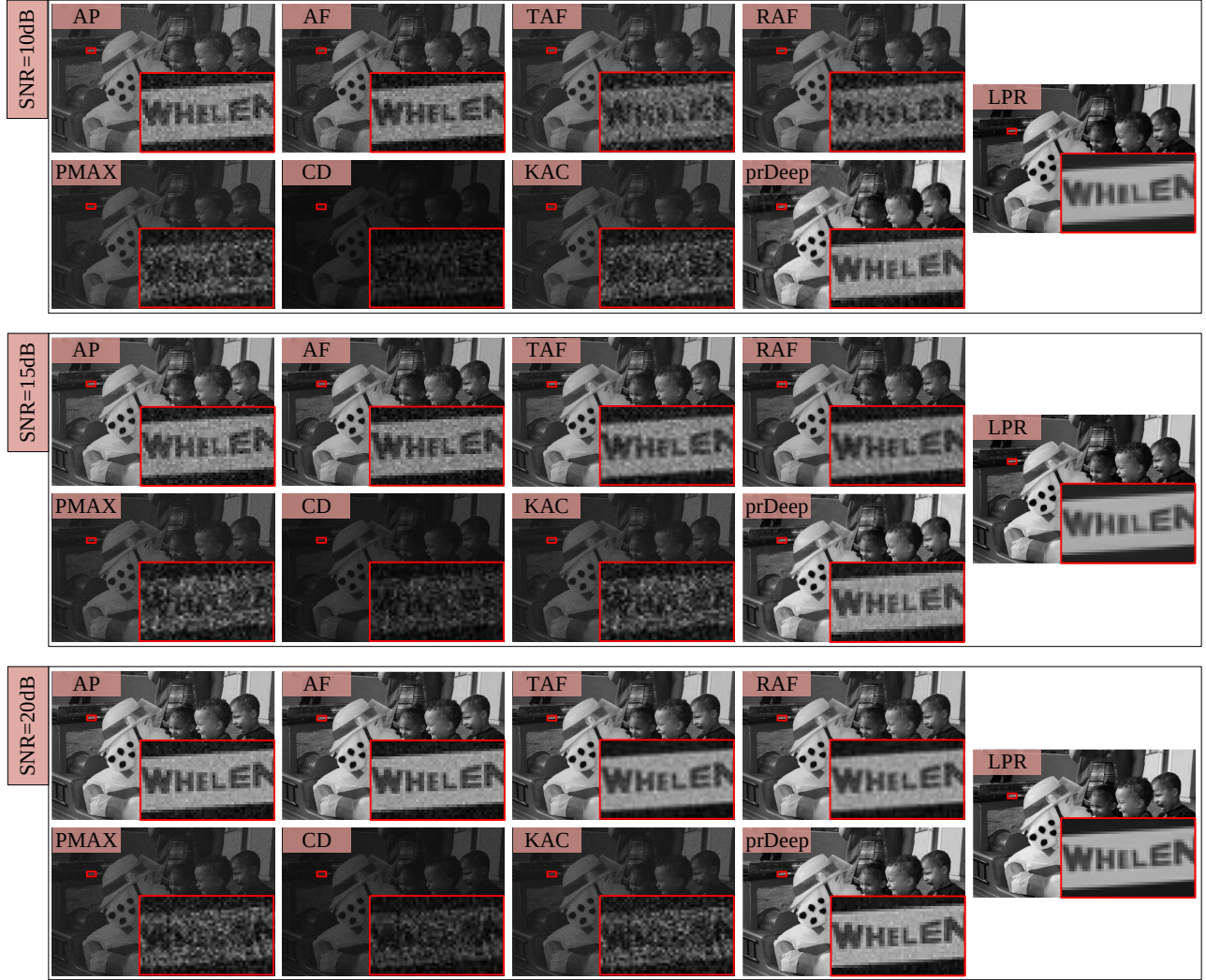


Figure S8: **Visual comparison of simulation results under the CDP modality (5 modulations).**

The reconstruction of AP is sensitive to measurement noise, and that of the other conventional algorithms are even worse due to limited sampling ratio and noise corruption. The target information is buried. In comparison, *LPR* can retrieve more details even in heavy noise, and maintains strong robustness to measurement noise.

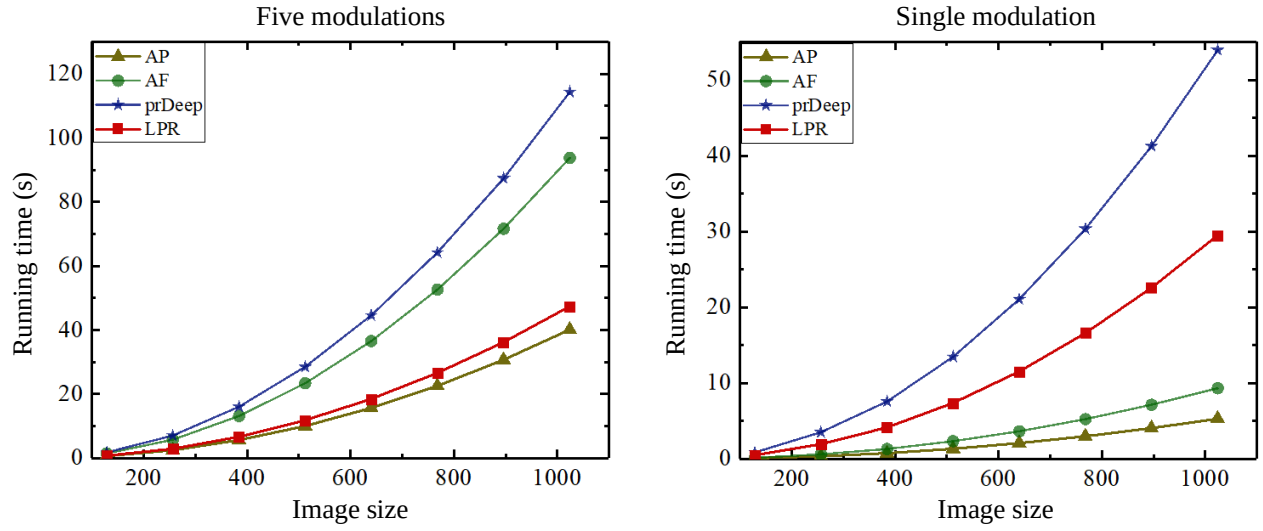


Figure S9: **The relationship between running time and image size under the CDP modality.**

The horizontal axis represents the image size in one dimension, and the vertical axis shows the running time (s). As the image size enlarges, the reported *LPR* technique obtains a lower computational complexity than the prDeep algorithm.

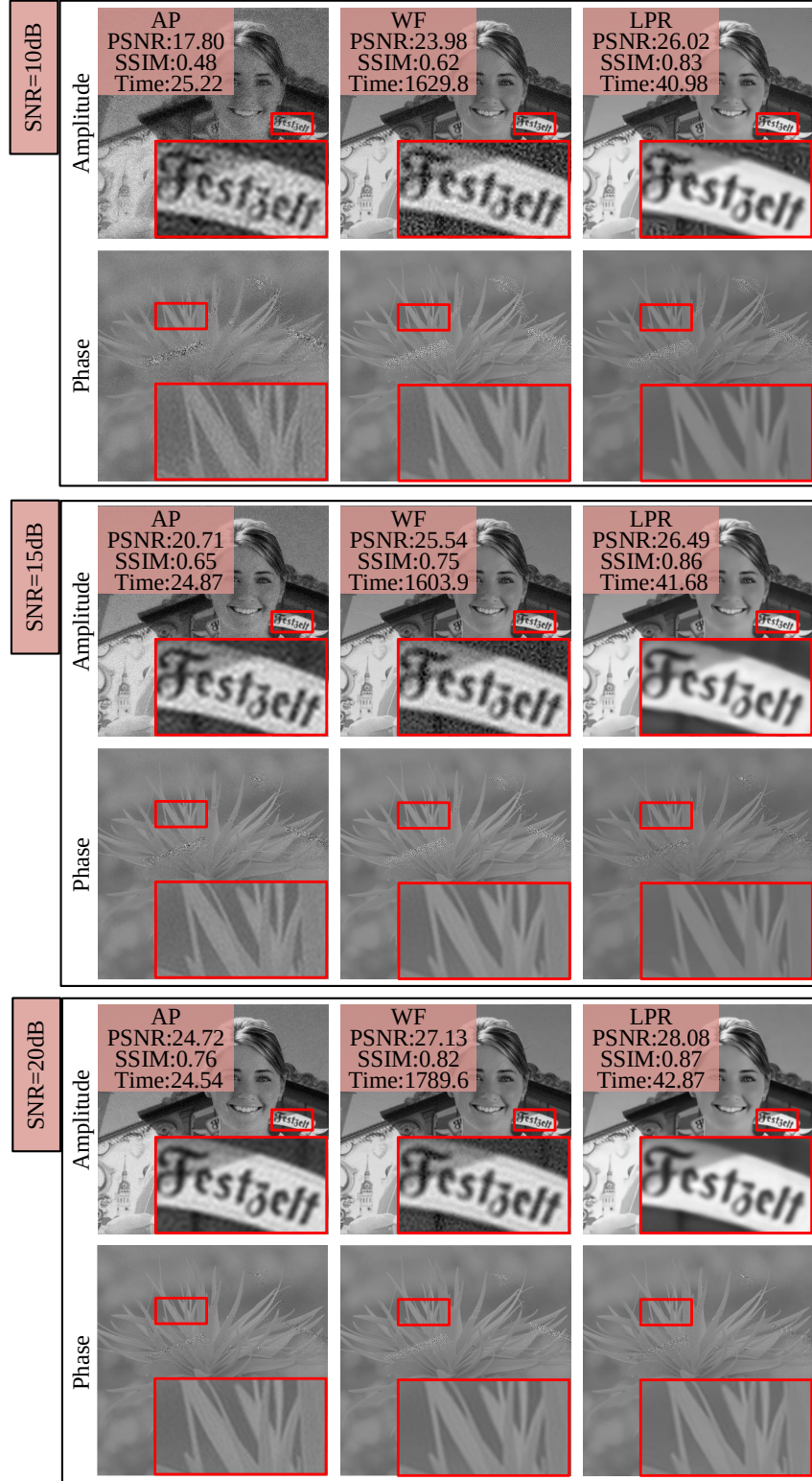


Figure S10: **Comparison of simulation results under the FPM modality.** The reported *LPR* technique provides the highest reconstruction quality.

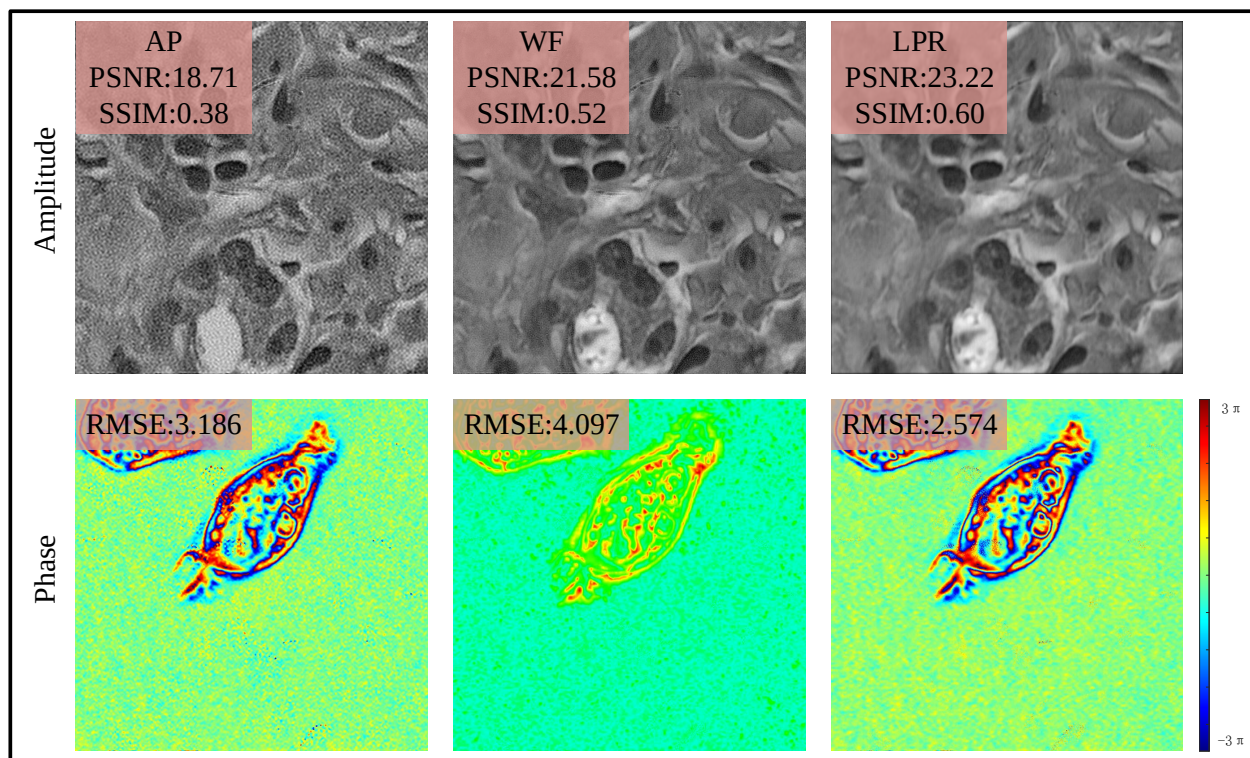


Figure S11: **Comparison of large-phase-range phase retrieval results under the FPM modality.** The phase ranges from -3π to 3π , and we added 10dB white Gaussian noise in the measurements. The quantitative evaluations validate that *LPR* produces higher fidelity with less reconstruction error than the other techniques.

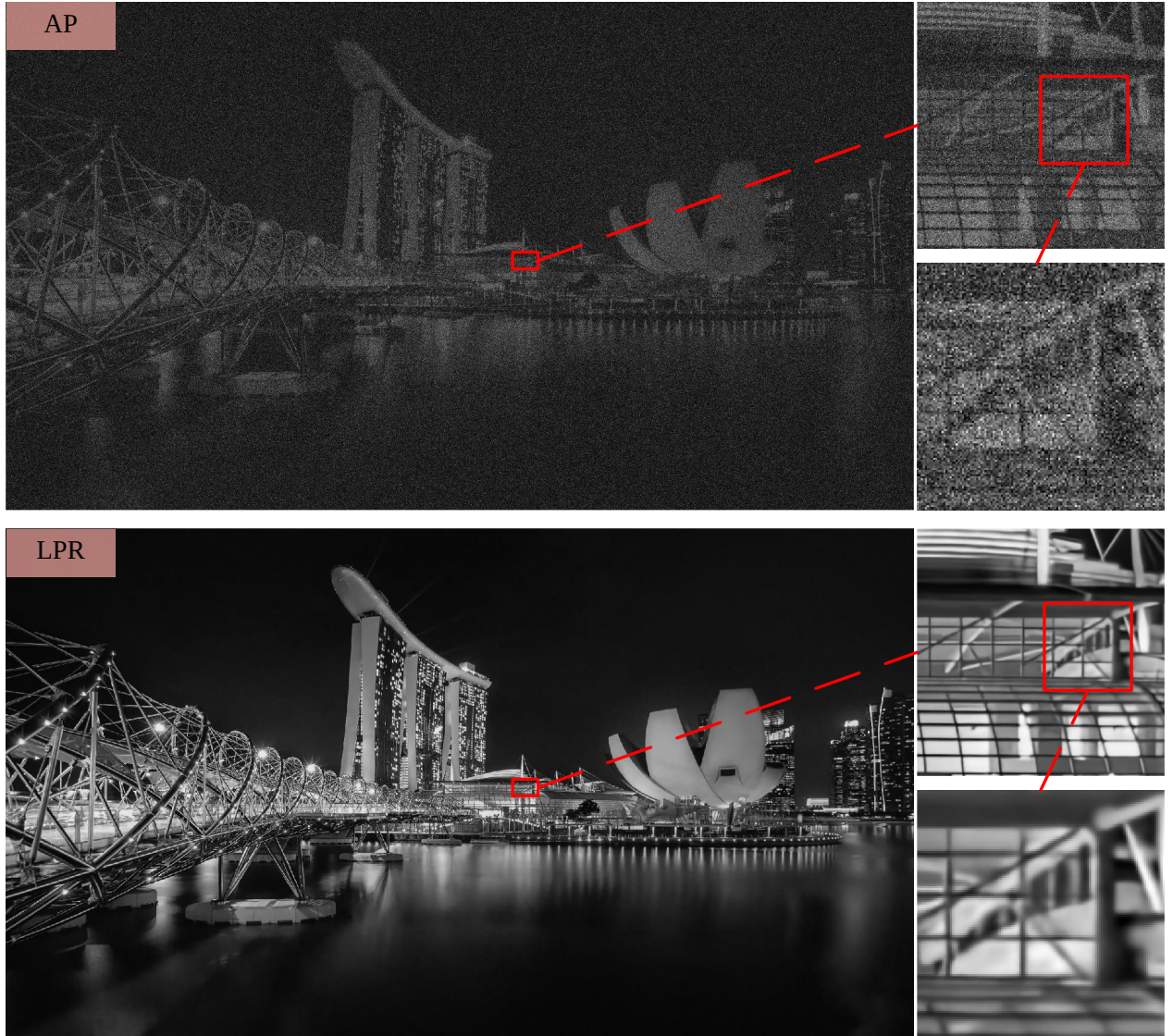


Figure S12: **Ultra-large-scale phase retrieval at the 8K level (7680×4320 pixels).** The reconstruction is implemented under the CDP modality (5 modulations). We use an 8K image (7952×4472 , Marina Bay Sands) as ground truth, and add white Gaussian noise ($\text{SNR} = 5\text{dB}$) to measurements. The reconstruction result of AP is buried in measurement noise, losing crucial target details (see close-up). In comparison, *LPR* obtains much better reconstruction performance, with more details and less background noise.

Algorithm 1 Large-scale phase retrieval algorithm (LPR).

Input: Intensity-only measurements I ,Forward and backward operators A / A^{-1} ,Initial estimate u_0 , and set $u^{(0)} = v^{(0)} = u_0$,Hyper-parameters λ .**Outer loop:****For** $k = 1, 2, \dots$ **do**

$$I' = I - |Av^{(k)}|^2$$

• Inner loop (AP solver):**For** $j = 1, 2, \dots$ **do**1) Forward transform: $\mathcal{U}^{(j,k)} \leftarrow Au^{(j,k)}$ 2) Impose intensity constraint: $\mathcal{U}^{(j,k)} \leftarrow I' \cdot \frac{\mathcal{U}^{(j,k)}}{|\mathcal{U}^{(j,k)}|}$ 3) Backward transform: $u^{(j+1,k)} \leftarrow A^{-1}\mathcal{U}^{(j,k)}$ **End**

$$u^{(k+1)} = v^{(k)} + \lambda u^{(k)}$$

$$v^{(k+1)} = EN(u^{(k+1)})$$

End**Output:** Reconstructed image u .
