

¹Values of β and γ

²An ordinary differential model with the same mechanism as the system (1) in main²
³text is constructed to fit the actual positive data to obtain the values of β and γ .³

⁵0.1 Material

⁶*Research object:* A typical large-scale pig farm in China.⁶

⁷ *Time interval:* 2016 – 2019⁷

⁸ *Time unit:* Year⁸

⁹ *Data types and sources:* The sampling test is carried out to obtain the pathogenic⁹
¹⁰positive rate of the farm. According to the field survey, the positive rate of pathogen¹⁰
¹¹monitoring can be seen in Table S3.¹¹

Table S3 The positive rate in this farm from 2016 to 2019.

Years	2016	2017	2018	2019
The positive rate	4.286%	3.505%	1.538%	0.83%

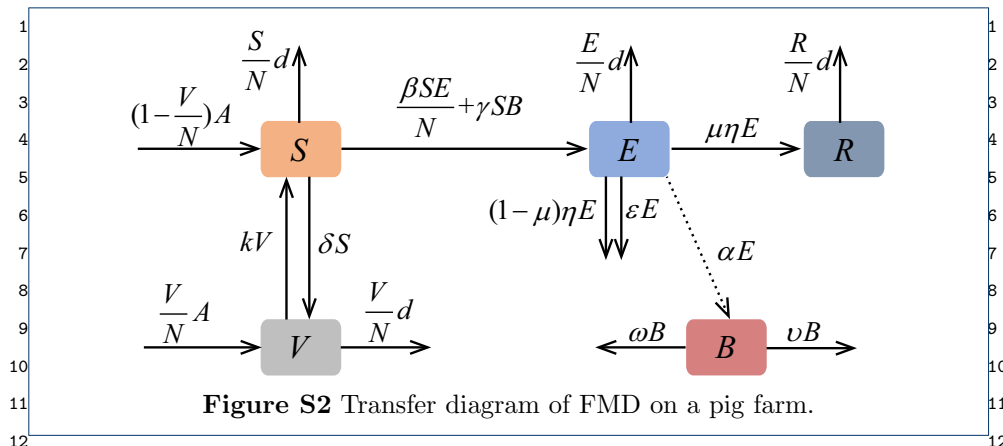
¹⁷0.2 Method

¹⁸At time t , the total number of pigs in this farm is denoted by $N(t)$. The transmission¹⁸
¹⁹modes of FMD are direct contact transmission and environmental transmission.¹⁹
²⁰Pigs infected with FMD need 3 – 4 weeks to remove the virus. During the infection²⁰
²¹period, some pigs that show clinical symptoms are culled, while the remaining pigs²¹
²²are self-healing and clear the virus. According to the transmission mechanism, the²²
²³pigs in the farm are divided into four compartments: susceptible group, immunized²³
²⁴group, infection group and recovery group. The number of four subgroups at time²⁴
²⁵ t is expressed as $S(t)$, $V(t)$, $E(t)$ and $R(t)$. The total number of viruses in the²⁵
²⁶environment of the farm at time t is denoted by $B(t)$. The transfer diagram of²⁶
²⁷FMD transmission among compartments can be seen in Figure S2. The system²⁷
²⁸corresponding to the transfer diagram is as follows:²⁸

$$\begin{cases}
 \frac{dS(t)}{dt} = (1 - \frac{V}{N})A + kV - \delta S - \frac{\beta SE}{N} - \gamma SB - \frac{S}{N}d, \\
 \frac{dV(t)}{dt} = \frac{V}{N}A + \delta S - kV - \frac{V}{N}d, \\
 \frac{dE(t)}{dt} = \frac{\beta SE}{N} + \gamma SB - \eta E - \varepsilon E - \frac{E}{N}d, \\
 \frac{dR(t)}{dt} = \mu\eta E - \frac{R}{N}d, \\
 \frac{dB(t)}{dt} = \alpha E - \omega B - \nu B.
 \end{cases} \quad (0.1)$$

³⁶ The model is interpreted as follows:³⁶

³⁷ (1) For susceptible group, the changes over time include birth of piglets, natural³⁷
³⁸death or sale, removal due to being infected, removal due to being immunized,³⁸
³⁹and import due to antibody loss. The number of newborn piglets per unit time in³⁹
⁴⁰the farm is A , in which the number of susceptible pigs is $(1 - \frac{V}{N})A$. The number⁴⁰
⁴¹of natural death or sale per unit time in the farm is d , in which the number of⁴¹
⁴²susceptible pigs is $\frac{S}{N}d$. The immunization rate coefficient per unit time is δ . The⁴²
⁴³protection period of the vaccine is $\frac{1}{k}$, then the antibody loss rate coefficient per⁴³
⁴⁴unit time is k . β is the coefficient of effective contact rate of infected pigs, which is⁴⁴
⁴⁵the product of the number of pigs contacted by an infected pig and the infection⁴⁵
⁴⁶probability of each contact with the infected. The proportion of susceptible pigs in⁴⁶



the farm is $\frac{S}{N}$, the average effective contact number of an infected pig to susceptible pig per unit time is $\beta \frac{SE}{N}$. Thus, the number of pigs infected by direct contact per unit time is $\frac{\beta SE}{N}$. The γ is the coefficient of infection rate of foot and mouth disease virus in the environment. The number of pigs infected by environmental contact in unit time is γSB (bilinear incidence rate).

(2) For infection group, the changes over time include the import of new infected pigs, natural death or sale, surveillance elimination, culling due to showing clinical symptoms, and removal to the recovery group after self recovery. η is the virus clearance rate coefficient. $1 - \mu$ the proportion of pigs that are culled for showing clinical symptoms. μ is the proportion of pigs with no clinical symptoms. ε is the monitoring rate coefficient. The number of infected pigs that die naturally or are sold per unit time is $\frac{E}{N}d$.

(3) For the recovery group, its changes over time include the import of the infected pigs, natural death or sale. The number of recovered pigs that die naturally or are sold per unit time is $\frac{R}{N}d$.

(4) For the immunized group, its changes over time include birth of piglets with maternal antibody, import of pigs immunized successfully, removal of immune failure, natural death or sale. $\frac{V}{N}A$ is the number of newborn piglets with maternal antibody. The number of immunized pigs that die naturally or are sold per unit time is $\frac{V}{N}d$.

(5) For the virus in environment, its changes over time include the import of the virus released by the infected person, the natural decay death, and the death caused by disinfection. α is the amount of virus excreted by an infected pig per unit time. ω is the natural decay rate coefficient of the virus in the environment per unit time. ν is disinfection rate coefficient of the virus in the environment per unit time.

The initial time of the system $t_0 = 2016$. In 2016, the amount of livestock on hand was $N_0 = 15063$. According to the reference and actual situation, the values and sources of parameters and initial values of variables are shown in Table S4. In addition, the values of parameters β and γ are estimated by applying least square method. The objective function is taken as

$$\min \sum_{t=2016}^{2019} [\tilde{x}(t) - x(t)]^2,$$

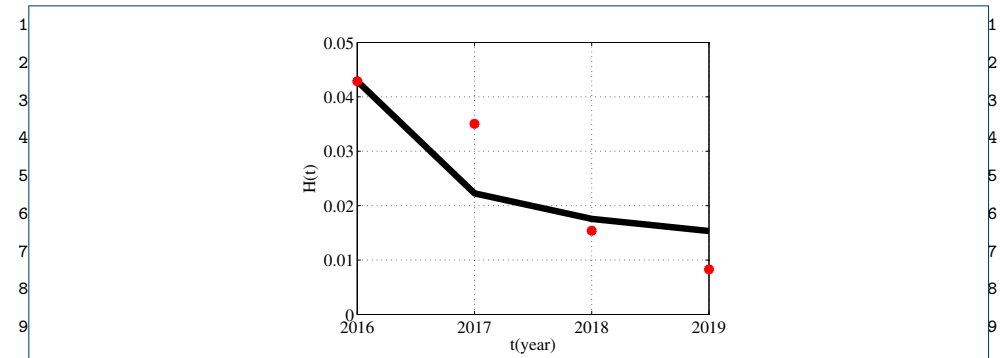


Figure S3 The fitting result of pathogen positive rate from 2016 to 2019. The black curve is the $x(t)$ obtained from the solution of system (0.1), and the red asterisks are the actual data $\tilde{x}(t)$ in Table S3.

Table S4 The values and sources of parameters and initial values of variables in the system (0.1).

Parameter values	Sources
$A = 54921$	The reproductive rhythm of this farm is 1 week. Approximately 1056 piglets are produced each week. There are 52 weeks in a year. So $A = 1056 \times 52 = 54912$.
$d = 54921$	According to the actual situation, the breeding quantity of the farm keeps a dynamic balance. So $A = d$.
$k = 2$	According to the vaccine use instruction, the protection period of the vaccine is 6 months, that is 0.5 years. so $k = \frac{1}{0.5} = 2$.
$\delta = 12.52$	According to the actual monitoring data, the average qualified rate of immune antibody is 86.23%. So, $\frac{V}{N} = 0.8623$. For the simplified system $\begin{cases} \frac{dS(t)}{dt} = kV - \delta S \\ \frac{dV(t)}{dt} = \delta S - kV \end{cases}$, when it reaches a steady state, $kV = \delta S$. Therefore $\delta = \frac{kV}{S} \approx 12.52$.
$\eta = 13$	Pigs infected with FMD need 3 – 4 weeks to remove the virus. So, $\eta = \frac{1}{4/52} = 13$.
$\mu = 1$	μ is the proportion of infected people without clinical symptoms. According to the actual situation, no pigs with symptoms were culled at this farm from 2016 to 2019. So $\mu = 1$.
$\varepsilon = 0.0925$	According to the calculation method of ε in reference (Ren et al., 2021), the monitoring rate coefficient ε is equal to the ratio of the number of sampled monitoring to the total number of pigs in the farm. By dividing the actual sampling monitoring data of 2016 – 2019 by the total number of pigs respectively and then calculating the mean value, $\varepsilon = 0.0925$ can be obtained.
$\alpha = 365 \times 10^{6.1}$	The average peak amount of virus excreted by a pig per day reaches $10^{6.1}$ TCID ₅₀ . Then $\alpha = 365 \times 10^{6.1}$.
$\omega = 12.167$	The survival time of the virus in the environment is about 30 days. Therefore, $\omega = \frac{1}{30/365} \approx 12.167$.
$\nu = 12$	The farm is disinfected once a month. Assume that each disinfection rate reaches 100%. So $\nu = \frac{1}{1/12} = 12$.
$V_0 = 11176$	According to the actual situation, the qualified rate of immune antibody in 2016 was 0.742. It is obvious that $V_0 = 15063 \times 0.742 \approx 11176$.
$E_0 = 646$	According to Table S3, the positive rate of pathogen in 2016 was 4.286%. Then, $E_0 = 4.286\% \times 15063 \approx 646$.
$S_0 = 1785$	Let $\frac{dV(t)}{dt} = \frac{V}{N}A + \delta S - kV - \frac{V}{N}d = 0$. Since $A = d$, $S = \frac{kV}{\delta}$. So, $S_0 = \frac{kV_0}{\delta} \approx 1785$.
$R_{00} = 122$	$R_{00} = N_0 - S_0 - E_0 - V_0 = 1456$.
$B_0 = 1.2283 \times 10^{10}$	Let $\frac{dB(t)}{dt} = \alpha E - \omega B - \nu B = 0$, then $B = \frac{\alpha E}{\omega + \nu}$. So, $B_0 = \frac{\alpha E_0}{\omega + \nu} \approx 1.2283 \times 10^{10}$.

where $\tilde{x}(t)$ is the actual reported positive rate, $x(t)$ is the theoretical positive rate of the system (0.1), and $x(t) = \frac{E(t)}{N(t)}$. The fitting result of pathogen positive rate from

¹2016 to 2019 is given in Figure S3. Meanwhile, $\beta = 0.9948$ and $\gamma = 4.5029 \times 10^{-10}$
²can be obtained.

³**Author details**

⁴**References**

⁵Ren, H.R., Li, M.T., Wang, Y.M., Jin, Z., Zhang, J.: The risk factor assessment of the spread of foot-and-mouth
⁶disease in mainland china. *Journal of Theoretical Biology* **512**(1630), 110558 (2021).
⁷doi:[10.1016/j.jtbi.2020.110558](https://doi.org/10.1016/j.jtbi.2020.110558)