Supporting Information for

Bio-inspired and Computer-Supported Design of Modulated Shape Changes in Polymer Materials

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Supporting method

The equations used to calculate the deformation of the already curved beams were taken from Chen et al. [1]. Here a beam is divided into N subdivisions. Through the Curved Beam Constraint Model (CBCM) equations below it describes the bending shape of the beam and resulting forces involved. The equations used are for the beams where curvature is not constant along the whole beam, which requires each subdivision of the length of the beam, L_i, to be computed, i.e. unequal discretization. Thus only equations 21-36 are quoted below, as this describes the whole system of non-linear equations.

The y-coordinate defined by the x-coordinate defined by some function

22. $y_i = f(x_i)$

The slope, β_i

23. $\beta_i = \arctan f'(x_i)$

Curvature, K_i

24. $K_i = \frac{f''(x_i)}{\left[1+f'^2(x_i)\right]^{\frac{3}{2}}}$

The length of each element, L_i

25. $L_{xi} = x_{i+1} - x_i, L_{yi} = y_{i+1} - y_i$
26. $L_i = L_{xi} \cos\beta_i + L_{yi} \sin\beta_i$

Thickness and approximate curvature normalized to L_i

27. $t_i = \frac{T}{L_i}, \kappa_i \approx \frac{K_i + K_{i+1}}{2} L_i$

Here $P_i, F_i, M_i, \Lambda_i, \Delta_i$, and $\alpha_i$are the radial force, tangential force, moment, tangential deflection, radial deflection and the end slope. The lowercase variables $p_i, f_i, m_i, \lambda_i, \delta_i$, and α_i correspond to their non-dimensionalised forms.

28. $p_i = \frac{P_i L_i^2}{EI}, f_i = \frac{F_i L_i^2}{EI}, m_i = \frac{M_i L_i}{EI}, \lambda_i = \frac{\Lambda_i}{L_i}, \delta = \frac{\Delta_i}{L_i}, \alpha_i = \alpha_i$

Beam constraint model equations (3N equations)

29. $\begin{bmatrix} f_i \\ m_i \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} \delta \\ \alpha \end{bmatrix} + p_i \begin{bmatrix} \frac{6}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{15} \end{bmatrix} \begin{bmatrix} \delta \\ \alpha \end{bmatrix} + p_i^2 \begin{bmatrix} -1/700 & 1/1400 \\ 1/1400 & -11/6300 \end{bmatrix} \begin{bmatrix} \delta \\ \alpha \end{bmatrix} + p_i \begin{bmatrix} \kappa/2 \\ \kappa/12 \end{bmatrix}$

30. $\lambda_i = \frac{t_i^2 p_i}{12} - \frac{\kappa_i}{2}\delta_i - \frac{\kappa_i}{12}\alpha_i - \frac{1}{2}\begin{bmatrix} \delta_i & \alpha_i \end{bmatrix} \begin{bmatrix} \frac{6}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{15} \end{bmatrix} \begin{bmatrix} \delta_i \\ \alpha_i \end{bmatrix} - p_i \begin{bmatrix} \delta_i & \alpha_i \end{bmatrix} \begin{bmatrix} -\frac{1}{700} & \frac{1}{1400} \\ \frac{1}{1400} & -\frac{11}{6300} \end{bmatrix} \begin{bmatrix} \delta_i \\ \alpha_i \end{bmatrix} +$
$p_i \frac{\kappa}{360}\alpha_i + p_i \frac{\kappa_i^2}{720}$

Static equilibrium equations (3N equations)

31. $P_0 = \frac{p_1 EI}{L_1^2}, F_0 = \frac{f_1 EI}{L_1^2}, P_0 = \frac{m_N EI}{L_N}$

32. $\begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ (1+\ \lambda_i) & -(0.5\kappa_i + \delta_i) & 1 \end{bmatrix} \begin{bmatrix} f_i \\ p_i \\ m_i \end{bmatrix} = \begin{bmatrix} \frac{f_1 L_i^2}{L_1^2} \\ \frac{p_1 L_i^2}{L_1^2} \\ \frac{m_{i-1} L_i}{L_{i-1}} \end{bmatrix}$

33. $\theta_1 = 0, \theta_i = \beta_1 + \sum_{k=1}^{i-1} \alpha_k (i = 2, 3, \dots, N)$

Geometric constraint equations (3 equations)

34. $\sum_{i-1}^{N} [(1+\lambda_i) L_i \cos\theta_i - (0.5\kappa_i + \delta_i) L_i \sin\theta_i)] = X_0$
35. $\sum_{i-1}^{N} [(1+\lambda_i) L_i \sin\theta_i + (0.5\kappa_i + \delta_i) L_i \cos\theta_i)] = Y_0$
36. $\beta_{N+1} + \sum_{i=1}^{N} \alpha_i = \theta_0$

Through these 6N + 3 equations and given three of the six parameters p_0, f_0, m_0, x_0, y_0, and θ_0, one could solve for the other three parameters. For the modelling in the manuscript, x_0, y_0 and θ_0 were used as inputs to solve for the other variables p_0, f_0, m_0. The values for the axial force given in the manuscript is F_0.

1. G. Chen, F. Ma, G. Hao and W. Zhu: Modeling Large Deflections of Initially Curved Beams in Compliant Mechanisms Using Chained Beam Constraint Model. *Journal of Mechanisms and Robotics* **11**, 011002 (2019).

Supporting figures

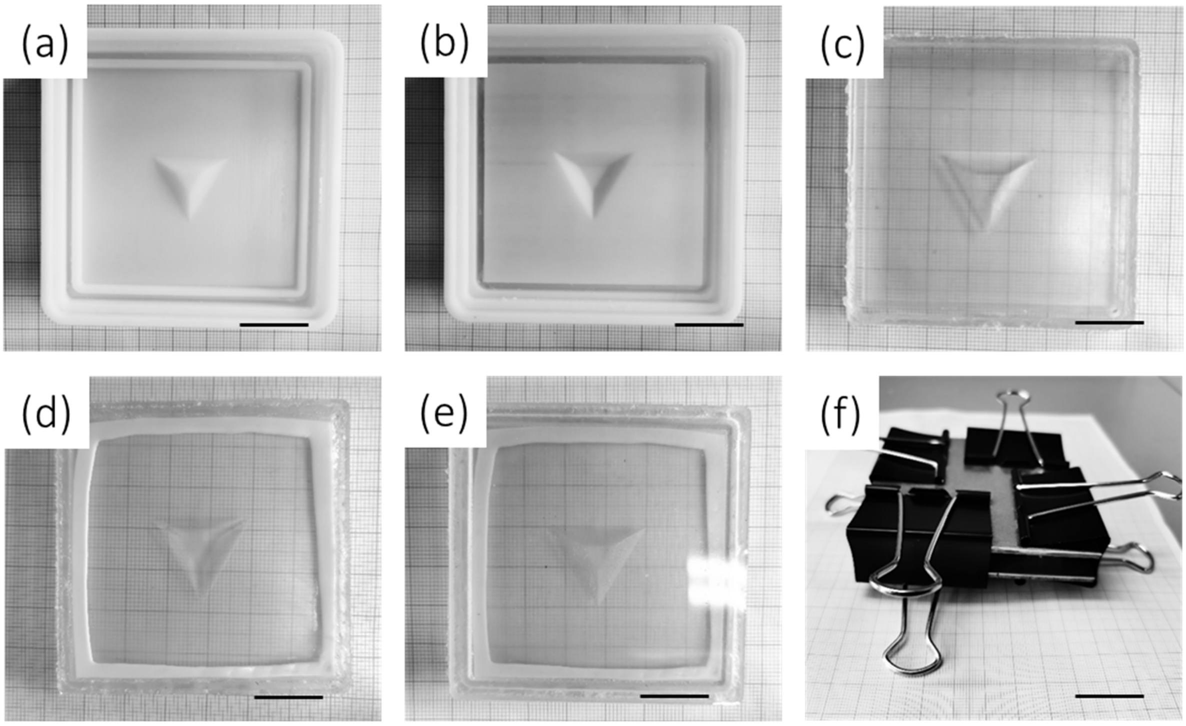

Figure S1 - Photos of different molds and casting device. (a, b) 3D printed counter parts for casting PDMS molds. (c, d) PDMS molds replicated from 3D printed counter parts. A Teflon spacer of 0.5 mm thickness was attached to one of the PDMS molds (d), in order to create the desired thickness of the cPEVA film. (e) PDMS molds sandwiched with PEVA blended film. (f) Casting device: sandwiched PDMS molds compressed between two metal clamps and fixed with four clamps. The scale bar in each picture is 20 mm.

Supporting videos

Supplementary video S1 – A Video of the digital model of the 30 degree truncated tetrahedron. (Left) The undeformed beam (black) and deformed beam (blue). (Right) The force-displacement curve with respect to the deformed beam. The parameters used for the digital modelling: Number of elements, N = 15; Deformation step, $\Delta Y = 0.05$ mm; Young's modulus = 40 MPa; Width = 7.215 mm; Thickness = 0.5 mm

Supplementary video S2 – A Video of the digital model of the 45 degree truncated tetrahedron. (Left) The undeformed beam (black) and deformed beam (blue). (Right) The force-displacement curve with respect to the deformed beam. The parameters used for the digital modelling: Number of elements, N = 15; Deformation step, $\Delta Y = 0.05$ mm; Young's modulus = 40 MPa; Width = 7.215 mm; Thickness = 0.5 mm

Supplementary video S3 – A video showing the temperature-activated snapping movement by pushing a programmed sample from shape A to shape B and resetting the structure by heating up the temperature to 80 °C