

Supplementary Material for Factors for an Abrupt Increase in Track Forecast Error of Typhoon Hagibis (2019)

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This supplementary material provides the brief review of the Ensemble Singular Vector Sensitivity Analysis (EnSVSA) following Enomoto et al. (2015) and the detailed procedure of the analysis, especially on how to obtain Fig. 4 in the main text.

1. Formulation of the EnSVSA

In this section, n is defined as the degrees of freedom and m as ensemble size. In the sensitivity analysis, it is assumed that the time evolution of the initial perturbations $\mathbf{y} \in \mathbf{R}^n$ is linear:

$$\mathbf{z} = \mathbf{M}\mathbf{y} \quad (1)$$

where $\mathbf{z} \in \mathbf{R}^n$ is the perturbations at the verification time t and $\mathbf{M} \in \mathbf{R}^{n \times n}$ is the tangent linear model. The goal of the analysis is to find the initial perturbations which maximize the perturbation growth at the verification time measured by a specific norm $\|\cdot\|_t$. This norm is expressed by the inner product:

$$\begin{aligned} \|y\|_0 &= \sqrt{\mathbf{y}^T \mathbf{G}_0 \mathbf{y}} \\ \|z\|_t &= \sqrt{\mathbf{z}^T \mathbf{G}_t \mathbf{z}} \end{aligned} \quad (2)$$

where $\mathbf{G}_0$ and $\mathbf{G}_t$ are arbitrary non-negative symmetric matrices that denote the weight in their respective definitions of norm.

With the ensemble formulation, the perturbations at the verification time t of each ensemble member i is expressed as follows:

$$\mathbf{z}_i = M(\mathbf{x} + \mathbf{y}_i) - M(\mathbf{x}) \quad (3)$$

where M is the nonlinear model and $\mathbf{x}$ is the reference state, for which we take the ensemble mean. Assume that an initial perturbations can be expressed in a linear combination of the ensemble initial perturbations, then under the assumption of linear time evolution, the corresponding perturbations at the verification time t can be expressed in a linear combination with the same coefficients:

$$\mathbf{y} = p_1 \mathbf{y}_1 + p_2 \mathbf{y}_2 + \cdots + p_m \mathbf{y}_m \quad (4)$$

$$\mathbf{z} = p_1 \mathbf{z}_1 + p_2 \mathbf{z}_2 + \cdots + p_m \mathbf{z}_m \quad (5)$$

In the matrix-vector notation:

$$\mathbf{y} = \mathbf{Y}\mathbf{p} \quad (6)$$

$$\mathbf{z} = \mathbf{Z}\mathbf{p} \quad (7)$$

where,

$$\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_m) \quad (8)$$

$$\mathbf{Z} = (\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_m) \quad (9)$$

$$\mathbf{p}^T = (p_1, p_2, \cdots, p_m) \quad (10)$$

In the EnSVSA, we find the coefficients (10) that maximize $\mathbf{p}^T \mathbf{Z}^T \mathbf{G}_t \mathbf{Z} \mathbf{p}$ with the constraint of $\mathbf{p}^T \mathbf{Y}^T \mathbf{G}_0 \mathbf{Y} \mathbf{p}$. This is reduced to the eigenvalue problem:

$$(\mathbf{Y}^T \mathbf{G}_0 \mathbf{Y})^{-1} \mathbf{Z}^T \mathbf{G}_t \mathbf{Z} \mathbf{p} = \Lambda \mathbf{p} \quad (11)$$

where Λ is the diagonal matrix whose diagonal elements are eigenvalues. For detailed derivation, see Enomoto et al. (2015) Section 2b.

If the initial perturbations $\mathbf{y}_i$ are normalized and orthogonal (we assume it is true), then the problem (11) is solved by the singular decomposition of matrix $\mathbf{Z}$

$$\mathbf{Z} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (12)$$

and the coefficients $\mathbf{p}$ is equivalent to the right singular vectors $\mathbf{V}$.

2. Procedure of the EnSVSA

In this section, we provide the detailed procedure for the EnSVSA. First, the verification time, the verification region, and the norm are defined. As indicated in the main text, we conduct the analysis with the moist total energy (TE) norm (Ehrendorfer et al., 1999),

$$\begin{aligned} TE = & \frac{1}{2} \frac{1}{D} \int_D \int_0^1 \left[u'^2 + v'^2 + \frac{c_p}{T_r} T'^2 \right. \\ & \left. + RT_r \left(\frac{p'_s}{p_r} \right)^2 + \epsilon \frac{L^2}{c_p T_r} q'^2 \right] d\sigma dD \end{aligned} \quad (13)$$

where $\sigma = p/p_r$ is the vertical coordinate. D is the verification area. u , v , T , p_s , and q are the zonal wind, meridional wind, temperature, surface pressure, and specific humidity, respectively. The prime denotes the perturbation from the ensemble mean. $c_p = 1005.7 \text{ J kg}^{-1} \text{ K}^{-1}$, $R = 287.04 \text{ J kg}^{-1} \text{ K}^{-1}$ and $L = 2.5104 \times 10^6 \text{ J kg}^{-1}$ are the specific heat at constant pressure, the gas constant of

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dry air, and the latent heat for vaporization, respectively. T_r , p_r , and ϵ are the reference temperature, pressure, and a factor for the latent heat term, respectively. Here, we use $T_r = 270$ K, $p_r = 1000$ hPa, $\epsilon = 1.0$.

Second, the components of $\mathbf{Z}$ matrix are calculated as follows. Note that followed calculation includes the norm definition ($\mathbf{G}_t$).

1. Calculate perturbation of prognostic variables (u, v, T, p_s, q) in the verification region at the verification time by subtracting ensemble mean values from ensemble member values.
2. Multiply the perturbations by the areal factor ($\cos\phi$, where ϕ is the latitude) and the layer thickness factor.
3. Multiply T' , p' and q' by the square root of coefficients in (13), i.e., $\sqrt{c_p/T_r}$, $\sqrt{RT_r/p_r}$, and $L\sqrt{\epsilon/c_p T_r}$, respectively.

The calculated values for member i become the i th column values of matrix $\mathbf{Z}$.

Then, we conduct the singular value decomposition of $\mathbf{Z}$ and obtain the singular values and the left and right singular vectors. The right singular vector which corresponds to the largest singular value is the first mode linear combination coefficients (the principal component score). The first mode perturbations are obtained by projecting the matrix $\mathbf{Z}$ to the first mode coefficients. The horizontal distribution of the moist total energy is obtained by calculation of (13) with the projected perturbations except for the areal integration.

Since we assume that linear time development over the time range between the initial and the verification time, the same coefficients can be used to obtain the first mode perturbations corresponding to any valid time between the initial and the verification time. Therefore we obtain the time development of the energy by the projection of ensemble forecast data at each valid time with the same initial date.

References

- Ehrendorfer, M., R. M. Errico, and K. D. Raeder, 1999: Singular-Vector perturbation growth in a primitive equation model with moist physics. *J. Atmos. Sci.*, **56**(11), 1627–1648.
- Enomoto, T., S. Yamane, and W. Ohfuchi, 2015: Simple sensitivity analysis using ensemble forecasts. *J. Meteor. Soc. Japan*, **93**(2), 199–213.