

Optimal Metacognitive Decision Strategies in Signal Detection Theory

Brian Maniscalco*, Lucie Charles*, & Megan Peters

Supplementary Material S2

Derivations of Optimal Type 1 and Type 2 Criteria

1. Optimizing Accuracy

1.1. Optimal Type 1 Criterion for Maximizing Type 1 Accuracy

Suppose an observer with perceptual sensitivity d' wishes to place the type 1 criterion c_1 so as to maximize the proportion of correct responses in discriminating two stimulus classes, S1 and S2, where $p(S1)$ and $p(S2)$ denote the prior probabilities of S1 and S2 being presented on a given trial. The measure to be optimized, proportion correct, can be expressed as

$$p(\text{correct}_1) = p(S2) \text{HR}_1 + p(S1) (1 - \text{FAR}_1) \quad (\text{S1})$$

where HR_1 = type 1 hit rate = $p(\text{response} = \text{"S2"} \mid \text{stimulus} = \text{S2})$ and FAR_1 = type 1 false alarm rate = $p(\text{response} = \text{"S2"} \mid \text{stimulus} = \text{S1})$. For a given d' and c_1 , HR_1 and FAR_1 can be expressed in the equal variance signal detection theory model as

$$\text{HR}_1 = 1 - \Phi\left(c_1 \mid \frac{d'}{2}\right) \quad (\text{S2})$$

$$\text{FAR}_1 = 1 - \Phi\left(c_1 \mid -\frac{d'}{2}\right) \quad (\text{S3})$$

where $\Phi(x | \mu)$ is the cumulative distribution function of the normal distribution with mean μ and standard deviation $\sigma = 1$ evaluated at x . (Here we set $\sigma = 1$ for convenience, without loss of generality, as d' corresponds to the signal-to-noise ratio $\frac{\mu_{S2} - \mu_{S1}}{\sigma}$.)

To find the criterion c_1 that maximizes $p(\text{correct}_1)$, we set the derivative of $p(\text{correct}_1)$ with respect to c_1 equal to zero and solve for c_1 . The derivative is given by

$$\frac{d}{dc_1} [p(\text{correct}_1)] = -p(S2) \varphi\left(c_1 \left| \frac{d'}{2} \right.\right) + p(S1) \varphi\left(c_1 \left| -\frac{d'}{2} \right.\right) \quad (S4)$$

where $\varphi(x | \mu)$ is the probability density function of the normal distribution with mean μ and standard deviation $\sigma = 1$ evaluated at x :

$$\varphi(x | \mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}} \quad (S5)$$

Setting the derivative equal to zero and solving for c_1 gives

$$\begin{aligned} p(S2) \varphi\left(c_1 \left| \frac{d'}{2} \right.\right) &= p(S1) \varphi\left(c_1 \left| -\frac{d'}{2} \right.\right) \\ \ln \frac{p(S1)}{p(S2)} &= \ln \frac{\varphi\left(c_1 \left| \frac{d'}{2} \right.\right)}{\varphi\left(c_1 \left| -\frac{d'}{2} \right.\right)} = \frac{-\left(c_1 - \frac{d'}{2}\right)^2}{2} - \frac{-\left(c_1 + \frac{d'}{2}\right)^2}{2} = c_1 d' \\ c_1^{A*} &= \frac{\ln \frac{p(S1)}{p(S2)}}{d'} \end{aligned} \quad (S6)$$

where the A^* superscript denotes that Equation S6 defines the optimal c_1 for maximizing type 1 accuracy (see **Table 1**, main manuscript).

1.2.1. Optimal Type 2 Criterion for Maximizing Type 2 Accuracy for “S2” Responses

Next, suppose the observer wishes to place the type 2 criterion for “S2” responses, $c_{2,“S2”}$, so as to maximize the proportion of correct type 2 responses when responding “S2.” By way of analogy to the type 1 case, type 2 responses are considered to be correct when “high confidence” is reported for correct type 1 responses and when “low confidence” is reported for incorrect type 1 responses (see **Table 2**, main manuscript). More formally, the observer must set $c_{2,“S2”}$ so as to maximize $p(\text{correct}_{2,“S2”})$:

$$p(\text{correct}_{2,“S2”}) = p(\text{correct}_{1,“S2”}) \text{HR}_{2,“S2”} + p(\text{incorrect}_{1,“S2”}) (1 - \text{FAR}_{2,“S2”}) \quad (\text{S7})$$

where $\text{HR}_{2,“S2”}$ and $\text{FAR}_{2,“S2”}$ correspond to type 2 hit rate and false alarm rate for “S2” responses:

$$\text{HR}_{2,“S2”} = \frac{1 - \Phi\left(c_{2,“S2”} \mid \frac{d'}{2}\right)}{\text{HR}_1} \quad (\text{S8})$$

$$\text{FAR}_{2,“S2”} = \frac{1 - \Phi\left(c_{2,“S2”} \mid -\frac{d'}{2}\right)}{\text{FAR}_1} \quad (\text{S9})$$

and where $p(\text{correct}_{1,“S2”})$ and $p(\text{incorrect}_{1,“S2”})$ correspond to the probability of a correct or incorrect type 1 decision, given that the type 1 response was “S2”:

$$p(\text{correct}_{1,“S2”}) = p(\text{stim} = \text{S2} \mid \text{resp} = \text{“S2”}) \quad (\text{S10})$$

$$p(\text{incorrect}_{1,“S2”}) = p(\text{stim} = \text{S1} \mid \text{resp} = \text{“S2”}) \quad (\text{S11})$$

Eqs. S10 and S11 can be re-expressed as

$$p(\text{correct}_{1,“S2”}) = \frac{p(\text{stim} = \text{S2} \cap \text{resp} = \text{“S2”})}{p(\text{resp} = \text{“S2”})} = \frac{p(\text{S2}) \text{HR}_1}{p(\text{resp} = \text{“S2”})} \quad (\text{S12})$$

$$p(\text{incorrect}_{1,“S2”}) = \frac{p(\text{stim} = \text{S1} \cap \text{resp} = \text{“S2”})}{p(\text{resp} = \text{“S2”})} = \frac{p(\text{S1}) \text{FAR}_1}{p(\text{resp} = \text{“S2”})} \quad (\text{S13})$$

Thus, substituting Eqs. S8-9 and S12-13 into Eq. S7, $p(\text{correct}_{2,“S2”})$ can be written

$$\begin{aligned}
p(\text{correct}_{2,"S2"}) &= \frac{p(S2) \text{HR}_1}{p(\text{resp} = "S2")} \frac{1 - \phi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right)}{\text{HR}_1} \\
&+ \frac{p(S1) \text{FAR}_1}{p(\text{resp} = "S2")} \left(1 - \left(\frac{1 - \phi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right)}{\text{FAR}_1} \right) \right)
\end{aligned}$$

Simplifying,

$$\begin{aligned}
p(\text{correct}_{2,"S2"}) &= \frac{p(S2)}{p(\text{resp} = "S2")} \left(1 - \phi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right) \right) \\
&+ \frac{p(S1)}{p(\text{resp} = "S2")} \left(\text{FAR}_1 - 1 + \phi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right) \right)
\end{aligned}$$

Differentiating with respect to $c_{2,"S2"}$ gives

$$\frac{d}{dc_{2,"S2"}} [p(\text{correct}_{2,"S2"})] = -\frac{p(S2)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right) + \frac{p(S1)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right) \quad (\text{S14})$$

Setting the derivative equal to zero and solving for $c_{2,"S2"}$ gives

$$\frac{p(S2)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right) = \frac{p(S1)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right)$$

$$\log \frac{p(S1)}{p(S2)} = \ln \frac{\phi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right)}{\phi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right)} = c_{2,"S2"} d'$$

$$c_{2,"S2"} = \frac{\ln \frac{p(S1)}{p(S2)}}{d'} \quad (S15)$$

Enforcing the constraint that $c_{2,"S2"} \geq c_1$ yields a final solution for $c_{2,"S2"}$ as

$$c_{2,"S2"}^{A*} = \max \left(\frac{\ln \frac{p(S1)}{p(S2)}}{d'}, c_1 \right) \quad (S16)$$

where the A* superscript denotes that Equation S16 defines the optimal $c_{2,"S2"}$ for maximizing type 2 accuracy (see **Table 1**, main manuscript).

1.2.2. Optimal Type 2 Criterion for Maximizing Type 2 Accuracy for “S1” Responses

For “S1” responses, the observer must set the type 2 criterion for “S1” responses, $c_{2,"S1"}$, so as to maximize $p(\text{correct}_{2,"S1"})$:

$$p(\text{correct}_{2,"S1"}) = p(\text{correct}_{1,"S1"}) \text{HR}_{2,"S1"} + p(\text{incorrect}_{1,"S1"}) (1 - \text{FAR}_{2,"S1"}) \quad (S17)$$

where

$$\text{HR}_{2,"S1"} = \frac{\Phi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)}{(1 - \text{FAR}_1)} \quad (S18)$$

$$\text{FAR}_{2,"S1"} = \frac{\Phi \left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{(1 - \text{HR}_1)} \quad (S19)$$

$$p(\text{correct}_{1,"S1"}) = \frac{p(S1) (1 - \text{FAR}_1)}{p(\text{resp} = S1)} \quad (S20)$$

$$p(\text{incorrect}_{1,"S1"}) = \frac{p(S2) (1 - HR_1)}{p(\text{resp} = S1)} \quad (S21)$$

Substituting Eqs. S18-21 into Eq. S17, $p(\text{correct}_{2,"S2"})$ can be written

$$p(\text{correct}_{2,"S1"}) = \frac{p(S1) (1 - FAR_1)}{p(\text{resp} = "S1")} \frac{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)}{(1 - FAR_1)} + \frac{p(S2) (1 - HR_1)}{p(\text{resp} = "S1")} \left(1 - \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{(1 - HR_1)} \right)$$

which simplifies to

$$p(\text{correct}_{2,"S1"}) = \frac{p(S1)}{p(\text{resp} = "S1")} \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) + \frac{p(S2)}{p(\text{resp} = S1)} \left((1 - HR_1) - \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right) \right)$$

Differentiating with respect to $c_{2,"S1"}$ gives

$$\frac{d}{dc_{2,"S1"}} [p(\text{correct}_{2,"S1"})] = \frac{p(S1)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) - \frac{p(S2)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right) \quad (S22)$$

Setting the derivative equal to zero and solving for $c_{2,"S1"}$ gives

$$\frac{p(S1)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) = \frac{p(S2)}{p(\text{resp} = "S2")} \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)$$

$$\log \frac{p(S1)}{p(S2)} = \ln \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)} = c_{2,"S1"} d'$$

$$c_{2,"S1"} = \frac{\ln \frac{p(S1)}{p(S2)}}{d'} \quad (S23)$$

Enforcing the constraint that $c_{2,"S1"} \leq c_1$ yields a final solution for $c_{2,"S1"}$ as

$$c_{2,"S1"}^{A*} = \min\left(\frac{\ln \frac{p(S1)}{p(S2)}}{d'}, c_1\right) \quad (S24)$$

where the A* superscript denotes that Equation S24 defines the optimal $c_{2,"S1"}$ for maximizing type 2 accuracy (see **Table 1**, main manuscript).

2. Optimizing Reward

2.1. Optimal Type 1 Criterion for Maximizing Type 1 Reward Contingencies

Suppose the observer wishes to place the type 1 criterion so as to maximize the reward earned via an environmental structure that differentially rewards different type 1 outcomes. The observer must set c_1 so as to maximize expected reward from a type 1 reward contingency table:

$$E(\text{reward}_1) = p(S2)[R_{\text{hit}1}HR_1 + R_{\text{miss}1}(1 - HR_1)] + p(S1)[R_{\text{CR}1}(1 - FAR_1) + R_{\text{FA}1}FAR_1] \quad (S25)$$

where $R_{\text{hit}1}$, $R_{\text{miss}1}$, $R_{\text{CR}1}$, and $R_{\text{FA}1}$ correspond to the number of points gained or lost following type 1 hits, misses, correct rejections, and false alarms (see **Table 3**, main manuscript). To find the criterion c_1 that maximizes $E(\text{reward}_1)$, we substitute for HR_1 and FAR_1 using Eqs. S2 and S3, set the derivative of $E(\text{reward}_1)$ with respect to c_1 equal to zero, and solve for c_1 . The derivative is given by

$$\begin{aligned} \frac{d}{dc_1}[E(\text{reward}_1)] = & p(S2) \left[-R_{\text{hit}1} \varphi\left(c_1 \left| \frac{d'}{2} \right.\right) + R_{\text{miss}1} \varphi\left(c_1 \left| \frac{d'}{2} \right.\right) \right] \\ & + p(S1) \left[R_{\text{CR}1} \varphi\left(c_1 \left| -\frac{d'}{2} \right.\right) - R_{\text{FA}1} \varphi\left(c_1 \left| -\frac{d'}{2} \right.\right) \right] \end{aligned} \quad (S26)$$

where $\varphi(x | \mu)$ is the probability density function of the normal distribution with mean μ and variance $\sigma = 1$ evaluated at x , as given in Eq. S5.

Setting the derivative equal to zero and solving for c_1 gives

$$p(S1) \left[(R_{CR1} - R_{FA1}) \varphi \left(c_1 \left| -\frac{d'}{2} \right. \right) \right] = p(S2) \left[(R_{hit1} - R_{miss1}) \varphi \left(c_1 \left| \frac{d'}{2} \right. \right) \right]$$

$$\ln \frac{p(S1)(R_{CR1} - R_{FA1})}{p(S2)(R_{hit1} - R_{miss1})} = \ln \frac{\varphi \left(c_1 \left| \frac{d'}{2} \right. \right)}{\varphi \left(c_1 \left| -\frac{d'}{2} \right. \right)} = c_1 d'$$

$$c_1^{R^*} = \frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{(R_{CR1} - R_{FA1})}{(R_{hit1} - R_{miss1})}}{d'} \quad (S27)$$

where the R^* superscript denotes that Equation S27 defines the optimal c_1 for maximizing type 1 reward (see **Table 1**, main manuscript).

2.2.1. Optimal Type 2 Criterion for Maximizing Type 2 Reward Contingencies for “S2” Responses

Now suppose the observer wishes to maximize the reward earned via an environmental structure that differentially rewards different type 2 outcomes. For “S2” responses, the observer must set the type 2 criterion for “S2” responses, $c_{2,“S2”}$, so as to maximize $E(\text{reward}_{2,“S2”})$:

$$\begin{aligned} E(\text{reward}_{2,“S2”}) = & p(\text{hit}_1, \text{LC} | “S2”) R_{miss2} + p(\text{hit}_1, \text{HC} | “S2”) R_{hit2} \\ & + p(\text{FA}_1, \text{LC} | “S2”) R_{CR2} + p(\text{FA}_1, \text{HC} | “S2”) R_{FA2} \end{aligned} \quad (S28)$$

where R_{hit2} , R_{miss2} , R_{CR2} , and R_{FA2} correspond to the number of points gained or lost following type 2 hits, misses, correct rejections, and false alarms (see **Table 4**, main manuscript), and where e.g. $p(\text{hit}_1, \text{LC} | “S2”)$ is the probability of a low confidence hit given that the observer responded “S2,” or more formally,

$$\begin{aligned} p(\text{hit}_1, \text{LC} | “S2”) &= p(\text{stim} = S2, \text{conf} = \text{low} | \text{resp} = “S2”) \\ &= \frac{p(\text{stim} = S2 \cap \text{resp} = “S2” \cap \text{conf} = \text{low})}{p(\text{resp} = “S2”)} \end{aligned}$$

This expression can be re-written as

$$p(\text{hit}_1, \text{LC} | "S2") = \frac{p(S2) \text{HR}_1 (1 - \text{HR}_{2,"S2"})}{p(\text{resp} = "S2")} \quad (\text{S29})$$

and similarly,

$$p(\text{hit}_1, \text{HC} | "S2") = \frac{p(S2) \text{HR}_1 \text{HR}_{2,"S2"}}{p(\text{resp} = "S2")} \quad (\text{S30})$$

$$p(\text{FA}_1, \text{LC} | "S2") = \frac{p(S1) \text{FAR}_1 (1 - \text{FAR}_{2,"S2"})}{p(\text{resp} = "S2")} \quad (\text{S31})$$

$$p(\text{FA}_1, \text{HC} | "S2") = \frac{p(S1) \text{FAR}_1 \text{FAR}_{2,"S2"}}{p(\text{resp} = "S2")} \quad (\text{S32})$$

Rearranging the equation for $E(\text{reward}_{2,"S2"})$ (Eq. S28) and using Eqs. S8-9 to substitute for the $\text{HR}_{2,"S2"}$ and $\text{FAR}_{2,"S2"}$ terms gives

$$\begin{aligned} E(\text{reward}_{2,"S2"}) &= \frac{p(S2) \text{HR}_1 (1 - \text{HR}_{2,"S2"})}{p(\text{resp} = "S2")} R_{\text{miss2}} + \frac{p(S2) \text{HR}_1 \text{HR}_{2,"S2"}}{p(\text{resp} = "S2")} R_{\text{hit2}} \\ &+ \frac{p(S1) \text{FAR}_1 (1 - \text{FAR}_{2,"S2"})}{p(\text{resp} = "S2")} R_{\text{CR2}} + \frac{p(S1) \text{FAR}_1 \text{FAR}_{2,"S2"}}{p(\text{resp} = "S2")} R_{\text{FA2}} \end{aligned}$$

$$\begin{aligned} E(\text{reward}_{2,"S2"}) &= \frac{p(S2) \text{HR}_1}{p(\text{resp} = "S2")} [(1 - \text{HR}_{2,"S2"}) R_{\text{miss2}} + \text{HR}_{2,"S2"} R_{\text{hit2}}] \\ &+ \frac{p(S1) \text{FAR}_1}{p(\text{resp} = "S2")} [(1 - \text{FAR}_{2,"S2"}) R_{\text{CR2}} + \text{FAR}_{2,"S2"} R_{\text{FA2}}] \end{aligned}$$

$$E(\text{reward}_{2, "S2"})$$

$$= \frac{p(S2) \text{HR}_1}{p(\text{resp} = "S2")} \left[\left(1 - \frac{1 - \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right)}{\text{HR}_1} \right) R_{\text{miss}2} + \frac{1 - \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right)}{\text{HR}_1} R_{\text{hit}2} \right] \\ + \frac{p(S1) \text{FAR}_1}{p(\text{resp} = "S2")} \left[\left(1 - \frac{1 - \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right)}{\text{FAR}_1} \right) R_{\text{CR}2} + \frac{1 - \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right)}{\text{FAR}_1} R_{\text{FA}2} \right]$$

$$E(\text{reward}_{2, "S2"})$$

$$= \frac{p(S2) \text{HR}_1}{p(\text{resp} = "S2")} \left[\frac{\text{HR}_1 - 1 + \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right)}{\text{HR}_1} R_{\text{miss}2} + \frac{1 - \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right)}{\text{HR}_1} R_{\text{hit}2} \right] \\ + \frac{p(S1) \text{FAR}_1}{p(\text{resp} = "S2")} \left[\frac{\text{FAR}_1 - 1 + \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right)}{\text{FAR}_1} R_{\text{CR}2} + \frac{1 - \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right)}{\text{FAR}_1} R_{\text{FA}2} \right]$$

$$E(\text{reward}_{2, "S2"})$$

$$= \frac{p(S2)}{p(\text{resp} = "S2")} \left[\left(\text{HR}_1 - 1 + \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right) \right) R_{\text{miss}2} + \left(1 - \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right) \right) R_{\text{hit}2} \right] \\ + \frac{p(S1)}{p(\text{resp} = "S2")} \left[\left(\text{FAR}_1 - 1 + \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right) \right) R_{\text{CR}2} \right. \\ \left. + \left(1 - \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right) \right) R_{\text{FA}2} \right]$$

Differentiating with respect to $c_{2, "S2"}$ gives

$$\frac{d}{dc_{2, "S2"}} [E(\text{reward}_{2, "S2"})] = \frac{p(S2)}{p(\text{resp} = "S2")} \left[\phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right) R_{\text{miss}2} - \phi\left(c_{2, "S2"} \left| \frac{d'}{2} \right.\right) R_{\text{hit}2} \right] \\ + \frac{p(S1)}{p(\text{resp} = "S2")} \left[\phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right) R_{\text{CR}2} - \phi\left(c_{2, "S2"} \left| -\frac{d'}{2} \right.\right) R_{\text{FA}2} \right] \quad (\text{S33})$$

Setting the derivative equal to zero and solving for $c_{2,"S2"}$ gives

$$\begin{aligned}
 -p(S2) \varphi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right) (R_{\text{miss}2} - R_{\text{hit}2}) &= p(S1) \varphi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right) (R_{\text{CR}2} - R_{\text{FA}2}) \\
 \ln \frac{p(S1)(R_{\text{CR}2} - R_{\text{FA}2})}{p(S2)(R_{\text{hit}2} - R_{\text{miss}2})} &= \ln \frac{\varphi\left(c_{2,"S2"} \left| \frac{d'}{2} \right.\right)}{\varphi\left(c_{2,"S2"} \left| -\frac{d'}{2} \right.\right)} = c_{2,"S2"} d' \\
 c_{2,"S2"} &= \frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{(R_{\text{CR}2} - R_{\text{FA}2})}{(R_{\text{hit}2} - R_{\text{miss}2})}}{d'} \tag{S34}
 \end{aligned}$$

Enforcing the constraint that $c_{2,"S2"} \geq c_1$ yields a final solution for $c_{2,"S2"}$ as

$$c_{2,"S2"}^{R*} = \max\left(\frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{(R_{\text{CR}2} - R_{\text{FA}2})}{(R_{\text{hit}2} - R_{\text{miss}2})}}{d'}, c_1\right) \tag{S35}$$

where the R^* superscript denotes that Equation S35 defines the optimal $c_{2,"S2"}$ for maximizing type 2 reward (see **Table 1**, main manuscript).

2.2.2. Optimal type 2 Criterion for Maximizing Type 2 Reward Contingencies for “S1” Responses

For “S1” responses, the observer must set the type 2 criterion for “S1” responses, $c_{2,"S1"}$, so as to maximize $E(\text{reward}_{2,"S1"})$:

$$\begin{aligned}
 E(\text{reward}_{2,"S1"}) &= p(\text{CR}_1, \text{LC} | "S1") R_{\text{miss}2} + p(\text{CR}_1, \text{HC} | "S1") R_{\text{hit}2} \\
 &\quad + p(\text{miss}_1, \text{LC} | "S1") R_{\text{CR}2} + p(\text{miss}_1, \text{HC} | "S1") R_{\text{FA}2} \tag{S36}
 \end{aligned}$$

where e.g. $p(\text{CR}_1, \text{LC} | "S1")$ is the probability of a low confidence correct rejection given that the observer responded “S1.” By similar reasoning for the derivation of Eq. S29, we can write

$$p(\text{CR}_1, \text{LC} | "S1") = \frac{p(S1) (1 - \text{FAR}_1) (1 - \text{HR}_{2,"S1"})}{p(\text{resp} = "S1")} \quad (\text{S37})$$

$$p(\text{CR}_1, \text{HC} | "S1") = \frac{p(S1) (1 - \text{FAR}_1) \text{HR}_{2,"S1"}}{p(\text{resp} = "S1")} \quad (\text{S38})$$

$$p(\text{miss}_1, \text{LC} | "S1") = \frac{p(S2) (1 - \text{HR}_1) (1 - \text{FAR}_{2,"S1"})}{p(\text{resp} = "S1")} \quad (\text{S39})$$

$$p(\text{miss}_1, \text{HC} | "S1") = \frac{p(S2) (1 - \text{HR}_1) \text{FAR}_{2,"S1"}}{p(\text{resp} = "S1")} \quad (\text{S40})$$

Rearranging the equation for $E(\text{reward}_{2,"S1"})$ (Eq. S36) and using Eqs. S18-19 to substitute for the $\text{HR}_{2,"S1"}$ and $\text{FAR}_{2,"S1"}$ terms gives

$$\begin{aligned} E(\text{reward}_{2,"S1"}) &= \frac{p(S1) (1 - \text{FAR}_1) (1 - \text{HR}_{2,"S1"})}{p(\text{resp} = "S1")} R_{\text{miss2}} + \frac{p(S1) (1 - \text{FAR}_1) \text{HR}_{2,"S1"}}{p(\text{resp} = "S1")} R_{\text{hit2}} \\ &+ \frac{p(S2) (1 - \text{HR}_1) (1 - \text{FAR}_{2,"S1"})}{p(\text{resp} = "S1")} R_{\text{CR2}} + \frac{p(S2) (1 - \text{HR}_1) \text{FAR}_{2,"S1"}}{p(\text{resp} = "S1")} R_{\text{FA2}} \end{aligned}$$

$$\begin{aligned} E(\text{reward}_{2,"S1"}) &= \frac{p(S1) (1 - \text{FAR}_1)}{p(\text{resp} = "S1")} [(1 - \text{HR}_{2,"S1"}) R_{\text{miss2}} + \text{HR}_{2,"S1"} R_{\text{hit2}}] \\ &+ \frac{p(S2) (1 - \text{HR}_1)}{p(\text{resp} = "S1")} [(1 - \text{FAR}_{2,"S1"}) R_{\text{CR2}} + \text{FAR}_{2,"S1"} R_{\text{FA2}}] \end{aligned}$$

$$E(\text{reward}_{2,"S1"})$$

$$= \frac{p(S1) (1 - \text{FAR}_1)}{p(\text{resp} = "S1")} \left[\left(1 - \frac{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)}{(1 - \text{FAR}_1)} \right) R_{\text{miss2}} + \frac{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)}{(1 - \text{FAR}_1)} R_{\text{hit2}} \right] \\ + \frac{p(S2) (1 - \text{HR}_1)}{p(\text{resp} = "S1")} \left[\left(1 - \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{(1 - \text{HR}_1)} \right) R_{\text{CR2}} + \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{(1 - \text{HR}_1)} R_{\text{FA2}} \right]$$

$$E(\text{reward}_{2,"S1"})$$

$$= \frac{p(S1)}{p(\text{resp} = "S1")} \left[\left((1 - \text{FAR}_1) - \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right) \right) R_{\text{miss2}} + \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right) R_{\text{hit2}} \right] \\ + \frac{p(S2)}{p(\text{resp} = "S1")} \left[\left((1 - \text{HR}_1) - \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right) \right) R_{\text{CR2}} + \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right) R_{\text{FA2}} \right]$$

Differentiating with respect to $c_{2,"S1"}$ gives

$$\frac{d}{dc_{2,"S1"}} [E(\text{reward}_{2,"S1"})] = \frac{p(S1)}{p(\text{resp} = "S1")} \left[-\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right) R_{\text{miss2}} + \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right) R_{\text{hit2}} \right] \\ + \frac{p(S2)}{p(\text{resp} = "S1")} \left[-\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right) R_{\text{CR2}} + \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right) R_{\text{FA2}} \right] \quad (\text{S41})$$

Setting the derivative equal to zero and solving for $c_{2,"S1"}$ gives

$$p(S1) \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right) (R_{\text{hit2}} - R_{\text{miss2}}) = -p(S2) \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right) (R_{\text{FA2}} - R_{\text{CR2}})$$

$$\ln \frac{p(S1)(R_{\text{hit2}} - R_{\text{miss2}})}{p(S2)(R_{\text{CR2}} - R_{\text{FA2}})} = \ln \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)} = c_{2,"S1"} d'$$

$$c_{2,"S1"} = \frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{(R_{hit2} - R_{miss2})}{(R_{CR2} - R_{FA2})}}{d'} \quad (S42)$$

Enforcing the constraint that $c_{2,"S1"} \leq c_1$ yields a final solution for $c_{2,"S1"}$ as

$$c_{2,"S1"}^{R*} = \min \left(\frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{(R_{hit2} - R_{miss2})}{(R_{CR2} - R_{FA2})}}{d'}, c_1 \right) \quad (S43)$$

where the R^* superscript denotes that Equation S43 defines the optimal $c_{2,"S1"}$ for maximizing type 2 reward (see **Table 1**, main manuscript).

3. Optimizing Calibration

(In the below we intentionally decline to include a **Section 3.1**, as optimizing type 2 calibration does not have a natural analogue in the type 1 case.)

Suppose the observer wishes to place high confidence ratings to reflect that some benchmark of type 1 accuracy has been achieved. For instance, the observer may choose to rate “high confidence” only when the estimated probability of a correct type 1 choice, $p(\text{correct}_1)$, exceeds 0.8.

More formally, let $p(\text{correct}_1)_T$ be the threshold value of accuracy needed to report high confidence (where T denotes threshold), and let $p(\text{correct}_1|x)$ be the observer’s estimate of being correct on the current trial based upon the evidence sample x . Then the observer seeking to calibrate confidence to accuracy uses the following decision policy for rating confidence:

$$\text{confidence} = \begin{cases} \text{high,} & \text{if } p(\text{correct}_1 | x) > p(\text{correct}_1)_T \\ \text{low,} & \text{if } p(\text{correct}_1 | x) \leq p(\text{correct}_1)_T \end{cases} \quad (S44)$$

The formulae for the type 2 criteria that achieve this decision policy differ for “S1” and “S2” responses.

3.2.1. Optimal Type 2 Criterion for Calibrating “S2” Responses

On trials where the observer responds “S2,” the quantity the observer must consider in order to calibrate confidence to accuracy is $p(\text{correct}_{1, \text{“S2”}} | x)$, where $p(\text{correct}_{1, \text{“S2”}})$ denotes probability of a correct type 1 response, given that the response was “S2”. Since “S2” responses are correct whenever the stimulus is S2, it follows that $p(\text{correct}_{1, \text{“S2”}} | x) = p(\text{stim}=\text{S2} | x)$, where $p(\text{stim}=\text{S2} | x)$ is given by the relative likelihood that x was generated by S2 rather than S1:

$$p(\text{correct}_{1, \text{“S2”}} | x) = p(\text{stim} = \text{S2} | x) = \frac{p(\text{S2}) \varphi\left(x \left| \frac{d'}{2} \right.\right)}{p(\text{S1}) \varphi\left(x \left| -\frac{d'}{2} \right.\right) + p(\text{S2}) \varphi\left(x \left| \frac{d'}{2} \right.\right)} \quad (\text{S45})$$

Following Eq. S44, the observer must report high confidence for “S2” responses when $p(\text{correct}_{1, \text{“S2”}} | x) > p(\text{correct}_1)_T$. This objective can be accomplished by setting the type 2 criterion $c_{2, \text{“S2”}}$ at the location of the decision axis value x where $p(\text{correct}_{1, \text{“S2”}} | x) = p(\text{correct}_1)_T$:

$$p(\text{correct}_1)_T = \frac{p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)}{p(\text{S1}) \varphi\left(c_{2, \text{“S2”}} \left| -\frac{d'}{2} \right.\right) + p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)} \quad (\text{S46})$$

Solving for $c_{2, \text{“S2”}}$,

$$\frac{1}{p(\text{correct}_1)_T} = \frac{p(\text{S1}) \varphi\left(c_{2, \text{“S2”}} \left| -\frac{d'}{2} \right.\right) + p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)}{p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)} = \frac{p(\text{S1}) \varphi\left(c_{2, \text{“S2”}} \left| -\frac{d'}{2} \right.\right)}{p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)} + 1$$

$$\ln \frac{1 - p(\text{correct}_1)_T}{p(\text{correct}_1)_T} = \ln \frac{p(\text{S1}) \varphi\left(c_{2, \text{“S2”}} \left| -\frac{d'}{2} \right.\right)}{p(\text{S2}) \varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)} = \ln \frac{p(\text{S1})}{p(\text{S2})} + \ln \frac{\varphi\left(c_{2, \text{“S2”}} \left| -\frac{d'}{2} \right.\right)}{\varphi\left(c_{2, \text{“S2”}} \left| \frac{d'}{2} \right.\right)}$$

$$\ln \frac{p(\text{S1})}{p(\text{S2})} + \ln \frac{p(\text{correct}_1)_T}{1 - p(\text{correct}_1)_T} = c_{2, \text{“S2”}} d'$$

$$c_{2, \text{“S2”}} = \frac{\ln \frac{p(\text{S1})}{p(\text{S2})} + \ln \frac{p(\text{correct}_1)_T}{1 - p(\text{correct}_1)_T}}{d'} \quad (\text{S47})$$

Enforcing the constraint that $c_{2,"S2"} \geq c_1$ yields a final solution for $c_{2,"S2"}$ as

$$c_{2,"S2"}^{C*} = \max \left(\frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{p(\text{correct}_1)_T}{1 - p(\text{correct}_1)_T}}{d'}, c_1 \right) \quad (\text{S48})$$

where the C^* superscript denotes that Equation S48 defines the optimal $c_{2,"S2"}$ for maximizing type 2 calibration (see **Table 1**, main manuscript).

3.2.2. Optimal Type 2 Criterion for Calibrating “S1” Responses

Following similar logic as for “S2” responses, the probability of being correct given a particular evidence value x for an “S1” response is given by

$$p(\text{correct}_{1,"S1"} | x) = p(\text{stim} = S1 | x) = \frac{p(S1) \varphi \left(x \left| -\frac{d'}{2} \right. \right)}{p(S1) \varphi \left(x \left| -\frac{d'}{2} \right. \right) + p(S2) \varphi \left(x \left| \frac{d'}{2} \right. \right)} \quad (\text{S49})$$

Following Eq. S44, the observer must report high confidence for “S1” responses when $p(\text{correct}_{1,"S1"} | x) > p(\text{correct}_1)_T$. This objective can be accomplished by setting the type 2 criterion $c_{2,"S1"}$ at the location of the decision axis value x where $p(\text{correct}_{1,"S1"} | x) = p(\text{correct}_1)_T$:

$$p(\text{correct}_1)_T = \frac{p(S1) \varphi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)}{p(S1) \varphi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) + p(S2) \varphi \left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)} \quad (\text{S50})$$

Solving for $c_{2,"S1"}$,

$$\frac{1}{p(\text{correct}_1)_T} = \frac{p(S1) \varphi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) + p(S2) \varphi \left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{p(S1) \varphi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)} = \frac{p(S2) \varphi \left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{p(S1) \varphi \left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)} + 1$$

$$\begin{aligned}
\ln \frac{1 - p(\text{correct}_1)_T}{p(\text{correct}_1)_T} &= \ln \frac{p(S2) \varphi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{p(S1) \varphi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)} = \ln \frac{p(S2)}{p(S1)} + \ln \frac{\varphi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{\varphi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)} \\
\ln \frac{p(S1)}{p(S2)} + \ln \frac{1 - p(\text{correct}_1)_T}{p(\text{correct}_1)_T} &= c_{2,"S1"} d' \\
c_{2,"S1"} &= \frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{1 - p(\text{correct}_1)_T}{p(\text{correct}_1)_T}}{d'} \tag{S51}
\end{aligned}$$

Enforcing the constraint that $c_{2,"S1"} \leq c_1$ yields a final solution for $c_{2,"S1"}$ as

$$c_{2,"S1"}^{C*} = \min \left(\frac{\ln \frac{p(S1)}{p(S2)} + \ln \frac{1 - p(\text{correct}_1)_T}{p(\text{correct}_1)_T}}{d'}, c_1 \right) \tag{S52}$$

where the C* superscript denotes that Equation S52 defines the optimal $c_{2,"S1"}$ for maximizing type 2 calibration (see **Table 1**, main manuscript).

4. Optimizing HR – FAR

4.1. Optimal Type 1 Criterion for Maximizing $HR_1 - FAR_1$

Suppose the observer has the objective of maximizing the difference between type 1 hit rate and type 1 false alarm rate:

$$D_1 = HR_1 - FAR_1 \tag{S53}$$

where HR_1 and FAR_1 are defined as in Eqs. S2 and S3, respectively. Thus, D_1 can be written

$$D_1 = 1 - \phi\left(c_1 \left| \frac{d'}{2} \right.\right) - \left(1 - \phi\left(c_1 \left| -\frac{d'}{2} \right.\right)\right) = -\phi\left(c_1 \left| \frac{d'}{2} \right.\right) + \phi\left(c_1 \left| -\frac{d'}{2} \right.\right)$$

Differentiating with respect to c_1 gives

$$\frac{d}{dc_1} [D_1] = -\phi\left(c_1 \left| \frac{d'}{2} \right.\right) + \phi\left(c_1 \left| -\frac{d'}{2} \right.\right) \quad (\text{S54})$$

Setting the derivative equal to zero and solving for c_1 gives

$$\phi\left(c_1 \left| \frac{d'}{2} \right.\right) = \phi\left(c_1 \left| -\frac{d'}{2} \right.\right)$$

$$\ln \frac{\phi\left(c_1 \left| \frac{d'}{2} \right.\right)}{\phi\left(c_1 \left| -\frac{d'}{2} \right.\right)} = \ln 1$$

$$c_1 d' = 0$$

$$c_1^{\text{HF}^*} = 0 \quad (\text{S55})$$

where the HF* superscript denotes that Equation S55 defines the optimal c_1 for maximizing the difference between type 1 hit rate and type 1 false alarm rate (see **Table 1**, main manuscript).

4.2.1. Optimal Type 2 Criterion for Maximizing $\text{HR}_2 - \text{FAR}_2$ for “S2” Responses

Suppose the observer has the objective of maximizing the difference between type 2 hit rate and type 2 false alarm rate:

$$D_2 = \text{HR}_2 - \text{FAR}_2 \quad (\text{S56})$$

For “S2” responses, the observer must set the type 2 criterion for “S2” responses, $c_{2,“S2”}$, so as to maximize $D_{2,“S2”}$:

$$D_{2,“S2”} = HR_{2,“S2”} - FAR_{2,“S2”} \quad (S57)$$

where $HR_{2,“S2”}$ and $FAR_{2,“S2”}$ are defined as in Eqs. S8 and S9, respectively. Thus, $D_{2,“S2”}$ can be written

$$D_{2,“S2”} = \frac{1 - \phi\left(c_{2,“S2”} \left| \frac{d'}{2} \right. \right)}{HR_1} - \frac{1 - \phi\left(c_{2,“S2”} \left| -\frac{d'}{2} \right. \right)}{FAR_1} \quad (S58)$$

Differentiating with respect to $c_{2,“S2”}$ gives

$$\frac{d}{dc_{2,“S2”}} [D_{2,“S2”}] = -\frac{1}{HR_1} \phi\left(c_{2,“S2”} \left| \frac{d'}{2} \right. \right) + \frac{1}{FAR_1} \phi\left(c_{2,“S2”} \left| -\frac{d'}{2} \right. \right) \quad (S59)$$

Setting the derivative equal to zero and solving for $c_{2,“S2”}$ gives

$$\frac{1}{HR_1} \phi\left(c_{2,“S2”} \left| \frac{d'}{2} \right. \right) = \frac{1}{FAR_1} \phi\left(c_{2,“S2”} \left| -\frac{d'}{2} \right. \right)$$

$$\ln \frac{HR_1}{FAR_1} = \ln \frac{\phi\left(c_{2,“S2”} \left| \frac{d'}{2} \right. \right)}{\phi\left(c_{2,“S2”} \left| -\frac{d'}{2} \right. \right)} = c_{2,“S2”} d'$$

$$c_{2,“S2”} = \frac{\ln \frac{HR_1}{FAR_1}}{d'} \quad (S60)$$

Enforcing the constraint that $c_{2,“S2”} \geq c_1$ yields a final solution for $c_{2,“S2”}$ as

$$c_{2,"S2"}^{HF*} = \max\left(\frac{\ln \frac{HR_1}{FAR_1}}{d'}, c_1\right) \quad (S61)$$

where the HF* superscript denotes that Equation S61 defines the optimal $c_{2,"S2"}$ for maximizing the difference between type 2 hit rate and type 2 false alarm rate (see **Table 1**, main manuscript).

4.2.2. Optimal Type 2 Criterion for Maximizing $HR_2 - FAR_2$ for “S1” Responses

For “S1” responses, the observer must set the type 2 criterion for “S1” responses, $c_{2,"S1"}$, so as to maximize $D_{2,"S1"}$:

$$D_{2,"S1"} = HR_{2,"S1"} - FAR_{2,"S1"} \quad (S62)$$

where $HR_{2,"S1"}$ and $FAR_{2,"S1"}$ are defined as in Eqs. S18 and S19, respectively. Thus, $D_{2,"S2"}$ can be written

$$D_{2,"S1"} = \frac{\phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right)}{(1 - FAR_1)} - \frac{\phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)}{(1 - HR_1)} \quad (S63)$$

Differentiating with respect $c_{2,"S1"}$ gives

$$\frac{d}{dc_{2,"S1"}} [D_{2,"S1"}] = \frac{1}{(1 - FAR_1)} \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) - \frac{1}{(1 - HR_1)} \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right) \quad (S64)$$

Setting the derivative equal to zero and solving for $c_{2,"S1"}$ gives

$$\frac{1}{(1 - FAR_1)} \phi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right. \right) = \frac{1}{(1 - HR_1)} \phi\left(c_{2,"S1"} \left| \frac{d'}{2} \right. \right)$$

$$\ln \frac{(1 - \text{HR}_1)}{(1 - \text{FAR}_1)} = \ln \frac{\varphi\left(c_{2,"S1"} \left| \frac{d'}{2} \right.\right)}{\varphi\left(c_{2,"S1"} \left| -\frac{d'}{2} \right.\right)} = c_{2,"S1"} d'$$

$$c_{2,"S1"} = \frac{\ln \frac{1 - \text{HR}_1}{1 - \text{FAR}_1}}{d'} \quad (\text{S65})$$

Enforcing the constraint that $c_{2,"S1"} \leq c_1$ yields a final solution for $c_{2,"S1"}$ as

$$c_{2,"S1"}^{\text{HF}^*} = \min \left(\frac{\ln \frac{1 - \text{HR}_1}{1 - \text{FAR}_1}}{d'}, c_1 \right) \quad (\text{S66})$$

where the HF* superscript denotes that Equation S66 defines the optimal $c_{2,"S1"}$ for maximizing the difference between type 2 hit rate and type 2 false alarm rate (see **Table 1**, main manuscript).

5. Expressing Criterion as Likelihood Ratio β

An alternative way to express any decision criterion c is as the value of the likelihood ratio β of the probability density functions of decision axis values x for each stimulus, i.e. $\frac{f(x|S2)}{f(x|S1)}$, occurring at the location of the criterion (i.e. the likelihood ratio at $x = c$). In the equal variance signal detection theory model considered above, $f(x|S1) = \varphi\left(x \left| -\frac{d'}{2} \right.\right)$ and $f(x|S2) = \varphi\left(x \left| \frac{d'}{2} \right.\right)$, and so it follows that the likelihood ratio β occurring at c is given by

$$\beta = \frac{f(c|S2)}{f(c|S1)} = \frac{\varphi\left(c \left| \frac{d'}{2} \right.\right)}{\varphi\left(c \left| -\frac{d'}{2} \right.\right)} = e^{cd'} \quad (\text{S67})$$

Eq. S67 can be used to re-express any of the type 1 or type 2 decision criteria derived above in terms of likelihood ratio. For instance, as discussed in the main text, the type 1 criterion that maximizes type 1 accuracy (from Eq. S6 above)

$$c_1^{A*} = \frac{\ln \frac{p(S1)}{p(S2)}}{d'}$$

can be expressed in terms of the β value occurring at c_1^{A*} by plugging c_1^{A*} into Eq. S67:

$$\beta_1^{A*} = \frac{f(c_1^{A*} | S2)}{f(c_1^{A*} | S1)} = e^{c_1^{A*} d'} = \frac{p(S1)}{p(S2)}$$

Similarly, consider the type 2 criterion that maximizes type 2 reward for “S2” responses (from Eq. S35 above, using simplified notation where $Q_2 = \frac{(R_{CR2} - R_{FA2})}{(R_{hit2} - R_{miss2})}$ as in the main text):

$$c_{2, "S2"}^{R*} = \max \left(\frac{\ln \frac{p(S1)}{p(S2)} + \ln Q_2}{d'}, c_1 \right)$$

Re-expressing all elements of the left-hand side and right-hand side of the equation in terms of β and simplifying gives

$$\beta_{2, "S2"}^{R*} = \max \left(\frac{p(S1)}{p(S2)} Q_2, \beta_1 \right)$$

where $\beta_{2, "S2"}^{R*}$ is the likelihood ratio at which the type 2 criterion for “S2” responses maximizes type 2 reward, and β_1 is the likelihood ratio corresponding to the (possibly suboptimal) type 1 criterion actually used by the observer.

In a similar way, all other type 1 and type 2 decision criteria derived above can be re-expressed in terms of β .