

A Logistic model

Monitor-variable encoding. We encode both properties using the monitor-variable approach of Section 3.1. For $\varphi_{\log}^{(1)}$, a scalar monitor ϕ is initialised to 1 and updated at each step as:

$$\phi_t = \begin{cases} \phi_{t-1} \cdot \mathbf{1}[x_t < k] & \text{if } t \leq 20, \\ \phi_{t-1} \cdot \mathbf{1}[x_t > k] & \text{if } t > 20. \end{cases}$$

Note that the monitor ϕ implicitly encodes the conjunction of the two subformulae incrementally along the trajectory. In addition, we separately track the two subformulae via independent monitors, allowing us to report the individual satisfaction probabilities $\mathbb{P}(\forall t \leq 20 : x_t < k)$ and $\mathbb{P}(\forall t > 20 : x_t > k)$ alongside the joint estimate.

For $\varphi_{\log}^{(2)}$, we maintain two monitors: `all_less_10_so_far`, which tracks $H(x < k)$ and is set to 0 as soon as $x_t \geq k$, and `OH`, which tracks $O_{[0,9]}(H(x < k))$ and is set to 1 as soon as `all_less_10_so_far` = 1 at any step $t \in [21, 30]$. The satisfaction probabilities are estimated as $\mathbb{E}[\phi]$ and $\mathbb{E}[\text{OH}]$, respectively, from the marginals of the GM returned by DeGAS.

Direct-integration encoding. We also verify both properties using the direct-integration approach of Section 3.2. For $\varphi_{\log}^{(1)}$, we store the value of x_t at each time step in an array so that the joint GM covers all T time steps simultaneously. Since the two temporal subformulae range over disjoint time windows and share no temporal nesting, the joint probability factorises as:

$$\mathbb{P}(\varrho \models \varphi_{\log}^{(1)}) = \mathbb{P}(\forall t \leq 20 : x_t < k) \cdot \mathbb{P}(\forall t > 20 : x_t > k),$$

where each factor is computed from the marginal CDFs of the joint GM, without introducing any monitor variables.

For $\varphi_{\log}^{(2)}$, we exploit the fact that $H(x < k)$ up to time t is a monotone decreasing event in t : if the trajectory has stayed below k up to time t , it has also done so up to any earlier time $t' < t$. Therefore, among all $t \in [21, 30]$, the event $H(x < k)$ at $t = 21$ is the least restrictive, and the satisfaction event of the outer $O_{[0,9]}$ reduces to:

$$\mathbb{P}(\varrho \models \varphi_{\log}^{(2)}) = \mathbb{P}(\forall t \leq 21 : x_t < k),$$

which is the same rectangular query as the first factor of $\varphi_{\log}^{(1)}$, but extended by one time step to include $t = 21$, and evaluated from the marginal CDF of the joint GM.

B Double-well model

Monitor-variable encoding. We encode φ_{DW} using the monitor-variable approach of Section 3.1. Two scalar monitors `phipos` and `phineg` track whether the trajectory has ever exceeded δ or fallen below $-\delta$, respectively, and are combined

via the inclusion-exclusion formula for disjunction:

$$\phi_t = \phi_t^+ + \phi_t^- - \phi_t^+ \cdot \phi_t^-,$$

where $\phi_t^+ = \mathbf{1}[\exists j \leq t : x_j > \delta]$ and $\phi_t^- = \mathbf{1}[\exists j \leq t : x_j < -\delta]$. The satisfaction probability is estimated as $\mathbb{P}(\varrho, T \models \varphi_{\text{DW}}) \approx \mathbb{E}[\phi_T]$, read off from the marginal of the GM returned by DeGAS.

Direct-integration encoding. We also verify φ_{DW} using the direct-integration approach of Section 3.2. We compute the satisfaction probability via inclusion-exclusion:

$$\begin{aligned} \mathbb{P}(\varrho \models \varphi_{\text{DW}}) &= \mathbb{P}(O(x > \delta) \vee O(x < -\delta)) \\ &= \mathbb{P}(O(x > \delta)) + \mathbb{P}(O(x < -\delta)) - \mathbb{P}(O(x > \delta) \wedge O(x < -\delta)). \end{aligned}$$

Each term is computed by complementarity from rectangular queries on the joint GM over all $T = 30$ time steps. Specifically,

$$\begin{aligned} \mathbb{P}(O(x > \delta)) &= 1 - \mathbb{P}(\forall t \leq T : x_t \leq \delta), \\ \mathbb{P}(O(x < -\delta)) &= 1 - \mathbb{P}(\forall t \leq T : x_t \geq -\delta). \end{aligned}$$

For each mixture component k , we extract the marginal mean μ_k and covariance matrix Σ_k over all time indices and evaluate the corresponding one-sided rectangular query; the final estimates are obtained as weighted sums over components.