BACK MATTER

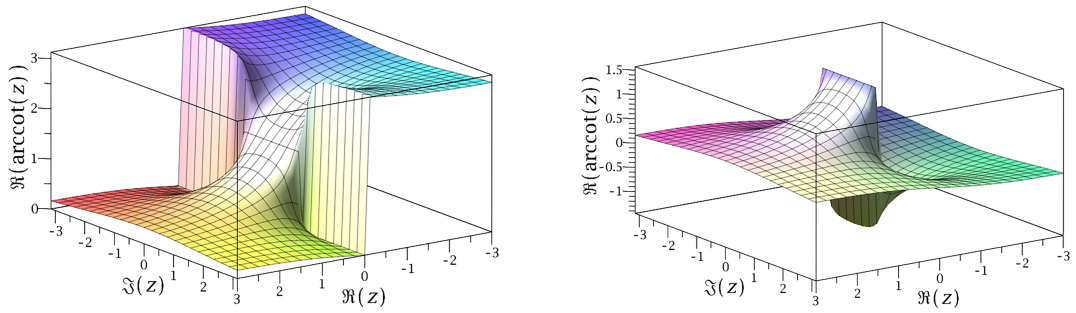
Appendix

Contents

This chapter contains additional explanations and examples for applications, problems, and evaluations that were presented throughout the thesis. In particular, Section A provides a more comprehensive explanation of branch cut issues and why those effect a correct translation between systems. Parts of this section has been published in a doctoral consortium and the AJIM journal [2, 5]. Section B provides an overview of the history of math formulae in Wikipedia articles. This Section has been presented as a poster at the JCDL conference [7]. Section C provides an example of a proof performed by the theorem prover Isabelle. This example illustrates the syntax and paradigm difference between Computer Algebra Systems (CAS) and theorem prover syntaxes. Section D summarizes the discovered issues and bugs in systems and libraries throughout the development and evaluation of LaCaSt. Section E provides result and test tables for evaluations of LaCaSt (and other converters) on Digital Library of Mathematical Functions (DLMF) formulae. Section F provides additional tables and explanations for the evaluation of LaCaSt on Wikipedia which was discussed in Chapter 5. The sections E and F are parts of the appendices for TACAS [3] and the TPAMI [4] submissions, respectively.

A Branch Cut Position Problems in CAS

Multi-valued functions map values from a domain to multiple values in a codomain and frequently appear in the complex analysis of elementary and special functions. Prominent examples are the inverse trigonometric functions, the complex logarithm, or the square root. A proper mathematical description of multi-valued functions requires the complex analysis of Riemann surfaces. Riemann surfaces are one-dimensional complex manifolds associated with a multi-valued function. One usually multiplies the complex domain into a many-layered covering space. The correct properties of multi-valued functions on the complex plane may no longer be valid by their counterpart functions on CAS, e.g., $(1/z)^w$ and $1/(z^w)$ for $z, w \in \mathbb{C}$ and $z \neq 0$. For example, consider $z, w \in \mathbb{C}$ such that $z \neq 0$. Then mathematically, $(1/z)^w$ always equals $1/(z^w)$ (when defined) for all points on the Riemann surface with fixed w. However, this should certainly not be assumed to be true in CAS, unless very specific assumptions are adopted (e.g., $w \in \mathbb{Z}, z > 0$). For all modern CAS[1], this equation is not true. Try, for instance, $w = 1/2$. Then $(1/z)^{1/2} - 1/z^{1/2} \neq 0$ on CAS, nor for w being any other rational non-integer number. The resulting ranges of multi-valued functions are referred to as branches, and the curves which separate these branches are called branch cuts. The restricted range which is associated with the range typically adopted using real numbers, is often referred to as the principal branch.



(a) The real part of arccotangent with a branch cut at $[-i\infty, -i] \cup [i, i\infty]$.

(b) The real part of arccotangent with a branch cut at $[-i, i]$.

Figure A.1: Two plots of the real part for the arccotangent function with a branch cut at $[-i\infty, -i] \cup [i, i\infty]$ in Figure a and at $[-i, i]$ in Figure b, respectively (plotted via Maple 2016). Figure a illustrates the principle branch in Maple, Axiom, or Reduce. Figure b illustrates the principle branch in Mathematica, SymPy, or the DLMF.

In order to compute multi-valued functions, CAS choose branch cuts for these functions so that they may evaluate them on their principal branches. Branch cuts may be positioned differently among CAS [10] as we have seen in the introduction in Table 1.4 of the thesis. This is certainly not an error and is usually well documented for specific CAS [9, 10, 11, 13, 17, 18]. However, there is no central database that summarizes branch cuts in different CAS or Digital Mathematical Library (DML). The DLMF as well, explains and defines their branch cuts carefully but does not carry the information within the info boxes of expressions. Our journal article [5] provides a more comprehensive explanation for the different computation results in Table 1.4 in the introduction of the thesis. If we plot the arccotangent function in Maple and Mathematica in the complex plain, the different principle branches become visible. Figure A.1 shows both principle

[1]The authors are not aware of any example of a CAS which treats multi-valued functions without adopting principal branches.

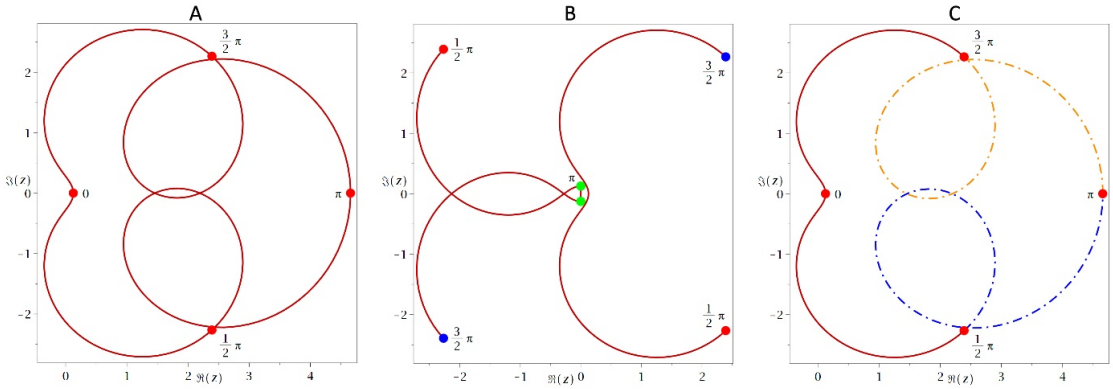


Figure A.2: Polar plot for the parabolic cylinder function with $z(\phi) = 2.5e^{i\phi}$ for $\phi \in [0, 2\pi]$. Subfigure A shows $U(0, z(\phi))$. Subfigure B shows the right-hand side of [12, (12.7.10)]. Subfigure C uses analytic continuation to allow computations on other branches.

branch versions according to the different branch cut positioning. While both versions compute the same values for $\Re(z) > 0$ (i.e., for example $\operatorname{arccot}(1) = \frac{1}{\pi}$ is valid in all systems in Table 1.4 of the thesis), the different sheets for $\Re(z) < 0$ are shifted by π. Figure a visualizes the branch cut position in Maple, Axiom, Reduce, and the 1st print of NIST's *Handbook of Mathematical Functions* [23]. Figure b, in contrast, visualizes the branch cut position in Mathematica, SymPy, or today's DLMF.

Due to complexity, it is also rather easy to lose track of branch cut positioning and evaluate expressions on incorrect values. A more complex example from [2] contains the parabolic cylinder function $U(a, z)$. $U(a, z)$ can also be represented by other functions, such as the modified Bessel function of the second kind $K_\nu(z)$. One of these relations between $U(a, z)$ and $K_\nu(z)$ is given in the DLMF [12, (12.7.E10)]

$$U(0, z) = \sqrt{\frac{z}{2\pi}} K_{\frac{1}{4}}\left(\frac{1}{4}z^2\right), \quad \text{for } z \in \mathbb{C}. \tag{A.1}$$

Figure A.2 visualizes the problem of this relation using polar plots, for fixed $r = 2.5$ in the polar representation of complex numbers $z = re^{i\phi}$. Figure A.2.A draws a path for $0 \le \phi \le 2\pi$ of $U(0, z(\phi))$. The path is continuous and closed. Figure A.2.B draws the same path for the principal values of the right-hand side of the equation as computed by a CAS. The function *jumps* over branch cuts at the angles $\phi \in \{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$. Based on the relation above, Figure A.2.B should yield to the same graph as in Figure A.2.A. The solution for this problem is analytic continuation which allows computations on other branches. Figure A.2.C shows the right-hand side of the equation with analytic continuation for $K_\nu(z)$. The orange $\pi \le \phi \le \frac{3}{2}\pi$ and blue $\frac{1}{2}\pi \le \phi \le \pi$ paths visualize the respective branches, computed with analytic continuation, while the red $0 \le \phi \le \frac{1}{2}\pi$, $\frac{3}{2}\pi \le \phi \le 2\pi$ path is the principal value. These examples illustrate the issue that a translation from one system to another might be correct on Riemann surfaces but are invalid due to the limit computation on principle branches in CAS. Strictly speaking, a direct translation of equation (A.1) to, for instance, Maple is only valid for $z \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

B History of Mathematics in Wikipedia

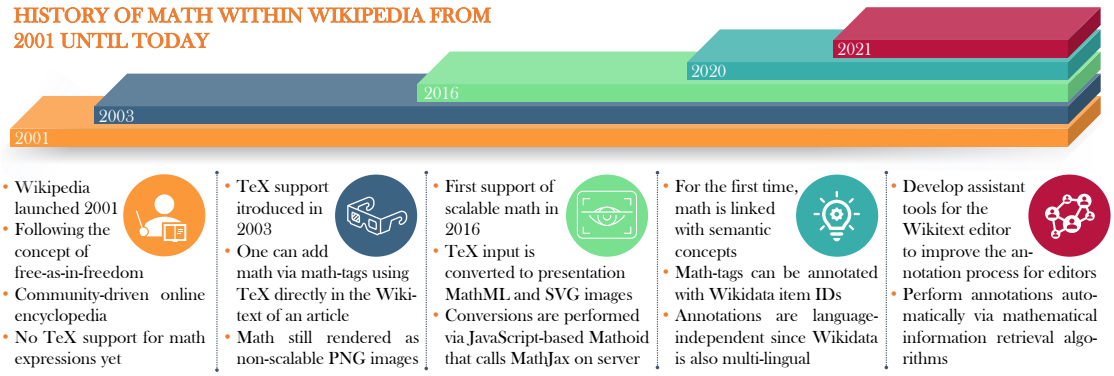


Figure B.3: History of the notation and rendering of mathematical formulae in Wikipedia articles from 2001 until today. Excerpt of the poster presented at the JCDL conference [7].

Mathematical formulae are an integral part of Wikipedia and other projects of the Wikimedia foundation[2]. The MediaWiki software is the technical backbone of many Wikimedia projects, including Wikipedia. Since 2003, wikitext – the markup language of MediaWiki – supports mathematical content [30]. For example, MediaWiki converts the wikitext code `<math>E=mc^2</math>` to the formula $E = mc^2$. While the markup for mathematical formulae has remained stable since 2003, MediaWiki's pipeline for processing wikitext to render formulae has changed significantly.

Initially, MediaWiki used LaTeX to convert math tags in wikitext to PNG images. The rendering process was slow, the images did not integrate well into the text, were inaccessible to screen readers for visually impaired users, and scaled poorly for both small and high-resolution screens. To alleviate these problems, we started developing a new JavaScript-based rendering backend called Mathoid in 2013 [30]. Mathoid invokes MathJax on the server-side to convert the LaTeX code to Mathematical Markup Language (MathML) and SVG output. The new rendering pipeline became available in production in 2016 [29].

To increase the availability of semantic information on mathematical formulae, we implemented the rendering of formulae in Wikidata – the central structured knowledge base for Wikimedia projects. The new functionality greatly facilitates the donation of semantically annotated mathematical formulae for volunteers.

The availability of semantic formula data in Wikidata has thus far enabled several research projects, e.g., on math question answering [28], semantic augmentation of mathematical content [25], and mathematical information retrieval [24]. However, the connection between formulae in Wikidata and Wikipedia had no immediate benefit for the average Wikipedia user until January 2020. Around that time, we started developing a popup system for semantic annotated (i.e., linked to Wikidata items) math formulae. This popup system is still work in progress and is, therefore, more closely discussed in Section 6.3.3 in Chapter 6 of the thesis.

[2]List of Wikimedia projects: https://meta.wikimedia.org/wiki/Wikimedia_projects

C An Example Proof by Isabelle

Listing C.1 provides an example proof via Isabelle for the irrationality of square roots of prime numbers. This example was taken from the top 100 proved theorems[3] by Isabelle[4].

Isabelle's proof that square roots of primes are irrational

```
theorem sqrt_prime_irrational:
  assumes "prime (p::nat)"
  shows "sqrt p ∉ ℂ"
proof
  from <prime p> have p: "1 < p" by (simp add: prime_nat_iff)
  assume "sqrt p ∈ ℂ"
  then obtain m n :: nat where
      n: "n ≠ 0" and sqrt_rat: "|sqrt p| = m/n"
    and "coprime m n" by (rule Rats_abs_nat_div_natE)
  have eq: "m^2 = p * n^2"
  proof -
    from n and sqrt_rat have "m = |sqrt p| * n" by simp
    then have "m^2 = (sqrt p)^2 * n^2"
      by (auto simp add: power2_eq_square)
    also have "(sqrt p)^2 = p" by simp
    also have "... * n^2 = p * n^2" by simp
    finally show ?thesis using of_nat_eq_iff by blast
  qed
  have "p dvd m ∧ p dvd n"
  proof
    from eq have "p dvd m^2" ..
    with <prime p> show "p dvd m" by (rule prime_dvd_power_nat)
    then obtain k where "m = p * k" ..
    with eq have "p * n^2 = p^2 * k^2" by (auto simp add: power2_eq_square
        ac_simps)
    with p have "n^2 = p * k^2" by (simp add: power2_eq_square)
    then have "p dvd n^2" ..
    with <prime p> show "p dvd n" by (rule prime_dvd_power_nat)
  qed
  then have "p dvd gcd m n" by simp
  with <coprime m n> have "p = 1" by simp
  with p show False by simp
qed
```

Listing C.1: A proof by Isabelle that square roots of primes are irrational numbers.

[3] http://www.cse.unsw.edu.au/~kleing/top100/ [accessed 24.08.2021]

[4] The proof and other alternatives come from Markus Wenzel and Tobias Nipkow and is available at https://isabelle.in.tum.de/dist/library/HOL/HOL-ex/Sqrt.html [accessed 24.08.2021]

D Summary of Bug Reports and Fixed Issues

Parts of this section appear also in our submission for the TACAS conference [3]. This section supports Chapter 5 of this thesis. Throughout the time of this thesis and with the help of LaCASt, numerous of minor and a couple major issues were discovered and reported to the CAS companies and the DLMF development team. Since we continuously worked together with developers from the DLMF, constant changes, tweaks, and updates has been implemented. In this section, we will only briefly cover the most prominent and severe examples that we discovered over the past years.

D.1 Digital Library of Mathematical Functions

The first error in the DLMF that we discovered with the help of LaCASt [5] was the sign error in [12, (14.5.14)]

$$\mathsf{Q}_{\nu}^{-1/2}(\cos\theta) = -\left(\frac{\pi}{2\sin\theta}\right)^{1/2}\frac{\cos\left(\left(\nu+\frac{1}{2}\right)\theta\right)}{\nu+\frac{1}{2}}. \tag{D.2}$$

This error also appeared in the original *Handbook of Special Functions* [23, p. 359] and was fixed with version 1.0.16 in September 2017. An entire group of equations [12, (10.19.10-11)] used semantic macros incorrectly and therefore yielded to wrong links and annotations visible in the attached information box next to the equation in the DLMF. In these formulae, $P_k(a)$ and $Q_k(a)$ are defined but had been incorrectly marked up as Legendre functions going all the way back to DLMF version 1.0.0. This error has been fixed due to our feedback with DLMF version 1.0.27 in June 2020. Minor discovered issues include a missing comma in the constraint [12, (10.16.7)] $2\nu \neq -1, -2, -3, \ldots$ which was also missing in the DLMF book [23, p. 228] (fixed with v. 1.0.19), unmarked [12, (2.4.2)] or erroneously marked definitions [12, (9.10.10)] and [12, (9.13.1)] (all remain unsolved), and wrong annotations of ν as *complex parameter* and x as *real* while *real value* and *positive real*, respectively, would be correct in [12, (10.24.4)] (remain unsolved). Additionally, due to LaCASt, the ambiguous semantic macro `\Wron` for Wronskians has been revised so that the variable which is differentiated against is precisely specified in 72 occasions [1].

D.2 Maple

Via LaCASt, we discovered a bug in Maple's 2016 `simplify` procedure. For the equation [12, (7.18.4)]

$$\frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(e^{z^2}\operatorname{erfc} z\right) = (-1)^n 2^n n! e^{z^2}\,\mathrm{i}^n\operatorname{erfc}(z), \quad n = 0, 1, 2, \ldots, \tag{D.3}$$

where e is the base of the natural logarithm, $\operatorname{erfc}(z)$ is the complementary error function, and $\mathrm{i}^n\operatorname{erfc}(z)$ the repeated integrals of the complementary error function, LaCASt correctly generated the following translation:

LaCASt translation of equation (D.3)

```
1  diff( exp(z^2)*erfc(z), [z$(n)] ) = (-1)^(n)*(2)^(n)*factorial(n)*exp
       (z^2)*erfc(n, z)
```

Redundant parentheses removed to improve readability.

Maple 2016 falsely returns 0 when we call the `simplify` procedure for the translated left-hand side of the equation. Maplesoft has confirmed this defect in Maple 2016 in private communications [5]. Although an updated behavior occurred in Maple 2018 and 2020, the issue still persists today[5]. Maple version 2020.2 automatically evaluates the left-hand side of equation (D.3) to the rather complex expression

$$\sum_{k=0}^{n} \frac{1}{\sqrt{\pi}} \binom{n}{k} \left(\sum_{m=0}^{k} e^{z^2} B_{k,m}\left((2)_1 z, \ldots, (2-k+m)_{k-m+1} z^{1-k+m}\right) \right) \left(-G^{1,2}_{2,3}\left(z; \begin{matrix} 0, \frac{1}{2} \\ 0, -\frac{1}{2} + \frac{n}{2} - \frac{k}{2}, \frac{n}{2} - \frac{k}{2} \end{matrix} \right) 2^{n-k} + (1-n+k)_{n-k} \sqrt{\pi} z^{n-k-1} \right) z^{1-n+k}, \tag{D.4}$$

where $G^{m,n}_{p,q}\left(z; \begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_q \end{matrix}\right)$ is the Meijer G-function [12, (16.17.1)], $B_{n,k}(x_1, \ldots, x_{n-k+1})$ the incomplete Bell polynomials [8], and $(x)_n$ the Pochhammer's symbol [12, (5.2.4)]. For small n and z, the difference of left- and right-hand side is indeed almost zero up to the machine accuracy. For large absolute values of z, however, the difference increases quickly. Figure D.4 plots the difference of left- and right-hand side of equation (D.3) for $n = 0$ and $z \in [-2, 2]$. For $|z| > 4$, the difference is already larger then 1.0.

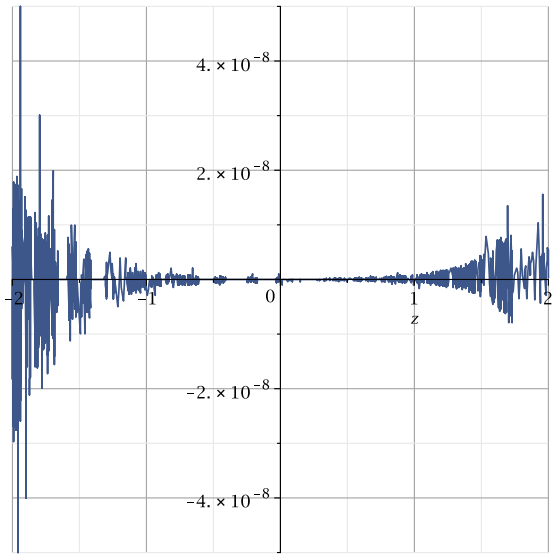


Figure D.4: The difference of the left- and right-hand side of equation (D.3) evaluated in Maple for $n = 0$ and $z \in [-2, 2]$.

D.3 Mathematica

As we pointed out in Section 5.1.4.1 of the thesis, we discovered some trouble with integrals in Mathematica and confusing behavior with rational numbers. After discussing these cases with Mathematica developers, some of them have been confirmed as bugs. Other cases, however, were the results of our testing methodology. First, we take a look at the confirmed errors. The

[5] As of Maple v.2020.2.

most crucial report was about [12, (18.17.14)]

$$\frac{x^{\alpha+\mu}L_n^{(\alpha+\mu)}(x)}{\Gamma(\alpha+\mu+n+1)} = \int_0^x \frac{y^{\alpha}L_n^{(\alpha)}(y)}{\Gamma(\alpha+n+1)} \frac{(x-y)^{\mu-1}}{\Gamma(\mu)}\,\mathrm{d}y. \tag{D.5}$$

For this equation, we calculated the difference of the left- and right-hand side as usual

$$\frac{x^{\alpha+\mu}L_n^{(\alpha+\mu)}(x)}{\Gamma(\alpha+\mu+n+1)} - \int_0^x \frac{y^{\alpha}L_n^{(\alpha)}(y)}{\Gamma(\alpha+n+1)} \frac{(x-y)^{\mu-1}}{\Gamma(\mu)}\,\mathrm{d}y \tag{D.6}$$

and computed numerical test values for this difference. Interestingly, for rational inputs (fractions) of test values, the difference was zero. For example, evaluating the difference on $x, a, \mu = \frac{3}{2}$ and $n = 2$ returns zero. However, the same test with floating point numbers, i.e., $x, a, \mu = 1.5$ and $n = 2$, result in a fatal segmentation fault[6] causing Mathematica to crash.

Segmentation Fault Example in Mathematica v. 12.1.1

```
In[1] := expr = -Integrate[((x - y)^(-1 + mu)*y^a*LaguerreL[n, a, y])
         / (Gamma[1 + n + a]*Gamma[mu]), {y, 0, x}] + (x^(a + mu)*
         LaguerreL[n, a + mu, x])/ Gamma[1 + n + a + mu];

In[2] := ReplaceAll[expr, {n -> 2, x -> 3/2, a -> 3/2, mu -> 3/2}]
Out[2] = 0

In[3] := ReplaceAll[expr, {n -> 2, x -> 1.5, a -> 1.5, mu -> 1.5}]
Segmentation fault (core dumped)
```

This issue was reported[7] and later fixed[8] with Mathematica version 12.2 (released November 2020). The developers told us this issue can be traced back to version 10.4 (released March 2016).

We further identified errors in the variable extraction procedure in Mathematica. For example, for [12, (24.4.26)]

$$E_n(0) = -E_n(1) = -\frac{2}{n+1}(2^{n+1}-1)B_{n+1}, \tag{D.7}$$

we expected to extract just n as the free variable. We reduced the issue to a minimal working example just for the most left-hand side of the equation. Here, Mathematica returned $E_n(0)$ as the free variable rather than n.

False Variable Extraction in Mathematica v. 12.0

```
In[1] := Reduce`FreeVariables[EulerE[n, 0]]
Out[1] = {EulerE[n, 0]}
```

This particular error was confirmed and has been fixed[9]. However, since the procedure `Reduce`FreeVariables` is not a publicly documented function in Mathematica, the method

[6] A *segmentation fault* is an access violation of protected memory. For example, an operating system can prevent a program `A` to change the memory of another program `B` and sends a *segmentation fault* to `A`. This signal generally causes program `A` to abnormally terminate, unless special error handling was implemented.

[7] Case ID: 4664157

[8] The fix was communicated to us via a new case ID: 4776927

[9] Case ID: 4373302

remain unstable. Especially in mathematical operators with bounded variables, such as sums, products, integrals, and limits, the procedure tend to generate inaccurate results.

In regard of the outlined issues with the `GenerateConditions` flag in integrals, most problematic cases were the result of using `ReplaceAll` to set numeric values for variables. Consider, for example [12, (10.43.8)]

$$\int_0^x e^{\pm t} t^{-\nu} I_\nu(t) \mathrm{d}t = -\frac{e^{\pm x} x^{-\nu+1}}{2\nu-1}(I_\nu(x) \mp I_{\nu-1}(x)) \mp \frac{2^{-\nu+1}}{(2\nu-1)\Gamma(\nu)}. \tag{D.8}$$

First, LACAST splits the expression in two test cases by resolving $\pm$ and $\mp$. For the first case, i.e., $\pm$ is replaced by $+$ and $\mp$ by $-$, Mathematica automatically evaluates the test expression, i.e., the difference of left- and right-hand side of the equation:

$$\left(\int_0^x e^t t^{-\nu} I_\nu(t)\,\mathrm{d}t\right) - \left(-\frac{e^x x^{-\nu+1}}{2\nu-1}(I_\nu(x) - I_{\nu-1}(x)) - \frac{2^{-\nu+1}}{(2\nu-1)\Gamma(\nu)}\right), \tag{D.9}$$

with the `GenerateConditions` set to `None` for the integral to

$$\begin{aligned}&\frac{e^x x^{-\nu}\left(-x+2\nu\right) I_\nu(x)}{-1+2\nu} + \frac{e^x x^{1-\nu}\left(-I_{-1+\nu}(x) + I_\nu(x)\right)}{-1+2\nu} + \\ &\frac{e^x x^{1-\nu} I_{1+\nu}(x)}{-1+2\nu} + \frac{2^{1-\nu}}{(-1+2\nu)\Gamma(z)} + \frac{2^\nu \sqrt{\pi}\nu \sec(\pi\nu)}{\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(1+2\nu)}.\end{aligned} \tag{D.10}$$

This happens because CAS automatically perform some computations on their inputs unless we prevent it (e.g., via `Hold`). However, evaluating this expression on $x, \nu = 1.5$ returns 0.398942 rather than the expected zero. For $x, \nu = \frac{3}{2}$, the return value is infinity (i.e., indeterminate). The issue was acknowledged by the developers, who explained that the last term causes the behaviour, because

$$\frac{2^\nu \sqrt{\pi}\nu \sec(\pi\nu)}{\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(1+2\nu)} \tag{D.11}$$

is `Infinity/Infinity` for $\nu = 3/2$. A workaround to this issue is to use `Limit` rather than `ReplaceAll` to evaluate the expression on specific values.

Limit Workaround for equation D.9

```
In[2] := N[ ReplaceAll[ expr, {Rule[x, 3/2], Rule[nu, 3/2]} ]]
Out[2] = Indeterminate

In[3] := Limit[ expr, {Rule[x, 3/2], Rule[nu, 3/2]} ]
Out[3] = 0
```

`expr` is the input of equation (D.9).

To the best of our knowledge, this issue still persists[10]. If this behavior is intended (or even desired) is up for debate. Yet, it is another characteristic of CAS to keep track of. The same workaround was suggested for [12, (11.5.2)] and [12, (11.5.8-10)]. In case of [12, (10.9.1)]

$$J_0(z) = \frac{1}{\pi}\int_0^\pi \cos(z\cos\theta)\,\mathrm{d}\theta, \tag{D.12}$$

[10] As of 2021-09-09

the right-hand side of the equation was evaluated to `BesselJ[0,z]` by Mathematica for `GenerateConditions -> False`. Which is correct. However, without this flag (or set to `None`), Mathematica returns `BesselJ[0, Abs[z]]` if $z \in \mathbb{R}$.

Conditional Flag Influence in Mathematica

```
In[1] := Divide[1,Pi]*Integrate[Cos[z*Sin[θ]], {θ, 0, Pi},
           GenerateConditions -> False]
Out[1] = BesselJ[0, z]

In[2] := Divide[1,Pi]*Integrate[Cos[z*Sin[θ]], {θ, 0, Pi}]
Out[2] = BesselJ[0, Abs[z]] if z ∈ ℝ
```

While confusing at the first glance, the output is not particularly wrong. Since $J_0(z)$ is even in the second argument and along the Real line, the absolute value is simply redundant.

In case of [12, (8.4.4)]

$$\Gamma(0,z) = \int_z^\infty t^{-1}e^{-t}\mathrm{d}t = E_1(z), \tag{D.13}$$

we have not received any feedback from the developers. For the difference between left- and right-hand side of the first equation

$$\Gamma(0,z) - \int_z^\infty t^{-1}e^{-t}\mathrm{d}t, \tag{D.14}$$

Mathematica conditionally returns 0 if $\Re(z) > 0$ and $\Im(z) = 0$. We would expect 0 without conditions. Setting the problematic `GenerateConditions` to `False` returns $-\ln(z)$.

Second Example of Conditional Flag Influence in Mathematica

```
In[1] := Gamma[0,z] - Integrate[(t)^(-1)*Exp[-t], {t,z,Infinity}]
Out[1] = 0 if Re[z] > 0 && Im[z] == 0

In[2] := Gamma[0,z] - Integrate[(t)^(-1)*Exp[-t], {t,z,Infinity},
           GenerateConditions -> False]
Out[2] = -Log[z]
```

We noticed that another initial computation hook on the input could cause the issue. For example, if we prevent instant evaluations on the input via `Hold` and evaluate the expression on $z = \mathrm{i}$, Mathematica returns $0. + 0.\mathrm{i}$. Without `Hold`, an evaluation on the same value returns `undefined` because replacing z in $\Im(z) = 0$ with $z = \mathrm{i}$ obviously results in an undefined state. Why `Hold` prevents Mathematica from ou

Hold Inputs in Mathematica

```
In[1] := expr = Gamma[0,z] - Integrate[(t)^(-1)*Exp[-t], {
          t,z,Infinity}]
Out[1] = 0 if Re[z] > 0 && Im[z] == 0

In[2] := N[ReplaceAll[expr, {z -> I}]]
Out[2] = Undefined

In[3] := expr = Hold[Gamma[0,z] - Integrate[(t)^(-1)*Exp[-t], {
          t,z,Infinity}]]
Out[3] = Hold[Gamma[0,z] - Integrate[(t)^(-1)*Exp[-t], {t,z,Infinity}
          ]]

In[4] := N[ReleaseHold[ReplaceAll[expr, {z -> I}]]]
Out[4] = 0. + 0. i
```

All cases were handled by the Mathematica team with the case ID 4664157.

D.4 Wikipedia

Since Wikipedia is a community-driven free encyclopedia, errors do not need to be reported but can simply be fixed by anyone at any time. Consequently, new errors may be introduced as fast as old issues are fixed. While we are actively working on a real-time scan of math edits in Wikipdia (see future work Section 6.3 of the thesis), we were only able to identify one malicious edit which was fixed three days later. In the *Bessel function* article, the left-hand side of the equation

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos\left(n\tau - x\sin\tau\right) d\tau \tag{D.15}$$

was changed[11] to $J_Z WE(x)$ on November 29th, 2020, and reverted three days later on December 2nd, 2020. So far, we have not detected other serious content errors in the Wikipedia articles we evaluated. However, several minor issues regarding the notation style has been detected. Especially a preceding index, such as in ${}_1F_2$, seem to cause confusions regularly among the Wikipedia editor's community. As we describe in the Appendix F.3, the index is often wrongly attached to preceding symbols, such as $=_3$ (entry 84), $*_2$ (entry 83), and $*_4$ (entry 75). In entry 85, an editor even cut the entire formula into two parts by adding `</math><math>` in between to be able to start the expression with an empty base subscript `</math><math>_{3}`. More issues like these are discussed in F.3.

[11] https://en.wikipedia.org/w/index.php?diff=991994767&oldid=991251002&title=Bessel_function&type=revision [accessed 2021-09-09]

E DLMF Evaluations

This section contains additional tables for evaluations on the DLMF dataset. The first section shows the 100 samples randomly selected from the DLMF for the evaluation of different LaTeX to CAS translators. The second section contains tables for the symbolic and numeric evaluation of LaCaST on the entire DLMF dataset.

E.1 DLMF Test Samples

This section contains a table with the 100 randomly selected sample equations from the DLMF. Throughout this thesis, we used these samples multiple times to compare different LaTeX to CAS translators. The Table contains the following information: the linked DLMF label of the formula (DLMF), the rendered LaTeX expression (LaTeX), and the results of the translations by Mathematica's LaTeX import[12] function [32] (MM), our machine translation approach (ML) presented in Section 4.1 in Chapter 4 of the thesis, SymPy's LaTeX import[13] function [22] (SP), and LaCaST (LCT).

LaCaST was evaluated twice. Once on the generic LaTeX expression of the formula ($\texttt{LCT}_\text{G}$) and once on the original semantic LaTeX expression ($\texttt{LCT}_\text{S}$). Hence, the second run ($\texttt{LCT}_\text{S}$) is not comparable to the previous results but can serve as a theoretical upper limit. On a manually annotated semantic dataset, the rule-based translator LaCaST achieved 51 correct translations out of 100 which underlines the difficulty of the task. Additionally, LaCaST was developed with a specific focus on the DLMF, which explains the good performance of the first run ($\texttt{LCT}_\text{G}$). No contextual information have been provided to LaCaST or any other translator in any way. The input was the pure LaTeX expression from the DLMF.

[12] `ToExpression` with `TeXForm`, https://reference.wolfram.com/language/tutorial/ImportingAndExporting.html#32618 Mathematica version 12.1.1

[13] `parse_latex`, https://docs.sympy.org/latest/modules/parsing.html SymPy version 1.8

Table E.1: Translation comparison of LaTeX to CAS converters on 100 randomly selected DLMF formulae.

DLMF	LaTeX	MM	ML	SP	LCT_G	LCT_S
1.5.E11	$\frac{\partial}{\partial x} = \cos\phi\frac{\partial}{\partial r} - \frac{\sin\phi}{r}\frac{\partial}{\partial\phi}$	✘	✘	✘	✘	✘
1.6.E17	$\mathbf{e}_j \times \mathbf{e}_k = \epsilon_{jk\ell}\mathbf{e}_\ell$	✘	✘	✘	✘	✘
1.9.E51	$\left(\sum_{n=0}^{\infty} a_n z^n\right)\left(\sum_{n=0}^{\infty} b_n z^n\right) = \sum_{n=0}^{\infty} c_n z^n$	✔	✘	✔	✔	✔
1.12.E16	$b'_n = d_n b_n$	✘	✘	✘	✘	✘
2.1.E17	$0 + 0\cdot z^{-1} + 0\cdot z^{-2} + \cdots$	✘	✘	✘	✔	✔
3.2.E22	$\mathbf{B} = \begin{bmatrix} \alpha_1 & \beta_2 & & & 0 \\ \beta_2 & \alpha_2 & \beta_3 & & \\ & \ddots & \ddots & \ddots & \\ & & \beta_{n-1} & \alpha_{n-1} & \beta_n \\ 0 & & & \beta_n & \alpha_n \end{bmatrix}$	✘	✘	✘	✘	✘
3.3.E40	$x = -2.2 + 1.44011\ 1973(f - 0.09614\ 53780) + 0.08865\ 85832(f - 0.09614\ 53780)(f - 0.02670\ 63331)$	✘	✘	✘	✔	✔
3.7#Ex19	$k_2 = hf(z_n + \frac{1}{2}h, w_n + \frac{1}{2}k_1)$	✘	✘	✘	✘	✘
3.11#E23Xf	$0 = c_{p+q}b_0 + c_{p+q-1}b_1 + \cdots + c_p b_q$	✔	✘	✘	✔	✔
4.4.E5	$e^{\pm\pi\mathrm{i}} = -1$	✘	✘	✘	✔	✔
4.4.E17	$\lim_{n\to\infty}\left(1 + \frac{z}{n}\right)^n = e^z$	✘	✔	✘	✔	✔
4.21.E1	$\sin u \pm \cos u = \sqrt{2}\sin\left(u \pm \frac{1}{4}\pi\right) = \pm\sqrt{2}\cos\left(u \mp \frac{1}{4}\pi\right)$	✘	✘	✘	✔	✔
4.23.E18	$\operatorname{arccot} z = \pm\frac{1}{2}\pi - \arctan z$	✘	✘	✘	✘	✔
4.33.E1	$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \cdots$	✘	✘	✔	✔	✔
5.4.E20	$x_n = -n + \frac{1}{\pi}\arctan\left(\frac{\pi}{\ln n}\right) + O\left(\frac{1}{n(\ln n)^2}\right)$	✘	✘	✘	✘	✘
5.6.E2	$\frac{1}{\Gamma(x)} + \frac{1}{\Gamma(1/x)} \leq 2$	✔	✘	✘	✘	✔
7.7.E12	$\mathrm{g}(z) + i\mathrm{f}(z) = e^{-\pi i z^2/2}\int_z^\infty e^{\pi i t^2/2}\mathrm{d}t$	✘	✘	✘	✘	✔
7.7.E16	$\int_0^\infty e^{-at}\sin\left(t^2\right)\mathrm{d}t = \sqrt{\frac{\pi}{2}}\mathrm{g}\left(\frac{a}{\sqrt{2\pi}}\right)$	✘	✘	✘	✘	✔
7.8.E2	$\frac{1}{x+\sqrt{x^2+2}} < \mathsf{M}(x) \leq \frac{1}{x+\sqrt{x^2+(4/\pi)}}$	✘	✘	✘	✘	✔
7.10#Ex1	$\frac{\mathrm{df}(z)}{\mathrm{d}z} = -\pi z\mathrm{g}(z)$	✘	✘	✘	✘	✔
7.13#Ex9	$x_n \sim \lambda + \frac{\alpha(\alpha\pi-4)}{8\pi\lambda^3} + \cdots$	✘	✘	✘	✘	✘
8.4.E3	$\gamma^*\left(\frac{1}{2}, -z^2\right) = \frac{2e^{z^2}}{z\sqrt{\pi}}F(z)$	✘	✘	✘	✘	✘
8.4.E13	$\Gamma(1-n, z) = z^{1-n}E_n(z)$	✘	✔	✘	✘	✔
8.6.E7	$\Gamma(a, z) = z^a\int_0^\infty \exp\left(at - ze^t\right)\mathrm{d}t$	✘	✔	✘	✘	✔

DLMF	LaTeX	MM	ML	SP	LCT$_G$	LCT$_S$
8.12#Ex18	$c_2(0) = \frac{25}{6048}$	✘	✘	✘	✘	✘
8.18.E4	$aF_{k+1} = (k+b-a\xi)F_k + k\xi F_{k-1}$	✘	✘	✘	✘	✘
9.5.E5	$\mathrm{Bi}\left(z\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty e^{\pi i/3}} \exp\left(\frac{1}{3}t^3 - zt\right) \mathrm{d}t + \frac{1}{2\pi}\int_{-\infty}^{\infty e^{-\pi i/3}} \exp\left(\frac{1}{3}t^3 - zt\right) \mathrm{d}t$	✘	✘	✘	✘	✔
9.13.E13	$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = \frac{1}{4}m^2 t^{m-2} w$	✘	✘	✘	✔	✔
10.18.E13	$x^2 M_\nu''\left(x\right) + xM_\nu'\left(x\right) + (x^2-\nu^2)M_\nu\left(x\right) = \frac{4}{\pi^2 M_\nu^3(x)}$	✘	✘	✘	✘	✘
10.19.E1	$J_\nu\left(z\right) \sim \frac{1}{\sqrt{2\pi\nu}}\left(\frac{ez}{2\nu}\right)^\nu$	✘	✘	✘	✘	✘
10.20.E5	$Y_\nu\left(\nu z\right) \sim -\left(\frac{4\zeta}{1-z^2}\right)^{\frac{1}{4}}\left(\frac{\mathrm{Bi}\left(\nu^{\frac{2}{3}}\zeta\right)}{\nu^{\frac{1}{3}}}\sum_{k=0}^{\infty}\frac{A_k(\zeta)}{\nu^{2k}} + \frac{\mathrm{Bi}'\left(\nu^{\frac{2}{3}}\zeta\right)}{\nu^{\frac{5}{3}}}\sum_{k=0}^{\infty}\frac{B_k(\zeta)}{\nu^{2k}}\right)$	✘	✘	✘	✘	✘
10.21.E43	$j'_{\nu,m} = \nu z(\zeta) + \frac{z(\zeta)(h(\zeta))^2 C_0(\zeta)}{2\zeta\nu} + O\left(\frac{1}{\nu}\right)$	✘	✘	✘	✘	✘
10.23.E16	$Y_0\left(z\right) = \frac{2}{\pi}\left(\ln\left(\frac{1}{2}z\right)+\gamma\right)J_0\left(z\right) - \frac{4}{\pi}\sum_{k=1}^{\infty}(-1)^k\frac{J_{2k}\left(z\right)}{k}$	✘	✘	✘	✘	✔
10.43.E19	$\int_0^\infty t^{\mu-1}K_\nu\left(t\right)\mathrm{d}t = 2^{\mu-2}\Gamma\left(\frac{1}{2}\mu-\frac{1}{2}\nu\right)\Gamma\left(\frac{1}{2}\mu+\frac{1}{2}\nu\right)$	✘	✘	✘	✘	✔
10.49.E10	$\mathrm{i}_n^{(2)}\left(z\right) = \frac{1}{2}e^z\sum_{k=0}^{n}(-1)^k\frac{a_k(n+\frac{1}{2})}{z^{k+1}} + (-1)^n\frac{1}{2}e^{-z}\sum_{k=0}^{n}\frac{a_k(n+\frac{1}{2})}{z^{k+1}}$	✘	✘	✘	✘	✔
10.58#Ex1	$a_{n,m} = j_{n+\frac{1}{2},m}$	✔	✘	✘	✘	✘
10.58#Ex2	$b_{n,m} = y_{n+\frac{1}{2},m}$	✔	✘	✘	✘	✘
10.70#Ex4	zeros of $\mathrm{kei}_\nu\, x \sim \sqrt{2}(t+f(-t))$	✘	✘	✘	✘	✘
11.11.E6	$\mathbf{E}_\nu\left(z\right) = \frac{2}{\pi\nu}\left(\sin^2\left(\frac{1}{2}\pi\nu\right) + \frac{\nu z}{\nu^2-1}\cos^2\left(\frac{1}{2}\pi\nu\right) + O\left(\frac{1}{\nu^2}\right)\right)$	✘	✘	✘	✘	✘
11.11#Ex2	$a_1(\lambda) = -\frac{\lambda}{2(1+\lambda)^4}$	✘	✘	✔	✘	✘
12.10#Ex26	$C_s(\zeta) = \chi(\zeta)A_s(\zeta) + A_s'(\zeta) + \zeta B_s(\zeta)$	✔	✘	✘	✘	✘

DLMF	LaTeX	MM	ML	SP	LCT$_G$	LCT$_S$
12.12.E2	$\int_0^\infty e^{-\frac{3}{4}t^2}t^{-a-\frac{3}{2}}U\left(a,t\right)\mathrm{d}t$ $= 2^{\frac{1}{4}+\frac{1}{2}a}\Gamma\left(-a-\frac{1}{2}\right)\cos\left(\left(\frac{1}{4}a+\frac{1}{8}\right)\pi\right)$	✘	✘	✘	✘	✔
12.14.E22	$c_{2r}+id_{2r}=\frac{\Gamma\left(2r+\frac{1}{2}+ia\right)}{\Gamma\left(\frac{1}{2}+ia\right)}$	✘	✘	✘	✘	✔
13.13.E4	$e^{y}\sum_{n=0}^{\infty}\frac{(b-a)_n(-y)^n}{(b)_n n!}M\left(a,b+n,x\right)$	✘	✘	✘	✘	✔
13.14.E5	$U\left(a,b,z\right)=e^{\frac{1}{2}z}z^{-\frac{1}{2}b}W_{\frac{1}{2}b-a,\frac{1}{2}b-\frac{1}{2}}\left(z\right)$	✘	✘	✘	✘	✔
13.15.E11	$W_{\kappa+1,\mu}\left(z\right)+(2\kappa-z)W_{\kappa,\mu}\left(z\right)$ $+(\kappa-\mu-\frac{1}{2})(\kappa+\mu-\frac{1}{2})$ $W_{\kappa-1,\mu}\left(z\right)=0$	✘	✘	✘	✘	✔
13.20.E6	$\alpha=\sqrt{2|\kappa-\mu|/\mu}$	✘	✘	✔	✔	✔
14.5.E1	$\mathsf{P}^{\mu}_{\nu}\left(0\right)=\frac{2^{\mu}\pi^{1/2}}{\Gamma\left(\frac{1}{2}\nu-\frac{1}{2}\mu+1\right)\Gamma\left(\frac{1}{2}-\frac{1}{2}\nu-\frac{1}{2}\mu\right)}$	✘	✘	✘	✘	✔
14.9.E7	$\frac{\sin\left((\nu-\mu)\pi\right)}{\Gamma\left(\nu+\mu+1\right)}\mathsf{P}^{\mu}_{\nu}\left(x\right)=\frac{\sin\left(\nu\pi\right)}{\Gamma\left(\nu-\mu+1\right)}\mathsf{P}^{-\mu}_{\nu}\left(x\right)$ $-\frac{\sin\left(\mu\pi\right)}{\Gamma\left(\nu-\mu+1\right)}\mathsf{P}^{-\mu}_{\nu}\left(-x\right)$	✘	✘	✘	✘	✔
14.17.E12	$\int_{-1}^{1}\mathsf{Q}_{\nu}\left(x\right)\mathsf{Q}_{\lambda}\left(x\right)\mathrm{d}x$ $=\frac{(\psi(\nu+1)-\psi(\lambda+1))(1+\cos(\nu\pi)\cos(\lambda\pi))}{(\lambda-\nu)(\lambda+\nu+1)}$ $+\frac{\frac{1}{2}\pi\sin\left((\lambda-\nu)\pi\right)}{(\lambda-\nu)(\lambda+\nu+1)}$	✘	✘	✘	✘	✔
14.20.E10	$\mathsf{P}_{-\frac{1}{2}+i\tau}\left(\cos\theta\right)=1+\frac{4\tau^2+1^2}{2^2}\sin^2\left(\frac{1}{2}\theta\right)$ $+\frac{(4\tau^2+1^2)(4\tau^2+3^2)}{2^2\cdot4^2}\sin^4\left(\frac{1}{2}\theta\right)$ $+\cdots$	✘	✘	✘	✘	✔
15.1.E2	$\frac{F\left(a,b;c;z\right)}{\Gamma\left(c\right)}=\mathbf{F}\left(a,b;c;z\right)=\mathbf{F}\left(\begin{matrix}a,b\\c\end{matrix};z\right)$ $={}_2\mathbf{F}_1\left(a,b;c;z\right)$	✘	✘	✘	✘	✔
15.5.E4	$\frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(z^{c-1}F\left(a,b;c;z\right)\right)$ $=(c-n)_n z^{c-n-1}F\left(a,b;c-n;z\right)$	✘	✘	✘	✘	✔
16.3.E1	$\frac{\mathrm{d}^n}{\mathrm{d}z^n}{}_pF_q\left(\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix};z\right)=\frac{(\mathbf{a})_n}{(\mathbf{b})_n}{}_pF_q\left(\begin{matrix}a_1+n,\ldots,a_p+n\\b_1+n,\ldots,b_q+n\end{matrix};z\right)$	✘	✘	✘	✘	✔
16.11.E7	$\left(\prod_{\ell=1}^{q}\Gamma\left(a_\ell\right)\Big/\prod_{\ell=1}^{q}\Gamma\left(b_\ell\right)\right){}_qF_q\left(\begin{matrix}a_1,\ldots,a_q\\b_1,\ldots,b_q\end{matrix};z\right)$ $\sim H_{q,q}(z\mathrm{e}^{\mp\pi\mathrm{i}})+E_{q,q}(z)$	✘	✘	✘	✘	✘

DLMF	LaTeX	MM	ML	SP	LCT$_G$	LCT$_S$
17.7.E16	${}_6\phi_5\left(\begin{matrix}a, qa^{\frac{1}{2}}, -qa^{\frac{1}{2}}, b, c, q^{-n}\\ a^{\frac{1}{2}}, -a^{\frac{1}{2}}, aq/b, aq/c, aq^{n+1}\end{matrix}; q, \frac{aq^{n+1}}{bc}\right) = \frac{(aq, aq/(bc); q)_n}{(aq/b, aq/c; q)_n}$	✘	✘	✘	✘	✔
18.2.E4	$\sum_{x\in X} x^{2n} w_x < \infty$	✔	✘	✘	✔	✔
18.15.E25	$\lambda_n = \begin{cases} \Gamma(n+1)\big/\Gamma\left(\frac{1}{2}n+1\right), & n \text{ even}, \\ \Gamma(n+2)\big/\left(\mu^{\frac{1}{2}}\Gamma\left(\frac{1}{2}n+\frac{3}{2}\right)\right), & n \text{ odd}, \end{cases}$	✘	✘	✘	✘	✘
18.27.E5	$P_n(x; a, b, c; q) = {}_3\phi_2\left(\begin{matrix}q^{-n}, abq^{n+1}, x\\ aq, cq\end{matrix}; q, q\right)$	✘	✘	✘	✘	✘
18.28.E14	$C_n(\cos\theta; \beta \mid q) = \frac{(\beta^2; q)_n}{(q; q)_n \beta^{\frac{1}{2}n}} {}_4\phi_3\left(\begin{matrix}q^{-n}, \beta^2 q^n, \beta^{\frac{1}{2}} e^{\mathrm{i}\theta}, \beta^{\frac{1}{2}} e^{-\mathrm{i}\theta}\\ \beta q^{\frac{1}{2}}, -\beta, -\beta q^{\frac{1}{2}}\end{matrix}; q, q\right)$	✘	✘	✘	✘	✘
18.30.E1	$A_n A_{n+1} C_{n+1} > 0$	✔	✘	✘	✔	✔
18.33.E1	$\frac{1}{2\pi\mathrm{i}} \int_{\lvert z\rvert=1} \phi_n(z)\overline{\phi_m(z)} w(z) \frac{\mathrm{d}z}{z} = \delta_{n,m}$	✘	✘	✘	✘	✘
19.2#Ex8	$F(m\pi \pm \phi, k) = 2mK(k) \pm F(\phi, k)$	✘	✘	✘	✘	✔
19.9.E11	$\phi \le F(\phi, k) \le \min(\phi/\Delta, \mathrm{gd}^{-1}(\phi))$	✘	✘	✘	✘	✘
19.21.E4	$R_F(0, z-1, z) = R_F(0, 1-z, 1) \mp \mathrm{i} R_F(0, z, 1)$	✘	✘	✘	✘	✔
19.25.E16	$\Pi\left(\phi,\alpha^2,k\right) = -\frac{1}{3}\omega^2 R_J\left(c-1, c-k^2, c, c-\omega^2\right) + \sqrt{\frac{(c-1)(c-k^2)}{(\alpha^2-1)(1-\omega^2)}} R_C\left(c(\alpha^2-1)(1-\omega^2), (\alpha^2-c)(c-\omega^2)\right)$	✘	✘	✘	✘	✘
19.30.E2	$s = a\int_0^\phi \sqrt{1 - k^2 \sin^2\theta}\,\mathrm{d}\theta$	✘	✘	✘	✘	✔
21.4.E1	$\boldsymbol{\Omega} = \begin{bmatrix} 1.690983006 + 0.951056516i & 1.5 + 0.363271264i \\ 1.5 + 0.363271264i & 1.309016994 + 0.951056516i \end{bmatrix}$	✘	✘	✘	✘	✘
22.10.E8	$\mathrm{cn}(z, k) = \mathrm{sech}\, z + \frac{k'^2}{4} (z - \sinh z \cosh z) \tanh z \,\mathrm{sech}\, z + O\left(k'^4\right)$	✘	✘	✘	✘	✘
23.3.E12	$\wp''(z) = 6\wp^2(z) - \frac{1}{2} g_2$	✘	✘	✘	✘	✘
23.6#Ex2	$\tau = \omega_3/\omega_1$	✔	✔	✔	✔	✔
23.9#Ex1	$c_2 = \frac{1}{20} g_2$	✘	✘	✘	✘	✘
24.10.E3	$\frac{B_m}{m} \equiv \frac{B_n}{n} \pmod p$	✘	✘	✘	✘	✘
24.14.E2	$\sum_{k=0}^n \binom{n}{k} B_k B_{n-k} = (1-n)B_n - nB_{n-1}$	✘	✘	✘	✘	✔
25.2.E10	$\zeta(s) = \frac{1}{s-1} + \frac{1}{2} + \sum_{k=1}^n \binom{s+2k-2}{2k-1} \frac{B_{2k}}{2k} - \binom{s+2n}{2n+1} \int_1^\infty \frac{\widetilde{B}_{2n+1}(x)}{x^{s+2n+1}} \mathrm{d}x$	✘	✘	✘	✘	✘

DLMF	LaTeX	MM	ML	SP	LCT$_G$	LCT$_S$
25.5.E3	$\zeta\left(s\right)=\frac{1}{(1-2^{1-s})\Gamma(s)}\int_0^\infty\frac{x^{s-1}}{e^x+1}\mathrm{d}x$	✘	✘	✘	✘	✔
26.11.E4	$\sum_{n=0}^\infty c_m\left(n\right)q^n=\frac{q^m}{(1-q)^m}$	✘	✘	✘	✘	✘
26.18.E1	$\lvert S\backslash(A_1\cup A_2\cup\cdots\cup A_n)\rvert=\lvert S\rvert$ $+\sum_{t=1}^n(-1)^t\sum_{1\leq j_1<j_2<\cdots<j_t\leq n}\lvert A_{j_1}\cap A_{j_2}\cap\cdots\cap A_{j_t}\rvert$	✘	✘	✘	✘	✘
27.8.E2	$\chi\left(mn\right)=\chi\left(m\right)\chi\left(n\right)$	✘	✘	✘	✘	✘
28.31#Ex2	$A=\eta-\frac{1}{8}\xi^2$	✘	✘	✘	✔	✔
29.6#Ex4	$\alpha_p=\frac{1}{2}(\nu-2p-1)(\nu+2p+2)k^2$	✘	✘	✘	✔	✔
29.10.E1	$h=\nu(\nu+1)-h'$	✘	✘	✘	✘	✘
30.5.E1	$\mathsf{Qs}_n^m\left(x,\gamma^2\right)$	✘	✘	✘	✘	✘
30.5.E3	$\mathsf{Qs}_n^m\left(x,0\right)=\mathsf{Q}_n^m\left(x\right)$	✘	✘	✘	✘	✘
30.9#Ex3	$2^{10}\beta_2=-5(q^4+26q^2+21)+384m^2(q^2+1)$	✘	✘	✔	✔	✔
31.12.E2	$\frac{\mathrm{d}^2w}{\mathrm{d}z^2}+\left(\frac{\delta}{z^2}+\frac{\gamma}{z}+1\right)\frac{\mathrm{d}w}{\mathrm{d}z}+\frac{\alpha z-q}{z^2}w=0$	✘	✘	✘	✔	✔
32.2.E25	$w(z;\alpha)=\epsilon W(\zeta)+\frac{1}{\epsilon^5}$	✘	✘	✘	✘	✘
32.2#Ex19	$z=\zeta^2$	✔	✘	✔	✔	✔
32.2#Ex21	$\beta=-\frac{1}{8}c\epsilon^{-2}$	✘	✔	✘	✔	✔
32.6.E23	$p=(\sigma'+\kappa_0\kappa_\infty z)/(2\kappa_0)$	✘	✘	✘	✘	✘
32.9.E5	$w(z)=P_{n^2+1}(\zeta)/Q_{n^2}(\zeta)$	✘	✘	✘	✔	✘
32.9.E6	$z^2(w')^2+2zww'=(C+2\alpha zw+\gamma z^2w^2)w^2$	✔	✘	✘	✘	✘
32.11.E19	$w(x)=\sigma\sqrt{\frac{1}{2}x}+\sigma\rho(2x)^{-1/4}\cos\left(\psi(x)+\theta\right)$ $+O\left(x^{-1}\right)$	✘	✘	✘	✘	✘
33.16.E13	$h(\epsilon,\ell;r)=(-1)^\ell\nu^{\ell+1}A(\epsilon,\ell)\Gamma(\nu-\ell)\zeta_\ell(-\nu,r)/\pi$	✘	✘	✘	✘	✘
34.3.E2	$\mathit{3jjj}1m{-}m0=(-1)^{j-m}\frac{2m}{(2j(2j+1)(2j+2))^{\frac{1}{2}}}$	✘	✘	✘	✘	✘
35.3.E2	$\Gamma_m\left(s_1,\ldots,s_m\right)=\int_{\mathbf{\Omega}}\mathrm{etr}\left(-\mathbf{X}\right)\lvert\mathbf{X}\rvert^{s_m-\frac{1}{2}(m+1)}$ $\prod_{j=1}^{m-1}\lvert(\mathbf{X})_j\rvert^{s_j-s_{j+1}}\mathrm{d}\mathbf{X}$	✘	✘	✘	✘	✘
35.3.E3	$\mathrm{B}_m(a,b)=\int_{\mathbf{0}<\mathbf{X}<\mathbf{I}}\lvert\mathbf{X}\rvert^{a-\frac{1}{2}(m+1)}\lvert\mathbf{I}-\mathbf{X}\rvert^{b-\frac{1}{2}(m+1)}\mathrm{d}\mathbf{X}$	✘	✘	✘	✘	✘
35.7.E5	${}_2F_1\left(\begin{smallmatrix}a,b\\c\end{smallmatrix};\mathbf{T}\right)=\frac{1}{\mathrm{B}_m(a,c-a)}\int_{\mathbf{0}<\mathbf{X}<\mathbf{I}}\lvert\mathbf{X}\rvert^{a-\frac{1}{2}(m+1)}$ $\lvert\mathbf{I}-\mathbf{X}\rvert^{c-a-\frac{1}{2}(m+1)}\lvert\mathbf{I}-\mathbf{T}\mathbf{X}\rvert^{-b}\mathrm{d}\mathbf{X}$	✘	✘	✘	✘	✘
36.4.E11	$x+iy=-z^2\exp\left(\frac{2}{3}i\pi m\right)$	✘	✘	✘	✔	✔
36.5#Ex10	$Y=\frac{1}{2}+\left((x+y)/z^2\right)$	✘	✘	✘	✔	✔
	Σ ✘	89	95	93	78	49
	Σ ✔	11	5	7	22	51

E.2 Qualitative Symbolic and Numeric Evaluations

This section is part of the appendix for our submission to the TACAS conference [3] and support our Section 5.1 in Chapter 5 of the thesis. This section contains three tables with additional information about our DLMF evaluation via LaCasT. All information in the tables are also available online[14]. This website also provides all translations for DLMF formulae, symbolic evaluation results, and numeric evaluation results including failed test values.

Table E.2, provides three examples of our evaluations on the DLMF with different degrees of complexity. The first entry [12, (1.4.8)], for example, illustrates the difficulty of translating formulae from LaTeX to CAS syntaxes even on a semantically enriched dataset like the DLMF. Often, the arguments of a function in derivative notations are omitted since they can be deduced from the variable of differentiation. For example, $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}$ the argument of the function $f(x)$ is omitted. However, in this case LaCasT is unable to correctly interpret f as a function and presumed it to be a variable. Unfortunately, not only caused this error a wrong translation but also produced a false positive evaluation because the symbolic simplification returned $0 = 0$ for

```
D[f, {x, 2}] == D[D[f, x], x].
```
(E.16)

The other two examples in Table E.2, even though more complex, illustrate the capability of LaCasT and our evaluation pipeline.

Additionally, Table E.4 and E.5 show the number of translated and evaluated expressions for each chapter of the DLMF. For overview reasons, Table E.5 only shows the results for translations to Mathematica. For Maple, see our website. For reference, Table E.3 shows the full name of each chapter and the total number of displayed formulae according to the released dataset by Youssef and Miller [33]. The actual number of functions may vary compared to Table E.4, because Youssef and Miller did not split multi-equations, and $\pm$ or $\mp$. Hence, our final dataset consists of $10,930$ formulae. Additionally, as described in our paper, we filter out non-semantic expressions, non-semantic macro definitions, ellipsis, approximations, and asymptotics. We ended up with $6,623$ test cases.

[14] https://lacast.wmflabs.org/ [accessed 2021-10-01]

Table E.2: The table shows three sample cases of our evaluation pipeline from the DLMF. The translation shows the performed translations to Mathematica. The numeric column contains the number of successfully computed test cases. The constraints column contains all applied constraints including global constraints from Figure 5.3 of the thesis.

[12, (1.4.8)]	$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)$			
Translation ✘	`D[f, {x, 2}] == D[D[f, x], x]` (A correct translation requires the argument for f, such as `D[f[x], {x, 2}] == D[D[f[x], x], x]`.)			
Symbolic	Numeric	Variables	Constraints	Test Values
✔	✔(30/30)	f ✘, x ✔	$x \in \mathbb{R}$, $\Re(x) > 0$	$x \in \left\{\frac{1}{2}, \frac{3}{2}, 2\right\}$, $f \in \left\{\pm\frac{1}{2}, \pm\frac{3}{2}, \pm 2, e^{2i\pi/3}, e^{i\pi/6}, e^{-i\pi/3}, e^{-5i\pi/6}\right\}$
[12, (11.5.2)]	$\mathbf{K}_\nu(z) = \frac{2(\frac{1}{2}z)^\nu}{\sqrt{\pi}\Gamma(\nu+\frac{1}{2})} \int_0^\infty e^{-zt}(1+t^2)^{\nu-\frac{1}{2}}\mathrm{d}t$			
Translation ✔	`StruveH[\[Nu],z]-BesselY[\[Nu],z]==Divide[2*(Divide[1,2]*z)^\[Nu], Sqrt[Pi]*Gamma[\[Nu]+Divide[1,2]]]*Integrate[Exp[-z*t]*(1+(t)^(2))^(\[Nu]-Divide[1,2]),{t,0,Infinity},GenerateConditions->None]`			
Symbolic	Numeric	Variables	Constraints	Test Values
✔	✘(10/25)	ν, z ✔	$-\pi < \mathrm{ph}(z) < \pi$, $\Re(z) > 0$, $\Re(\nu + \frac{1}{2}) > 0$, $\Re(\nu + k + 1) > 0$, $\Re(-\nu + k + 1) > 0$, $\Re(n + \nu + \frac{3}{2}) > 0$	$\nu, z \in \left\{\frac{1}{2}, \frac{3}{2}, 2, e^{i\pi/6}, e^{-i\pi/3}\right\}$
[12, (18.5.8)]	$P_n^{(\alpha,\beta)}(x) = 2^{-n}\sum_{\ell=0}^{n}\binom{n+\alpha}{\ell}\binom{n+\beta}{n-\ell}(x-1)^{n-\ell}(x+1)^\ell$			
Translation ✔	`JacobiP[n,\[Alpha],\[Beta],x]==(2)^(-n)*Sum[Binomial[n+\[Alpha],\[ScriptL]]*Binomial[n+\[Beta],n-\[ScriptL]]*(x-1)^(n-\[ScriptL])*(x+1)^\[ScriptL],{\[ScriptL],0,n},GenerateConditions->None]`			
Symbolic	Numeric	Variables	Constraints	Test Values
✘	✔(81/81)	n, α, β, x ✔	$n \in \{1, 2, 3\}$, $x, \alpha, \beta \in \mathbb{R}$, $x, \alpha, \beta > 0$	$n \in \{1, 2, 3\}$, $\alpha, \beta, x \in \left\{\frac{1}{2}, \frac{3}{2}, 2\right\}$

Table E.3: Summary of DLMF chapters with corresponding 2-letter codes (2C), chapter numbers (C#), chapter names of the DLMF chapters, and the total number of displayed formulas (i.e., without inline formulae) per chapter according to [33] (F).

2C	C#	Chapter Name	F
AL	1	Algebraic and Analytic Methods	571
AS	2	Asymptotic Approximations	349
NM	3	Numerical Methods	296
EF	4	Elementary Functions	513
GA	5	Gamma Function	161
EX	6	Exponential, Logarithmic, Sine, and Cosine Integrals	100
ER	7	Error Functions, Dawson's and Fresnel Integrals	137
IG	8	Incomplete Gamma and Related Functions	240
AI	9	Airy and Related Functions	230
BS	10	Bessel Functions	696
ST	11	Struve and Related Functions	149
PC	12	Parabolic Cylinder Functions	177
CH	13	Confluent Hypergeometric Functions	372
LE	14	Legendre and Related Functions	283
HY	15	Hypergeometric Function	198
GH	16	Generalized Hypergeometric Functions & Meijer G-Function	99
QH	17	q-Hypergeometric and Related Functions	181
OP	18	Orthogonal Polynomials	502
EL	19	Elliptic Integrals	464
TH	20	Theta Functions	113
MT	21	Multidimensional Theta Functions	59
JA	22	Jacobian Elliptic Functions	258
WE	23	Weierstrass Elliptic and Modular Functions	167
BP	24	Bernoulli and Euler Polynomials	188
ZE	25	Zeta and Related Functions	171
CM	26	Combinatorial Analysis	201
NT	27	Functions of Number Theory	132
MA	28	Mathieu Functions and Hill's Equation	353
LA	29	Lamé Functions	205
SW	30	Spheroidal Wave Functions	114
HE	31	Heun Functions	120
PT	32	Painlevé Transcendents	286
CW	33	Coulomb Functions	146
TJ	34	$3j$, $6j$, $9j$ Symbols	64
FM	35	Functions of Matrix Argument	62
IC	36	Integrals with Coalescing Saddles	137
		Σ	8,494

Table E.4: Overview of translations for DLMF chapters. Table headings are 2C: 2-letter chapter codes; C#: chapter numbers; F: number of formulae; T_{old}: number of translated expressions using old translator [1]; T_{Map}, T_{Math}: number of translations with improved translator—Map for Maple and Math for Mathematica; M_{Map}, M_{Math}: number of failed translations due to missing macro translation; E: number of other errors in the translation process. Best five performances are colored.

2C	C#	F	T_{old}		T_{Map}		T_{Math}		M_{Map}	M_{Math}	E
AL	1	227	60	(26.4%)	102	(44.9%)	103	(45.4%)	79	78	46
AS	2	136	33	(24.3%)	65	(47.8%)	65	(47.8%)	51	51	20
NM	3	53	36	(67.9%)	40	(75.5%)	40	(75.5%)	8	8	5
EF	4	569	353	(62.0%)	494	(89.3%)	564	(99.1%)	58	4	1
GA	5	144	38	(26.4%)	130	(93.5%)	139	(96.5%)	7	3	2
EX	6	107	21	(19.6%)	56	(52.3%)	77	(72.0%)	50	29	1
ER	7	149	35	(23.5%)	101	(67.8%)	120	(80.5%)	47	28	1
IG	8	204	84	(41.2%)	160	(78.4%)	163	(79.9%)	39	36	5
AI	9	235	36	(15.3%)	180	(76.6%)	179	(76.2%)	46	47	9
BS	10	653	143	(21.9%)	392	(60.0%)	486	(74.4%)	243	135	32
ST	11	124	48	(38.7%)	121	(97.6%)	112	(90.3%)	3	12	0
PC	12	106	33	(31.1%)	79	(74.5%)	90	(84.9%)	23	9	7
CH	13	260	126	(48.5%)	252	(96.9%)	254	(97.7%)	3	1	5
LE	14	238	166	(69.7%)	230	(96.6%)	229	(96.2%)	5	6	3
HY	15	206	148	(71.8%)	198	(96.1%)	197	(95.6%)	3	4	5
GH	16	53	20	(37.7%)	23	(43.4%)	25	(47.2%)	27	1	27
QH	17	175	1	(0.6%)	53	(30.3%)	124	(70.8%)	112	35	16
OP	18	468	132	(28.2%)	235	(50.2%)	288	(61.5%)	203	149	31
EL	19	516	103	(20.0%)	252	(48.8%)	416	(80.6%)	250	84	16
TH	20	128	52	(40.6%)	98	(76.6%)	98	(76.6%)	8	8	22
MT	21	32	0	(0.0%)	0	(0.0%)	0	(0.0%)	30	30	2
JA	22	264	115	(43.6%)	232	(87.9%)	238	(90.2%)	27	21	5
WE	23	164	7	(4.3%)	19	(11.6%)	34	(20.7%)	125	112	18
BP	24	175	31	(17.7%)	117	(67.2%)	148	(84.6%)	35	15	12
ZE	25	154	28	(18.2%)	124	(80.5%)	120	(77.9%)	26	30	4
CM	26	136	31	(22.8%)	78	(57.3%)	87	(64.0%)	54	42	7
NT	27	79	5	(6.3%)	26	(32.9%)	15	(19.0%)	38	49	15
MA	28	267	52	(19.5%)	97	(36.3%)	110	(41.2%)	138	125	32
LA	29	111	11	(9.9%)	23	(20.7%)	22	(19.8%)	79	80	9
SW	30	71	14	(19.7%)	19	(26.8%)	26	(36.6%)	47	39	6
HE	31	35	29	(82.8%)	22	(62.8%)	15	(42.8%)	9	16	4
PT	32	67	43	(64.2%)	57	(85.1%)	57	(85.1%)	0	0	10
CW	33	108	21	(19.4%)	14	(13.0%)	11	(10.2%)	80	86	11
TJ	34	57	0	(0.0%)	1	(1.8%)	37	(64.9%)	46	4	16
FM	35	46	0	(0.0%)	0	(0.0%)	0	(0.0%)	36	36	10
IC	36	106	12	(11.3%)	24	(22.6%)	24	(22.6%)	79	79	3
Σ		6,623	2,067	(31.2%)	4,114	(62.1%)	4,713	(71.2%)	2,114	1,492	418

Table E.5: Overview of symbolic and numeric evaluations for DLMF chapters as in Table E.4. Table headings are T_{Math}: number of successfully translations to Mathetmatica; $S_{success}$, S_{fail}: number of successful and failed symbolic verifications (for translated expressions only) respectively; $N_{success}$, N_{fail}: number of successful and remaining failed numeric (for failed symbolical tests only) respectively. P, T: number of partial (at least one test was successful) and total failed numeric tests. Best five performances are colored.

2C	C#	T_{Math}	$S_{success}$		S_{fail}	$N_{success}$		N_{fail}	[P / T]	A	E
AL	1	103	34	(33.0%)	69	14	(20.3%)	40	[9 / 31]	11	4
AS	2	65	6	(9.2%)	59	4	(6.8%)	38	[6 / 32]	7	9
NM	3	40	5	(12.5%)	35	0	(0.0%)	29	[8 / 21]	6	0
EF	4	564	304	(53.9%)	260	110	(42.3%)	146	[55 / 91]	2	0
GA	5	139	65	(46.8%)	74	30	(40.5%)	20	[9 / 11]	13	9
EX	6	77	18	(23.4%)	59	23	(39.0%)	32	[6 / 26]	4	0
ER	7	120	45	(37.5%)	75	21	(28.0%)	43	[13 / 30]	9	1
IG	8	163	65	(39.9%)	98	22	(22.4%)	44	[19 / 25]	16	15
AI	9	179	69	(38.5%)	110	30	(27.3%)	58	[38 / 20]	14	7
BS	10	486	115	(23.7%)	371	90	(24.2%)	151	[57 / 94]	92	18
ST	11	112	36	(32.1%)	76	21	(27.6%)	33	[8 / 25]	10	11
PC	12	90	18	(20.0%)	72	13	(18.0%)	43	[15 / 28]	12	3
CH	13	254	69	(27.2%)	185	23	(12.4%)	95	[59 / 36]	45	21
LE	14	229	30	(13.1%)	199	59	(29.6%)	92	[54 / 38]	41	5
HY	15	197	53	(26.9%)	144	23	(16.0%)	77	[52 / 25]	29	6
GH	16	25	2	(8.0%)	23	1	(4.3%)	10	[7 / 3]	9	2
QH	17	124	6	(4.8%)	118	13	(11.0%)	57	[52 / 5]	39	5
OP	18	288	101	(35.1%)	185	45	(24.3%)	68	[31 / 37]	52	12
EL	19	416	51	(12.2%)	365	18	(4.9%)	264	[49 /215]	61	15
TH	20	98	1	(1.0%)	97	33	(34.0%)	40	[25 / 15]	24	0
MT	21	0	-		-	-		-	-	-	-
JA	22	238	30	(12.6%)	206	22	(10.7%)	131	[39 / 92]	51	0
WE	23	34	4	(11.8%)	30	2	(6.7%)	23	[9 / 14]	2	3
BP	24	148	23	(15.5%)	125	78	(62.4%)	33	[22 / 11]	14	0
ZE	25	120	48	(40.0%)	72	22	(30.5%)	22	[6 / 16]	22	3
CM	26	87	19	(21.8%)	68	44	(64.7%)	18	[10 / 8]	5	1
NT	27	15	6	(40.0%)	9	3	(33.3%)	6	[3 / 3]	0	0
MA	28	110	7	(6.4%)	103	3	(2.9%)	48	[13 / 35]	33	17
LA	29	22	0	(0.0%)	22	0	(0.0%)	21	[1 / 20]	0	1
SW	30	26	0	(0.0%)	26	0	(0.0%)	19	[2 / 17]	5	1
HE	31	15	2	(13.3%)	13	0	(0.0%)	8	[0 / 8]	5	0
PT	32	57	3	(5.3%)	54	0	(0.0%)	41	[2 / 39]	8	5
CW	33	11	0	(0.0%)	11	0	(0.0%)	11	[2 / 9]	0	0
TJ	34	37	0	(0.0%)	37	14	(37.8%)	10	[5 / 5]	13	0
FM	35	0	-		-	-		-	-	-	-
IC	36	24	0	(0.0%)	24	3	(12.5%)	13	[1 / 12]	1	6
Σ		4,713	1,235	(26.2%)	3,474	784	(22.6%)	1,784	[687 / 1,097]	655	180

F Wikipedia Evaluations

This Section was submitted as supplementary materials for the Journal article [4] and supports Section 4.2 in Chapter 4 of the thesis.

F.1 Noun Phrase Definitions

Originally, we considered only nouns, noun sequences, adjectives followed by nouns, and Wikipedia links as candidates of definiens [26]. However, in the field of Orthogonal Polynomials and Special Functions (OPSF), such descriptions are generally insufficient. Hence, we include connective possessive endings and prepositions between noun phrases. Consider the Bessel functions of which there are two kinds, the first and the second. Additionally, there are modified and spherical versions, each with a first and second kind. Hence, in order to distinguish one from another, we need to know the entire name, such as the *modified Bessel function of the second kind*. Thence, we define our Mathematical Concept (MC) as noun phrases `NP` in terms of Part-of-Speech (POS) tag patterns used by CoreNLP. We use the following patterns[15]: (1) `JJ* NN+`, (2) `JJ* NN+ POSS NN+`, and (3) `JJ* NN+ IN DT? JJ* NN+`. The POS tags refer to the tags used by CoreNLP and `JJ` and `NN` include all specialized forms (such as plurals `NNS`). The symbols '`?`', '`+`', and '`*`' refer to the quantities *at most once*, *at least once*, and *many or none* of the successive tokens with that particular POS tag. These new noun phrase definitions allow us to distinguish special functions and orthogonal polynomials more adequately in contrast to the previously used nouns, noun sequences, and adjective-noun sequences. Note that a noun phrase may appear multiple times in a document. In that case, we adjusted the score as in [27]. While we have not found any improvement in the original project on single identifiers [27], this score adjustment performs better on our current use case. Note that Mathematical Language Processing (MLP) also tries to increase the score if a noun phrase appears multiple times in the same list. This happens if the same description occurs multiple times near the same Mathematical Objects of Interest (MOI). While Schubotz et al. [27] reported that this adjustment did not lead to better performance, we identified the opposite in our use cases and include the proposed adjustment for the score calculations.

F.2 Example Translation Process

Consider the English Wikipedia article about Jacobi polynomials as our exemplary use case. Figure F.5 shows the dependency graph overlay for the first equation in that article. Figure F.6 shows a larger part of the article with more MOI dependencies. Due to implementation constraints and performance reasons, the implementation of our demo page slightly differs from the actual pipeline we used internally. The only difference is that the given formula does not need to be in the given context. The reason is, that we can build a dependency graph of the given context first and then ask for a single formula translation and computation. This is more efficient compared to translating and computing an entire document at once. So the actual pipeline you can use in our demo works as follows. First, a dependency graph is generated for the page that serves as our context. Next, the given formula is integrated into the dependency graph. If the given formula is already in the graph, we can work with that node. If not, we generate a new node and set up the dependencies for this new node. Consider, we want to

[15] Note that possessive endings are tagged by `POS` with CoreNLP [21]. To distinguish possessive endings from part-of-speech tags, we use `POSS` here for possessive endings.

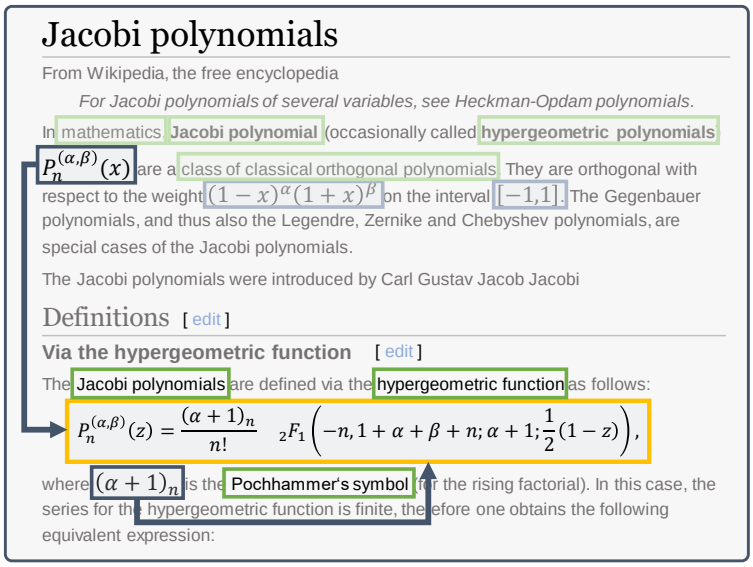


Figure F.5: The part of the dependency graph for the definition of the Jacobi polynomial (yellow). The definition has two ingoing dependencies (blue) and is annotated with three descriptive terms in the same sentence (green). The MOI $P_n^{(\alpha,\beta)}(x)$ would be annotated with all descriptive terms (green) from the sentence in the introduction and the sentence in the definition of the Jacobi polynomial.

translate the equation

$$P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n, 1+\alpha+\beta+n; \alpha+1; \tfrac{1}{2}(1-z)\right), \tag{F.17}$$

in the context of Jacobi polynomials. Since this equation does exist in the context, we take that node in the dependency graph for the following process. The dependency graph from Jacobi polynomials tells us, that the equation contains two other MOI, namely $P_n^{(\alpha,\beta)}(z)$ from earlier in the article and $(\alpha+1)_n$ right below the equation, while the definition itself is not part of any other MOI (no outgoing dependencies). Hence, the annotated descriptive terms for the equation are only the noun phrases extracted from the sentence the equation appears in. These noun phrases are *Pochhammer's symbol* (0.69), *hypergeometric function* (0.6), and *Jacobi polynomial* (0.6). Note that the term *rising factorial* at the end of the sentence is not included. Because of the aforementioned challenges in processing mathematical language (see Section 4.2.5 of the thesis), CoreNLP tagged *factorial* as an adjective instead of a noun and, therefore, the phrase was not considered as a noun phrase.

Next we search for semantic macros with the descriptive terms annotated to the definition in the dependency graph and find `\JacobipolyP` for *Jacobi polynomials*, `\Pochhammersym` for the *Pochhammer's symbol*, and `\genhyperF` for *hypergeometric function*. Each retrieved macro contains a list of possible replacement patterns. Finally, we score each replacement pattern by the score that was generated from MLP for the descriptive term, the search score from ES for retrieving the macro, and the likelihood value of the semantic macro version in the DLMF. Finally, we iterate over every in-going dependency of the MOI and recursively apply the same process to each of the dependant MOIs. For the equation above, this recursive behavior would

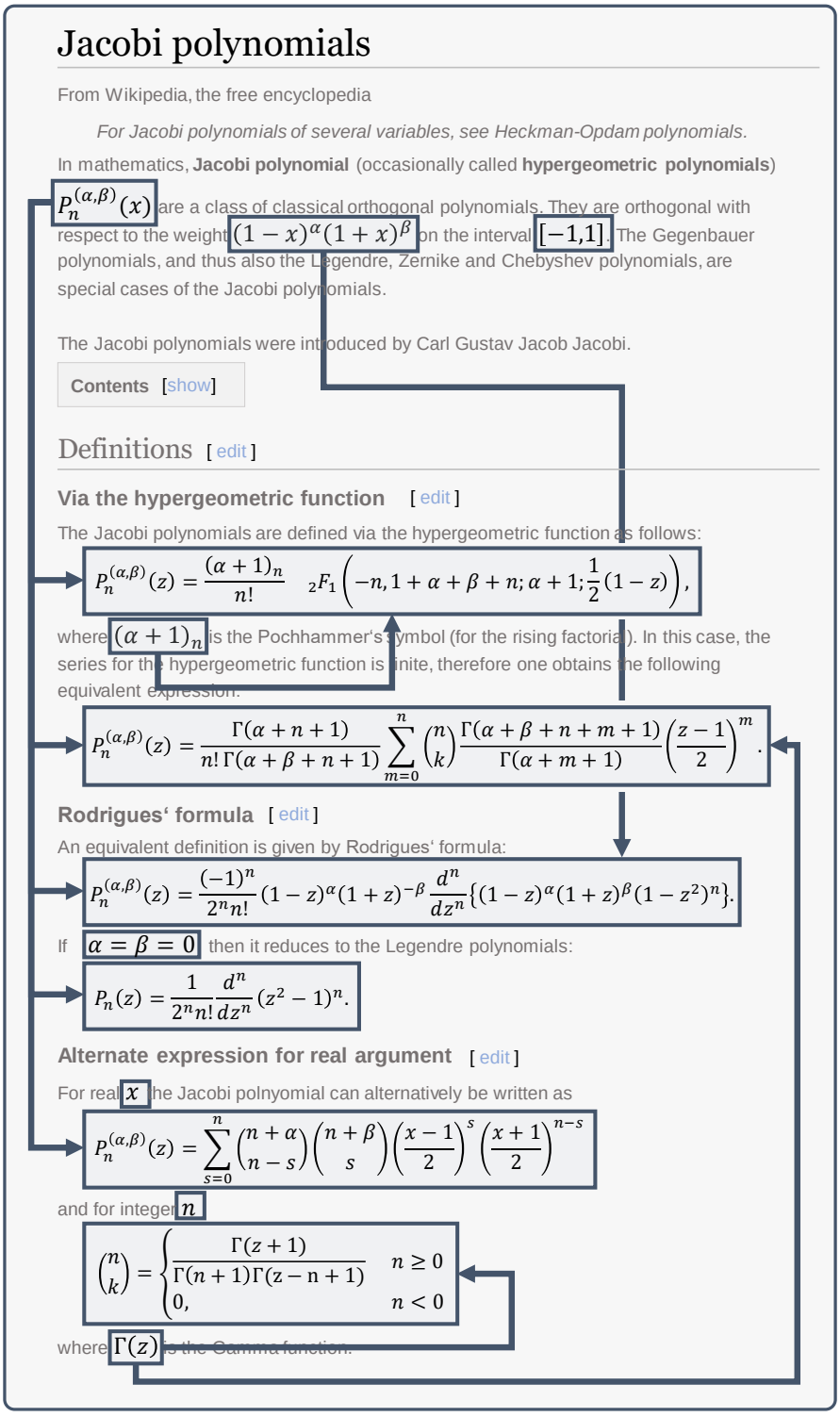


Figure F.6: Complete dependency graph of all MOI for the first sections in the Wikipedia article about *Jacobi polynomials*. Dependencies of the identifiers x and n are omitted due their appearance in almost every MOI.

not be necessary since every component of the equation is already mentioned in the same sentence. However, consider as a counterexample the next equation in the same article

$$P_n^{(\alpha,\beta)}(z) = \frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \sum_{m=0}^{n} \binom{n}{m} \frac{\Gamma(\alpha+\beta+n+m+1)}{\Gamma(\alpha+m+1)} \left(\frac{z-1}{2}\right)^m . \quad \text{(F.18)}$$

In the context of this equation, neither the Jacobi polynomial nor the Gamma function is mentioned in the sentence. However, iterating over the ingoing dependencies reveals $\Gamma(z)$ from later in the article and $P_n^{(\alpha,\beta)}(x)$ from the introduction. Both MOI are annotated with the necessary descriptions to retrieve the replacement patterns for the gamma function and the Jacobi polynomial. Finally, we order the list of retrieved replacement patterns according to their scores. The score for a single replacement pattern is the average of the scores from each part of the retrieval pipeline. Those are (1) the noun phrase score for the given formula calculated from MLP, (2) the relative Elasticsearch score of the retrieved semantic macro replacement patterns, and (3) the DLMF likelihood values for each generic LaTeX pattern. Table F.6 contains some of the retrieved replacement rules.

Table F.6: The retrieved replacement patterns for equation (F.17). The highlighted rows indicate the replacement patterns that matches parts of equation (F.17).

Score	Description	Replacement Pattern
0.8953	Pochhammer symbol (or shifted factorial)	`(par1)_{par2}` → `\Pochhammersym{par1}{par2}`
0.8953	q-multiple Pochhammer symbol	`(par1;par2)_{par3}` → `\qmultiPochhammersym {par1}{par2}{par3}`
0.8702	q-Pochhammer symbol	`(par1;par2)_{par3}` → `\qPochhammer{par1}{par2} {par3}`
0.8663	Jacobi polynomial	`P^{(par1,par2)}_{par3}(var1)` → `\JacobipolyP{par1}{par2} {par3}@{var1}`
0.8521	Associated Jacobi polynomial	`P^{(par1, par2)}_{par3} (var1; var2)` → `\assJacobipolyP{par1}{par2} {par3}@{var1}{var2}`
0.8521	Shifted Jacobi polynomial	`G_{par1} (var1,var2,var3)` → `\shiftJacobipolyG{par1}@ {var1}{var2}{var3}`
0.8506	q-hypergeometric (or basic hypergeometric) function	`{}_{par1}\phi_{par2} ({var1 \atop var2};var3,var4)` → `\qgenhyperphi{par1}{par2} @@{var1}{var2}{var3}{var4}`
0.8079	bilateral q-hypergeometric (or bilateral basic hypergeometric) function	`{}_{par1}\psi_{par2} ({var1 \atop var2};var3,var4)` → `\qgenhyperpsi{par1}{par2} @@{var1}{var2}{var3}{var4}`
0.7186	generalized hypergeometric function	`{}_{par1}F_{par2} ({var1 \atop var2};var3)` → `\genhyperF{par1}{par2} @@{var1}{var2}{var3}`

We apply each replacement pattern that we extracted (Table F.6). The replacement patterns for `\JacobipolyP`, `\Pochhammersym`, and `\genhyperF` matched parts in the original LaTeX (green backgrounds in Table F.6). The original LaTeX looked like this:

LaTeX of equation (F.17)

```
P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!} {}_2F_1\left
    (-n,1+\alpha+\beta+n;\alpha+1;\tfrac{1}{2}(1-z)\right)
```

The replacement patterns replaced each part with the semantic macros. Hence, the semantically enhanced (or simply semantic LaTeX) looks like this:

Enhanced semantic LaTeX of equation (F.17)

```
\JacobipolyP{\alpha}{\beta}{n}@{z} = \frac{\Pochhammersym{
    \alpha + 1}{n}}{n!}\genhyperF{2}{1}@{-n,1+\alpha+\beta+n}{
    \alpha+1}{\tfrac{1}{2}(1-z)}
```

Every semantic macro in this expression is supported by LaCaSt. Hence, we use LaCaSt to translate it to Maple:

Maple translation of equation (F.17)

```
JacobiP(n, alpha, beta, z) = (pochhammer(alpha + 1, n))/(factorial(n)
    )*hypergeom([- n , 1 + alpha + beta + n], [alpha + 1], (1)/(2)*(1
     - z))
```

and Mathematica:

Mathematica translation of equation (F.17)

```
JacobiP[n, \[Alpha], \[Beta], z] == Divide[Pochhammer[\[Alpha]+ 1, n]
    ,(n)!]*HypergeometricPFQ[{- n , 1 + \[Alpha]+ \[Beta]+ n}, {\[
    Alpha]+ 1}, Divide[1,2]*(1 - z)]
```

In addition, we automatically evaluate the equation numerically and symbolically as described in [4]. Essentially, LaCaSt extracted 4 free variables in the expression: α, β, n, and z. Further, it located the equal sign to split the expression in left- (LHS) and right-hand side (RHS). The symbolical test tries to simplify the difference of LHS and RHS to 0. For Mathematica, this simplification looks like this:

Symbolic evaluation of equation (D.3)

```
FullSimplify[
  (JacobiP[n, \[Alpha], \[Beta], z])
  - (Divide[Pochhammer[\[Alpha]+ 1, n],(n)!] * HypergeometricPFQ[{- n
      , 1 + \[Alpha]+ \[Beta]+ n}, {\[Alpha]+ 1}, Divide[1,2]*(1 - z
    )])
]
```

Since it is the common definition of Jacobi polynomials, Mathematica indeed returns 0 for this simplification. Hence, the symbolic test was successful. The numeric tests work similarly. We

replace every variable with actual numeric values and calculate the difference between LHS and RHS. If the numerically calculated difference is 0, the test was successful. Because of resource limitations, we limit the number of test calculations to 10. For example, in Maple, one of the test calculations was for $\alpha = \frac{3}{2}, \beta = \frac{3}{2}, n = 2, z = -\frac{1}{2} + \frac{\sqrt{3i}}{2}$. The difference of LHS and RHS for these values was $0 + 0i$, which is equal to 0. Hence the numeric test was successful for this particular case. All other test calculations were also successful. Conclusively, the entire numeric test case was successful. The evaluation results for this particular example can be found on the demo page for the equation above: https://tpami.wmflabs.org/wiki/Special:LaTeXTranslator?pid=123&eid=math.123.0.

F.3 Evaluation Analysis

In this section, we refer to a benchmark entry with the associated ID. The full benchmark dataset is given later in Section F.4 and online at https://tpami.wmflabs.org/wiki/Poject:GoldData. In the following, we discuss some issues we identified with the evaluation. This section is more comprehensive to the one in Section 5.2.4 of the thesis. It also contains some actual examples and direct links to the entries.

False Math Detection Since the benchmark contains randomly picked formulae from the articles, it also contains entries that might not have been properly annotated with math-templates or math-tags in the Wikitext. Four entries in the benchmark (28, 43, 78, and 85) were wrongly detected by our engine and contained only parts of the actual formula. For example, entry 28 was extracted as `_{1}(z) =` from the Polylogarithm because the formula $\mathrm{Li}_1(z) = -\ln(1-z)$ was given in plain text `Li<sub>1</sub>(''z'') = -ln(1-''z'')`. While our approach correctly identified `<sub>1</sub>(''z'') =` and `(1-''z'')`, it did not identify the plain text words Li and ln as part of the formula.

Inappropriate LaTeX Some translations also failed due to inappropriate LaTeX expressions. For example, the hypergeometric functions have an index prior to the identifier F, such as in ${}_1F_2$. In many cases, the leading subscript was attached to the previous token, such as equal signs (84) or asterisks for multiplications (75, 83), rather than using an empty expression `{}_{1}`. For example, entry 75

$$\begin{aligned} p_n(x;a,b,c|q) =& a^{-n}e^{-inu}(abe^{2iu},ac,ad;q)_n \\ & *_4\,\Phi_3(q^{-n},abcdq^{n-1},ae^{i(t+2u)},ae^{-it};abe^{2iu},ac,ad;q;q). \end{aligned} \tag{F.19}$$

Here the asterisk has an index: $*_4$ (`*_4`). Hence, we were not able to match ${}_4\Phi_3$ with `\qgenhyperphi{4}{3}@...` because the DLMF macro pattern presumed an empty expression with underscore rather than an asterisk.

In one case (85) (from Continuous q-Laguerre polynomials), the editor of the Wikipedia article even split the entire equation into two parts via `</math><math>` to avoid the problem with the leading underscore. Hence, our extracted formula was not complete and missed the second half of the expression.

Split formula to allow for an empty based subscript

```
<math>P_{n}^{(\alpha)}(x|q)=\frac{(q^\alpha+1;q)_{n}}{(q;q)_{n
    }}</math><math>_{3}\Phi_{2}(q^{-n},q^{\alpha/2+1/4}e^{
    i\theta},q^{\alpha/2+1/4}*e^{-i\theta};q^{\alpha+1},0|q,q)
    </math>
```

Definitions and Substitutions Recognizing an equation as a definition would have a great impact on performance. As a test, we manually annotated every definition in the benchmark by replacing the equal sign $=$ with the unambiguous notation $:=$ and extended LaCasT to recognize such combination as a definition of the left-hand side (the DLMF did not use this notation, hence LaCasT was not capable of translating $:=$ in the first place). This resulted in 18 more correct translations (e.g., 66, 68, and 75) and increased the performance from .28 to .47.

Identifying definitions in a document would also solve the problem of substitutions. Consider, for example, the benchmark entry 3 containing the elliptic integral of the first kind $F(x;k) = u$, from the article Elliptic integral. Here, x was defined as the Jacobian elliptic function $\mathrm{sn}(u,k)$ immediately above this equation. Hence, an appropriate translation of $F(x;k) = u$ to Mathematica would be

Appropriate translation of $F(x;k) = u$

```
EllipticF[ArcSin[JacobiSN[u, (k)^2]], (k)^2] == u
```

While LaCasT is capable of performing such complex translation from semantic LaTeX (note that the second arguments are squared and we must use $\sin^{-1}(x)$ for the first argument), we were not able to detect the substitution of x. Hence, our translation was not appropriate, as defined in our paper.

The dependency graph may provide beneficial information towards a definition recognition system for equations. However, rather than presuming that every equation symbol is indicating a definition [20], we propose a more selective approach. Considering one part of an equation (including multi-equations) as an extra MOI would establish additional dependencies in the dependency graph, such as a connection between $x = \mathrm{sn}(u,k)$ and $F(x;k) = u$. A combination with recent advances of definition recognition in NLP [14, 15, 16, 19, 31] may then allow us to detect x as the defining element. The already established dependency between x and $F(x;k) = u$ can finally be used to resolve the substitution. Hence, for future research, we will elaborate the possibility of integrating existing NLP techniques for definition recognition [14, 15] into our dependency graph concept.

Missing Information Another problem that causes translations to fail is missing information. For example, the gamma function seems to be considered common knowledge in most articles on special functions because it is often not specifically declared by name in the context (e.g., 19, 31, and 53). To test the impact of considering the gamma function as common knowledge, we added the description *Euler gamma function* to the list of noun phrases that fetch semantic macros from the database. Adding a low score ensures the pattern for the gamma function will be applied late in the list of replacement patterns. This indeed improved performance

slightly, providing us a successful translation of three more benchmark entries (19, 56, and 93). Additionally, it improved the semantic LaTeX in three other entries (22, 31, and 53) but these cases had other issues that prevented a successful translation. This naive approach, emphasizes the importance of knowing the domain knowledge for specific articles. Schubotz et al. [26] utilized namespaces of mathematical expressions to improve definiens extractions for identifiers. These namespaces may allow us to activate the feature we used to test the impact of the gamma function more precisely in certain cases and deactivate it in others.

Non-Matching Replacement Patterns An issue we would more regularly face in domains other than special functions and orthogonal polynomials are non-standard notations. For example, for six entries (18, 44, 59, 62, 77, and 87) we were not able to appropriately replace hypergeometric functions with semantic macros because they used the matrix and array environments in their arguments, while the DLMF only uses `\atop` for the same visualization. Consequently, none of our replacement patterns matched even though we correctly identified the expressions as hypergeometric functions. Entries 3 (from above) and 54, illustrate how minor errors already cause a failure in the semantic enhancement process. In the DLMF, the elliptic integral only uses a comma to separate the arguments. Hence, the appropriated semantic macro did not match $F(x;k)$. Similarly, the hypergeometric function in entry 54

$$ {}_1F_2\left(1, \tfrac{3}{2}, \alpha+\tfrac{3}{2}, -\tfrac{z^2}{4}\right) \tag{F.20} $$

inappropriately uses commas to split the arguments. Hence, again, none of the replacement patterns matched the expression. For an appropriate translation, one must derive the 1 and 2 from ${}_1F_2$ in order to extract the correct arguments.

One idea for solving these issues is to add more replacement rules to a macro. Greiner-Petter et al. [6] presented a search engine for MOI that allows searching for common notations for a given textual query. Searching for Jacobi polynomials in arXiv shows that different variants of $P_n^{(\alpha,\beta)}(x)$ are highly related or even equivalently used for Jacobi polynomials, such as p, H, or R rather than P. However, widening the replacement patterns further may harm performance, as seen in Table 2 (in the paper) for higher numbers of retrieved noun phrases and macros.

Derivatives and Prime Notations Related problems that we encountered with our implementation of replacement patterns are primes and derivative notations. Primes for differentiation are generally put in between the function identifier and its argument, e.g., $\mathrm{Ai}'(x)$. Unfortunately, this causes the replacement patterns to fail because `Ai(var1)` did not allow tokens in between. We can avoid this by adding support for optional tokens to our pattern matching implementation. Hence, we would rather generate the pattern `Ai opToken (var1)`. The DLMF adds primes between the semantic macro and its arguments as well, e.g., `\AiryAi'@{z}`. Hence, we can also simply put this optional token into the semantic LaTeX expression.

Synonyms One minor issue we faced were synonyms. For example, entry 51 discusses *Kravchuk polynomials* while the DLMF using the different transliteration *Krawtchouk polynomials*. Hence, our pipeline was unable to find the correct semantic macro for the polynomials. Since we already use Elasticsearch (ES) to search for semantic macros, and Elasticsearch (ES) is capable of handling synonyms, we only need to feed the database with common synonyms. In

this case, the Wikipedia article itself mentions the alternative transliteration at the beginning of the article. We could extract these synonyms while analyzing the Wikitext and adding them to the semantic macro database.

F.4 Benchmark Dataset

The following tables contain the 95 entries of the manually crafted benchmark dataset that we used to qualitatively evaluated our *semantification* and translations. Each entry is linked (on the top right) to a page on our online demo that provides further information and shows the actual translations. The hyperlink to each entry is `https://tpami.wmflabs.org/wiki/Gold_` followed by the entry ID, e.g., the page name is `Gold_1` for the first entry. The following tables do not include the translation results but only the manually crafted translations for a better overview. The translation results for each case can be reached via the hyperlink.

In addition, every entry shows if our semantification (translation to semantic LaTeX) and translation to Mathematica (CAS) was correct (✔) or wrong (✘). If the human annotator could not translate the expression manually, the translations to the CAS are empty. If the semantification or translation failed, the tables contain a *Reason for Failure* category and provide an additional explanation on why the translation failed. We derived five main reasons for failures: definition or substitution detection, failed pattern matching, ambiguous derivative and prime notations, missing information in the context, and untranslatable functions (e.g., because the CAS does not support that function).

Our math identification algorithm failed in four cases (28, 43, 78, and 85) because the formula was given in plain text. In that case, the human annotator fixed the formula to provide correct semantic LaTeX, Mathematica, and Maple translations. The original LaTeX was not fixed to see what our system detected. Since LaCaST was unable to detect the math correctly, the translations counted as failures.

Table F.7: The 95 benchmark entries to evaluate LaCaST translations and semantifications. The top right of each entry is a hyperlink to additional information available at https://tpami.wmflabs.org/Gold_ID where ID is the benchmark number, e.g., Gold_1.

Formula	$J_{-(m+\frac{1}{2})}(x) = (-1)^{m+1} Y_{m+\frac{1}{2}}(x),$ $Y_{-(m+\frac{1}{2})}(x) = (-1)^{m} J_{m+\frac{1}{2}}(x).$	**1:** Bessel function
LaTeX	`\begin{align}J_{-(m+\frac{1}{2})}(x) &= (-1)^{m+1} Y_{m+\frac{1}{2}}(x), \\Y_{-(m+\frac{1}{2})}(x) &= (-1)^m J_{m+\frac{1}{2}}(x).\end{align}`	
Semantic LaTeX	`\begin{align}\BesselJ{- (m + \frac{1}{2})}@{x} &= (- 1)^{m+1} \BesselY{m+\frac{1}{2}}@{x} , \\ \BesselY{- (m + \frac{1}{2})}@{x} &= (-1)^m \BesselJ{m+\frac{1}{2}}@{x} .\end{align}`	
Mathematica	`BesselJ[- (m +Divide[1,2]), x] == (- 1)^(m + 1)* BesselY[m +Divide[1,2], x]` `BesselY[- (m +Divide[1,2]), x] == (- 1)^(m)* BesselJ[m +Divide[1,2], x]`	
Maple	`BesselJ(- (m +(1)/(2)), x) = (- 1)^(m + 1)* BesselY(m +(1)/(2), x); BesselY(- (m +(1)/(2)), x) = (- 1)^(m)* BesselJ(m +(1)/(2), x)`	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-
Explanation	—					

Formula	$E(e) = \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2\theta}\, d\theta$	**2:** Ellipse
LaTeX	`E(e) \,=\, \int_0^{\pi/2}\sqrt {1 - e^2 \sin^2\theta}\ d\theta`	
Semantic LaTeX	`\compellintEk@{e} = \int_0^{\cpi / 2} \sqrt{1 - e^2 \sin^2 \theta} \diff{\theta}`	
Mathematica	`EllipticE[(e)^2] == Integrate[Sqrt[1 - (e)^(2)*(Sin[\[Theta]])^(2)], {\[Theta], 0, Pi/2}]`	
Maple	`EllipticE(e) = int(sqrt(1 - (e)^(2)*(sin(theta))^(2)), theta = 0..Pi/2)`	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-
Explanation	e was misinterpreted as Euler's number.					

Formula	$F(x;k) = u$	**3:** Elliptic integral
LaTeX	`F(x;k) = u`	
Semantic LaTeX	`\incellintFk@{\asin@{\Jacobiellsnk@@{u}{k}}}{k} = u`	
Mathematica	`EllipticF[ArcSin[JacobiSN[u, (k)^2]], (k)^2] == u`	
Maple	`EllipticF(JacobiSN(u, k), k) = u`	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	-	-

Explanation x should have been substituted.

Formula	$\frac{1}{\Gamma(z)} = \frac{i}{2\pi} \int_C (-t)^{-z} e^{-t}\,dt$	**4:** Gamma function
LaTeX	`\frac{1}{\Gamma(z)} = \frac{i}{2\pi}\int_C (-t)^{-z}e^{-t}\,dt`	
Semantic LaTeX	`\frac{1}{\EulerGamma@{z}} = \frac{\iunit}{2 \cpi} \int_C(- t)^{-z} \expe^{-t} \diff{t}`	
Mathematica		
Maple		

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	✘	-	-	-	-

Explanation Contour integrals cannot be translated.

Formula	$2^4 = 2 \times 2 \times 2 \times 2 = 16$	**5:** Logarithm
LaTeX	`2^{4} = 2 \times2 \times 2 \times 2 = 16`	
Semantic LaTeX	`2^{4} = 2 \times2 \times 2 \times 2 = 16`	
Mathematica	`(2)^(4) == 2 * 2 * 2 * 2 == 16`	
Maple	`(2)^(4) = 2 * 2 * 2 * 2 = 16`	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\psi(x) := \sum_{n=1}^{\infty} e^{-n^2 \pi x}$ **6:** Riemann zeta function
LaTeX	`\psi(x) := \sum_{n=1}^\infty e^{-n^2 \pi x}`
Semantic LaTeX	`\psi(x) : = \sum_{n=1}^\infty \expe^{- n^2 \cpi x}`
Mathematica	`\[Psi][x_] := Sum[Exp[-(n)^(2)*Pi*x], {n, 1, Infinity}]`
Maple	`psi := (x) -> sum(exp(-(n)^(2)*Pi*x), n=1..infinity)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\operatorname{li}(x) = \lim_{\varepsilon \to 0+} \left(\int_0^{1-\varepsilon} \frac{dt}{\ln t} + \int_{1+\varepsilon}^{x} \frac{dt}{\ln t} \right)$ **7:** Logarithmic integral function
LaTeX	`\operatorname{li}(x) = \lim_{\varepsilon \to 0+} \left( \int_0^{1-\varepsilon} \frac{dt}{\ln t} + \int_{1+\varepsilon}^x \frac{dt}{\ln t} \right)`
Semantic LaTeX	`\logint@{x} = \lim_{\varepsilon \to 0+}(\int_0^{1-\varepsilon} \frac{\diff{t}}{\ln t} + \int_{1+\varepsilon}^x \frac{\diff{t}}{\ln t})`
Mathematica	`LogIntegral[x] == Limit[Integrate[Divide[1,Log[t]], {t, 0, 1 - \[CurlyEpsilon]}]+ Integrate[Divide[1,Log[t]], {t, 1 + \[CurlyEpsilon], x}], \[CurlyEpsilon] -> 0, Direction -> "FromAbove"]`
Maple	`Li(x) = limit(int((1)/(ln(t)), t = 0..1 - varepsilon)+ int((1)/(ln(t)), t = 1 + varepsilon..x), varepsilon = 0, right)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$w_i = \frac{1}{p'_n(x_i)} \int_a^b \omega(x) \frac{p_n(x)}{x-x_i} dx$ **8:** Gaussian quadrature
LaTeX	`w_{i} = \frac{1}{p'_{n}(x_{i})}\int_{a}^{b}\omega(x)\frac{p_{n}(x)}{x-x_{i}} dx`
Semantic LaTeX	`w_{i} = \frac{1}{p'_{n}(x_{i})} \int_{a}^{b} \omega(x) \frac{p_{n}(x)}{x-x_{i}} \diff{x}`
Mathematica	`Subscript[w, i] = Divide[1, Subscript[p\[Prime], n][Subscript[x, i]]]* Integrate[\[Omega][x]*Divide[Subscript[p,n][x], x-Subscript[x,i]], {x, a, b}]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	✘	-	-
Explanation	Prime notations are ambiguous and lead to wrong translations.					

Formula	$x = ue^u,$ $\frac{dx}{du} = (u+1)e^u.$ **9:** Lambert W function
LaTeX	`\begin{align}x & =ue^u, \\[5pt]\frac{dx}{du} & =(u+1)e^u.\end{align}`
Semantic LaTeX	`\begin{align}x & =\LambertW@{x}\expe^{\LambertW@{x}}, \\ \deriv{x}{\LambertW@{x}} &=(\LambertW@{x} + 1) \expe^{\LambertW@{x}} .\end{align}`
Mathematica	`x == ProductLog[x]*(E)^(ProductLog[x]) D[x,ProductLog[x]] = (ProductLog[x] + 1)*Exp[ProductLog[x]]`
Maple	`x = LambertW(x)*exp(u); diff(x, [LambertW(x)$1]) = (LambertW(x) + 1)*exp(LambertW(x))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	-	-
Explanation	u should have been substituted.					

Formula	$\frac{1}{\left\vert \mathbf{x}-\mathbf{x}' \right\vert} = \frac{1}{\sqrt{r^2+r'^2-2rr'\cos\gamma}} = \sum_{\ell=0}^{\infty} \frac{r'^\ell}{r^{\ell+1}} P_\ell(\cos\gamma)$ **10:** Legendre polynomials
LaTeX	`\frac{1}{\left\vert \mathbf{x}-\mathbf{x}' \right\vert} = \frac{1}{\sqrt{r^2+r'^2-2r{r'}\cos\gamma}} = \sum_{\ell=0}^\infty \frac{{r'}^\ell}{r^{\ell+1}} P_\ell(\cos \gamma)`
Semantic LaTeX	`\frac{1}{\left\vert \mathbf{x} - \mathbf{x}' \right\vert} = \frac{1}{\sqrt{r^2+r'^2-2r{r'}\cos\gamma}} = \sum_{\ell=0}^\infty \frac{{r'}^\ell}{r^{\ell+1}} \LegendrepolyP{\ell}@{\cos \gamma}`
Mathematica	`Divide[1, Abs[x - x\[Prime]]] == Divide[1, Sqrt[r^2+(r\[Prime])^(2)-2*r*r\[Prime] Cos[\[Gamma]]]] == Sum[Divide[(r\[Prime])^(\[ScriptL]), r^(\[ScriptL]+1)]*LegendreP[\[ScriptL], Cos[\[Gamma]]], {\[ScriptL], 0, Infinity}]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	✘	-	-
Explanation	—					

Formula	$\operatorname{erf}^{(k)}(z) = \frac{2(-1)^{k-1}}{\sqrt{\pi}} H_{k-1}(z) e^{-z^2} = \frac{2}{\sqrt{\pi}} \frac{d^{k-1}}{dz^{k-1}} \left(e^{-z^2}\right), \quad k = 1, 2, \dots$ **11:** Error function
LaTeX	`\operatorname{erf}^{(k)}(z) = \frac{2 (-1)^{k-1}}{\sqrt{\pi}} \mathit{H}_{k-1}(z) e^{-z^2} = \frac{2}{\sqrt{\pi}} \frac{d^{k-1}}{dz^{k-1}} \left(e^{-z^2}\right),\qquad k=1, 2, \dots`
Semantic LaTeX	`\erf@@{(z)}^{(k)} = \frac{2 (-1)^{k-1}}{\sqrt{\cpi}} \HermitepolyH{k-1}@{z} \expe^{-z^2} = \frac{2}{\sqrt{\cpi}} \deriv [{k-1}]{ }{z}(\expe^{-z^2}) , \qquad k = 1 , 2 , \dots`
Mathematica	`D[Erf[z], {z, k}] == Divide[2*(- 1)^(k - 1),Sqrt[Pi]]*HermiteH[k - 1, z]*Exp[- (z)^(2)] == Divide[2,Sqrt[Pi]]*D[Exp[- (z)^(2)], {z, k - 1}]`
Maple	`diff(erf(z), [z$k]) = (2*(- 1)^(k - 1))/(sqrt(Pi))*HermiteH(k - 1, z)*exp(- (z)^(2)) = (2)/(sqrt(Pi))*diff(exp(- (z)^(2)), [z$(k - 1)])`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	✘	-	-
Explanation	$\operatorname{erf}^{(k)}(z)$ was not understood as k-th derivative but as power.					

Formula	$x_k = \cos\left(\frac{\pi(k+1/2)}{n}\right), \quad k = 0, \ldots, n-1$ **12:** Chebyshev polynomials
LaTeX	`x_k = \cos\left(\frac{\pi(k+1/2)}{n}\right),\quad k=0,\ldots,n-1`
Semantic LaTeX	`x_k = \cos(\frac{\cpi(k + 1 / 2)}{n}) , \quad k = 0 , \ldots , n - 1`
Mathematica	`Subscript[x, k] == Cos[Divide[Pi*(k + 1/2),n]]`
Maple	`x[k] = cos((Pi*(k + 1/2))/(n))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$E(x, y; u) := \sum_{n=0}^{\infty} u^n \, \psi_n(x) \, \psi_n(y) = \frac{1}{\sqrt{\pi(1-u^2)}} \exp\left(-\frac{1-u}{1+u}\frac{(x+y)^2}{4} - \frac{1+u}{1-u}\frac{(x-y)^2}{4}\right)$ **13:** Hermite polynomials
LaTeX	`E(x, y; u) := \sum_{n=0}^\infty u^n \, \psi_n (x) \, \psi_n (y) = \frac{1}{\sqrt{\pi (1 - u^2)}} \, \exp\left(-\frac{1 - u}{1 + u} \, \frac{(x + y)^2}{4} - \frac{1 + u}{1 - u} \, \frac{(x - y)^2}{4}\right)`
Semantic LaTeX	`E(x , y ; u) : = \sum_{n=0}^\infty u^n \psi_n(x) \psi_n(y) = \frac{1}{\sqrt{\cpi(1 - u^2)}} \exp(- \frac{1 - u}{1 + u} \frac{(x + y)^2}{4} - \frac{1 + u}{1 - u} \frac{(x - y)^2}{4})`
Mathematica	`\[CapitalEpsilon][x_, y_, u_] := Sum[(u)^(n)* Subscript[\[Psi], n][x]* Subscript[\[Psi], n][y], {n, 0, Infinity}] == Divide[1,Sqrt[Pi*(1 - (u)^(2))]]*Exp[-Divide[1 - u,1 + u]*Divide[(x + y)^(2),4]-Divide[1 + u,1 - u]*Divide[(x - y)^(2),4]]`
Maple	`Epsilon := (x, y, u) -> sum((u)^(n)* psi[n](x)* psi[n](y), n = 0..infinity) = (1)/(sqrt(Pi*(1 - (u)^(2))))*exp(-(1 - u)/(1 + u)*((x + y)^(2))/(4)-(1 + u)/(1 - u)*((x - y)^(2))/(4))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$x = \pm 1$ **14:** Legendre function
LaTeX	`x = \pm 1`
Semantic LaTeX	`x = \pm 1`
Mathematica	`x == \[PlusMinus]1`
Maple	`x = &+- 1`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation	—

Formula	$E_n = 2^n E_n(\frac{1}{2})$ **15:** Bernoulli polynomials
LaTeX	`E_n=2^nE_n(\tfrac{1}{2})`
Semantic LaTeX	`\EulernumberE{n} = 2^n\EulerpolyE{n}@{\tfrac{1}{2}}`
Mathematica	`EulerE[n] == (2)^(n)* EulerE[n, Divide[1,2]]`
Maple	`euler(n) = (2)^(n)* euler(n, (1)/(2))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-

Explanation	Both E were detected as Euler's number but the second E should have been Euler's polynomial.

Formula	$\mathrm{Si}(ix) = i\,\mathrm{Shi}(x)$ **16:** Trigonometric integral
LaTeX	`\operatorname{Si}(ix) = i\operatorname{Shi}(x)`
Semantic LaTeX	`\sinint@{\iunit x} = \iunit \sinhint@{x}`
Mathematica	`SinIntegral[I*x] == I*SinhIntegral[x]`
Maple	`Si(I*x) = I*Shi(x)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-

Explanation	The noun *integral* was not tagged as a noun by CoreNLP. Hence, the macro for hyperbolic sine function was retrieved too late (low score) and not considered for replacement. Hence, the semantification failed.

Formula	$f(z) = \frac{1}{B(x,y)}$ **17:** Beta function
LaTeX	`f(z)=\frac{1}{\Beta(x,y)}`
Semantic LaTeX	`f(x, y) = \frac{1}{\EulerBeta@{x}{y}}`
Mathematica	`f[x_, y_] := Divide[1,Beta[x, y]]`
Maple	`f := (x,y) -> (1)/(Beta(x, y))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	✘	-

Explanation The original formula contained $f(z)$ but should have been $f(x, z)$.
This was fixed in the Wikipedia article after we generated the dataset.

Formula	$\int x^m e^{ix^n}\,dx = \frac{x^{m+1}}{m+1}\,{}_1F_1\left(\begin{array}{c}\frac{m+1}{n}\\1+\frac{m+1}{n}\end{array}\middle\vert ix^n\right) = \frac{1}{n} i^{\frac{m+1}{n}}\gamma\left(\frac{m+1}{n}, -ix^n\right),$ **18:** Fresnel integral
LaTeX	`\begin{align}\int x^m e^{ix^n}\,dx & =\frac{x^{m+1}}{m+1}\,_1F_1\left(\begin{array}{c} \frac{m+1}{n}\\1+\frac{m+1}{n}\end{array}\mid ix^n\right) \\[6px]& =\frac{1}{n} i^\frac{m+1}{n}\gamma\left(\frac{m+1}{n},-ix^n\right),\end{align}`
Semantic LaTeX	`\begin{align}\int x^m \exp(\iunit x^n) \diff{x} &= \frac{x^{m+1}}{m+1}\ genhyperF{1}{1}@{\frac{m+1}{n}}{1+\frac{m+1}{n}}{\iunit x^n}\\ &=\frac{1}{n} \iunit^{(m+1)/n} \incgamma@{\frac{m+1}{n}}{-\iunit x^n}\end{align}`
Mathematica	`Integrate[(x)^(m)* Exp[I*(x)^(n)], x] == Divide[(x)^(m + 1),m + 1]* HypergeometricPFQ[{Divide[m + 1,n]}, {1 +Divide[m + 1,n]}, I*(x)^(n)] == Divide[1,n]*(I)^((m + 1)/n)* Gamma[Divide[m + 1,n], 0, - I*(x)^(n)]`
Maple	`int((x)^(m)* exp(I*(x)^(n)), x) = ((x)^(m + 1))/(m + 1)*hypergeom([(m + 1)/( n)], [1 +(m + 1)/(n)], I*(x)^(n)) = (1)/(n)*(I)^((m + 1)/n)* GAMMA((m + 1)/( n))-GAMMA((m + 1)/(n), - I*(x)^(n))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-

Explanation The DLMF does not use the matrix (array environment) for the argument of ${}_1F_1$. Hence, no pattern exists that matches the hypergeometric function.

Formula	$T_n(x) = \frac{\Gamma(1/2)\sqrt{1-x^2}}{(-2)^n\,\Gamma(n+1/2)} \frac{d^n}{dx^n}\left([1-x^2]^{n-1/2}\right)$ **19:** Classical orthogonal polynomials
LaTeX	`T_n(x) = \frac{\Gamma(1/2)\sqrt{1-x^2}}{(-2)^n\,\Gamma(n+1/2)} \ \frac{d^n}{dx^n}\left([1-x^2]^{n-1/2}\right)`
Semantic LaTeX	`\ChebyshevpolyT{n}@{x} = \frac{\EulerGamma{1/2}\sqrt{1-x^2}}{(-2)^n\ EulerGamma{n+1/2}} \deriv [n]{ }{x}([1 - x^2]^{n-1/2})`
Mathematica	`ChebyshevT[n, x] == Divide[Gamma[1/2]*Sqrt[1 - (x)^(2)],(- 2)^(n)* Gamma[n + 1/2]]*D[(1 - (x)^(2))^(n - 1/2), {x, n}]`
Maple	`ChebyshevT(n, x) = (GAMMA(1/2)*sqrt(1 - (x)^(2)))/((- 2)^(n)* GAMMA(n + 1/2))*diff((1 - (x)^(2))^(n - 1/2), [x$(n)])`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-
Explanation	No information about the Gamma function in the context.					

Formula	${}_1F_0(1;;z) = \sum_{n\geqslant 0} z^n = (1-z)^{-1}$ **20:** Generalized hypergeometric function
LaTeX	`{}_1F_0(1;;z) = \sum_{n \geqslant 0} z^n = (1-z)^{-1}`
Semantic LaTeX	`\genhyperF{1}{0}@{1}{}{z} = \sum_{n \geqslant 0} z^n = (1-z)^{-1}`
Mathematica	`HypergeometricPFQ[{1}, {}, z] == Sum[(z)^(n), {n, 0, Infinity}] == (1 - z)^(- 1)`
Maple	`hypergeom([1], [], z) = sum((z)^(n), n = 0..infinity) = (1 - z)^(- 1)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-
Explanation	Empty arguments currently does not match our semantic patterns (considered a bug).					

Formula	$\chi(-1) = 1$	**21:** Dirichlet L-function
LaTeX	`\chi(-1) = 1`	
Semantic LaTeX	`\Dirichletchar@@{- 1}{k} = 1`	
Mathematica	`DirichletCharacter[1, 1, -1] == 1`	
Maple		

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	-	✘	-

Explanation $\chi(-1)$ was translated to `DirichletCharacter[1,k,-1]`.
This is not accurate, even though the only valid option for k is 1.

Formula	$\mathrm{Bi}'(z) \sim \frac{z^{\frac{1}{4}} e^{\frac{2}{3}z^{\frac{3}{2}}}}{\sqrt{\pi}} \left[\sum_{n=0}^{\infty} \frac{1+6n}{1-6n} \frac{\Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})\left(\frac{3}{4}\right)^n}{2\pi n! z^{3n/2}}\right]$	**22:** Airy function
LaTeX	`\operatorname{Bi}'(z)\sim \frac{z^{\frac{1}{4}}e^{\frac{2}{3}z^{\frac{3}{2}}}}{\sqrt\pi\,}\left[ \sum_{n=0}^{\infty}\frac{1+6n}{1-6n} \dfrac{ \Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})\left(\frac{3}{4}\right)^n}{2\pi n! z^{3n/2}} \right]`	
Semantic LaTeX	`\AiryBi'@{z} \sim \frac{z^{\frac{1}{4}}} \expe^{\frac{2}{3}z^{\frac{3}{2}}}}{\sqrt{\cpi}} [\sum_{n=0}^{\infty} \frac{1+6n}{1-6n} \frac{\EulerGamma@{n + \frac{5}{6}} \EulerGamma@{n + \frac{1}{6}}(\frac{3}{4})^n{2 \cpi n! z^{3n /2}}}]`	
Mathematica		
Maple		

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	✘

Explanation `\sim` cannot be translated.

Formula	$F'(y) = 1 - 2yF(y)$	**23:** Dawson function
LaTeX	`F'(y)=1-2yF(y)`	
Semantic LaTeX	`\DawsonsintF'@{y}=1-2y\DawsonsintF@{y}`	
Mathematica	`D[DawsonF[y], {y, 1}] == 1 - 2*y*DawsonF[y]`	
Maple	`diff( dawson(y), y$(1) ) = 1 - 2*y*dawson(y)`	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	✘	✘	-

Explanation The dependency graph did not capture a dependency to *Dawson*. Hence we did not retrieve the necessary semantic macro.

Formula	$s \neq 1$	**24:** Hurwitz zeta function
LaTeX	`s\not =1`	
Semantic LaTeX	`s \neq 1`	
Mathematica	`s \[NotEqual] 1`	
Maple		

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$q = e^{i\pi\tau}$ **25:** Theta function
LaTeX	`q = e^{i\pi\tau}`
Semantic LaTeX	`q = \expe^{\iunit \cpi \tau}`
Mathematica	`q == Exp[I*Pi*\[Tau]]`
Maple	`q = exp(I*Pi*tau)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\frac{\mathrm{d}}{\mathrm{d}z}\operatorname{dn}(z) = -k^2\operatorname{sn}(z)\operatorname{cn}(z)$ **26:** Jacobi elliptic functions
LaTeX	`\frac{\mathrm{d}}{\mathrm{d}z} \operatorname{dn}(z) = - k^2 \operatorname{sn}(z) \operatorname{cn}(z)`
Semantic LaTeX	`\deriv [1]{ }{z} \Jacobielldnk@@{(z)}{k} = - k^2 \Jacobiellsnk@@{(z)}{k} \Jacobiellcnk@@{(z)}{k}`
Mathematica	`D[JacobiDN[z, (k)^2], {z, 1}] == - (k)^(2)* JacobiSN[z, (k)^2]*JacobiCN[z, (k)^2]`
Maple	`diff(JacobiDN(z, k), [z$(1)]) = - (k)^(2)* JacobiSN(z, k)*JacobiCN(z, k)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\int_{-\infty}^{\infty} \frac{\gamma\left(\frac{s}{2}, z^2\pi\right)}{(z^2\pi)^{\frac{s}{2}}} e^{-2\pi i k z} \mathrm{d}z = \frac{\Gamma\left(\frac{1-s}{2}, k^2\pi\right)}{(k^2\pi)^{\frac{1-s}{2}}}$ **27:** Incomplete gamma function
LaTeX	`\int_{-\infty}^\infty \frac {\gamma\left(\frac s 2, z^2 \pi \right)} {(z^2 \pi)^\frac s 2} e^{-2 \pi i k z} \mathrm d z = \frac {\Gamma\left(\frac {1-s} 2, k^2 \pi \right)} {(k^2 \pi)^\frac {1-s} 2}`
Semantic LaTeX	`\int_{-\infty}^\infty \frac{\incgamma@{\frac s 2}{z^2 \cpi}}{(z^2 \cpi)^\frac s 2} \expe^{- 2 \cpi \iunit k z} \diff{z} = \frac{\incGamma@{\frac {1-s} 2}{k^2 \cpi}}{(k^2 \cpi)^\frac {1-s} 2}`
Mathematica	`Integrate[Divide[Gamma[Divide[s,2], 0, (z)^(2)* Pi],((z)^(2)* Pi)^(Divide[s,2])]*Exp[- 2*Pi*I*k*z], {z, - Infinity, Infinity}] == Divide[Gamma[Divide[1 - s,2], (k)^(2)* Pi],((k)^(2)* Pi)^(Divide[1 - s,2])]`
Maple	`int((GAMMA((s)/(2))-GAMMA((s)/(2), (z)^(2)* Pi))/(((z)^(2)* Pi)^((s)/(2)))* exp(- 2*Pi*I*k*z), z = - infinity..infinity) = (GAMMA((1 - s)/(2), (k)^(2)* Pi))/(((k)^(2)* Pi)^((1 - s)/(2)))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	${}_1(z) =$ **28:** Polylogarithm
LaTeX	`_{1}(z) =`
Semantic LaTeX	`\polylog{1}@{z} = -\ln@{1-z}`
Mathematica	`PolyLog[1, z] = -Log[1 - z]`
Maple	`polylog(1, z) = -ln(1 - z)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	-	-

Explanation Our math detection algorithm failed because the formula was not properly given in math notation. Instead, `Li` was given in plain text: `Li<sub>-1</sub>("z")`.

Formula	$\int_{-\infty}^{\infty} \operatorname{sinc}(t)\, e^{-i2\pi f t}\, dt = \operatorname{rect}(f)$ **29:** Sinc function
LaTeX	`\int_{-\infty}^\infty \operatorname{sinc}(t) \, e^{-i 2 \pi f t}\,dt = \operatorname{rect}(f)`
Semantic LaTeX	`\int_{-\infty}^\infty \operatorname{sinc}(t) \expe^{- \iunit 2 \cpi f t} \diff{t} = \operatorname{rect}(f)`
Mathematica	`Integrate[sinc[(t)]*Exp[- I*2*Pi*f*t], {t, - Infinity, Infinity}] == rect[f]`
Maple	`int(sinc((t))*exp(- I*2*Pi*f*t), t = - infinity..infinity) = rect(f)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$N = 1$ **30:** Exponential integral
LaTeX	`N=1`
Semantic LaTeX	`N=1`
Mathematica	`N == 1`
Maple	`N = 1`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\sum_{n=0}^{\infty} \frac{n!\,\Gamma(\alpha+1)}{\Gamma(n+\alpha+1)} L_n^{(\alpha)}(x) L_n^{(\alpha)}(y) t^n = \frac{1}{(1-t)^{\alpha+1}} e^{-(x+y)t/(1-t)} \, {}_0F_1\left(;\alpha+1;\frac{xyt}{(1-t)^2}\right)$
LaTeX	`\sum_{n=0}^\infty \frac{n!\,\Gamma\left(\alpha + 1\right)}{\Gamma\left(n+\alpha+1\right)}L_n^{(\alpha)}(x)L_n^{(\alpha)}(y)t^n=\frac{1}{(1-t)^{\alpha + 1}}e^{-(x+y)t/(1-t)}\,_0F_1\left(;\alpha + 1;\frac{xyt}{(1-t)^2}\right)`
Semantic LaTeX	`\sum_{n=0}^\infty \frac{n! \EulerGamma@{\alpha + 1}}{\EulerGamma@{n + \alpha + 1}} \LaguerrepolyL[\alpha]{n}@{x} \LaguerrepolyL[\alpha]{n}@{x} t^n = \frac{1}{(1-t)^{\alpha + 1}} \expe^{-(x+y)t/(1-t)} \genhyperF{0}{1}@{}{\alpha + 1}{\frac{xyt}{(1-t)^2}}`
Mathematica	`Sum[Divide[(n)!*Gamma[\[Alpha]+ 1],Gamma[n + \[Alpha]+ 1]]*LaguerreL[n, \[Alpha], x]*LaguerreL[n, \[Alpha], x]*(t)^(n), {n, 0, Infinity}] == Divide[1,(1 - t)^(\[Alpha]+ 1)]*Exp[-(x + y)*t/(1 - t)]*HypergeometricPFQ[{}, {\[Alpha]+ 1}, Divide[x*y*t,(1 - t)^(2)]]`
Maple	`sum((factorial(n)*GAMMA(alpha + 1))/(GAMMA(n + alpha + 1))*LaguerreL(n, alpha, x)*LaguerreL(n, alpha, x)*(t)^(n), n = 0..infinity) = (1)/((1 - t)^(alpha + 1))*exp(-(x + y)*t/(1 - t))*hypergeom([], [alpha + 1], (x*y*t)/((1 - t)^(2)))`

31: Laguerre polynomials

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-

Explanation No infos about the Gamma function in the context.

Formula	$c_{lm} = (-1)^m \frac{(\ell-m)!}{(\ell+m)!}$
LaTeX	`c_{lm} = (-1)^m \frac{(\ell-m)!}{(\ell+m)!}`
Semantic LaTeX	`c_{lm} = (-1)^m \frac{(\ell-m)!}{(\ell+m)!}`
Mathematica	`Subscript[c, l, m] == (- 1)^(m)*Divide[(\[ScriptL]- m)!,(\[ScriptL]+ m)!]`
Maple	`c[l, m] = (- 1)^(m)*(factorial(ell - m))/(factorial(ell + m))`

32: Associated Legendre polynomials

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\mathrm{Gi}(x) = \frac{1}{\pi}\int_0^\infty \sin\left(\frac{t^3}{3} + xt\right)\,dt$ **33:** Scorer's function
LaTeX	`\mathrm{Gi}(x) = \frac{1}{\pi} \int_0^\infty \sin\left(\frac{t^3}{3} + xt\right)\, dt`
Semantic LaTeX	`\ScorerGi@{x} = \frac{1}{\cpi} \int_0^\infty \sin(\frac{t^3}{3} + xt) \diff{t}`
Mathematica	`ScorerGi[x] == Divide[1,Pi]*Integrate[Sin[Divide[(t)^(3),3]+ x*t], {t, 0, Infinity}]`
Maple	`AiryBi(x)*(int(AiryAi(t), t = (x) .. infinity))+AiryAi(x)*(int(AiryBi(t), t = 0 .. (x))) = (1)/(Pi)*int(sin(((t)^(3))/(3)+ x*t), t = 0..infinity)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\frac{\partial^2}{\partial x^2}V(x;\sigma,\gamma) = \frac{x^2-\gamma^2-\sigma^2}{\sigma^4}\frac{\mathrm{Re}[w(z)]}{\sigma\sqrt{2\pi}} - \frac{2x\gamma}{\sigma^4}\frac{\mathrm{Im}[w(z)]}{\sigma\sqrt{2\pi}} + \frac{\gamma}{\sigma^4}\frac{1}{\pi}$ **34:** Voigt profile
LaTeX	`\frac{\partial^2}{\partial x^2} V(x;\sigma,\gamma)= \frac{x^2-\gamma^2-\sigma^2}{\sigma^4} \frac{\operatorname{Re}[w(z)]}{\sigma\sqrt{2 \pi}}-\frac{2 x \gamma}{\sigma^4} \frac{\operatorname{Im}[w(z)]}{\sigma\sqrt{2 \pi}}+\frac{\gamma}{\sigma^4}\frac{1}{\pi}`
Semantic LaTeX	`\deriv[2]{}{x} V(x ; \sigma , \gamma) = \frac{x^2-\gamma^2-\sigma^2}{\sigma^4} \frac{\realpart [\Faddeevaw@{z}]}{\sigma \sqrt{2 \cpi}} - \frac{2 x \gamma}{\sigma^4} \frac{\imagpart [\Faddeevaw@{z}]}{\sigma \sqrt{2 \cpi}} + \frac{\gamma}{\sigma^4} \frac{1}{\cpi}`
Mathematica	`D[PDF[VoigtDistribution[\[Gamma], \[Sigma]], x], {x, 2}] == Divide[x^2 - \[Gamma]^2 - \[Sigma]^2, \[Sigma]^4] * Divide[Re[Exp[-(Divide[x+I*y,\[Sigma]*Sqrt[2]])^2]*Erfc[-I*(Divide[x+I*y,\[Sigma]*Sqrt[2]])]], \[Sigma]*Sqrt[2*Pi]] - Divide[2*x*y, \[Sigma]^4] * Divide[Im[Exp[-(Divide[x+I*y,\[Sigma]*Sqrt[2]])^2]*Erfc[-I*(Divide[x+I*y,\[Sigma]*Sqrt[2]])]], \[Sigma]*Sqrt[2*Pi]] + Divide[\[Gamma],\[Sigma]^4]*Divide[1,Pi]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	-	-	✘

Explanation The Voigt profile does not exist in the DLMF. Hence, no translatable macro exists for $V(x;\sigma,\gamma)$.

Formula	$\Phi(z,s,a) = \frac{1}{1-z}\frac{1}{a^s} + \sum_{n=1}^{N-1} \frac{(-1)^n \mathrm{Li}_{-n}(z)}{n!}\frac{(s)_n}{a^{n+s}} + O(a^{-N-s})$ **35:** Lerch zeta function
LaTeX	`\Phi(z,s,a) = \frac{1}{1-z} \frac{1}{a^{s}} + \sum_{n=1}^{N-1} \frac{(-1)^{n} \mathrm{Li}_{-n}(z)}{n!} \frac{(s)_{n}}{a^{n+s}} +O(a^{-N-s})`
Semantic LaTeX	`\Phi(z , s , a) = \frac{1}{1-z} \frac{1}{a^{s}} + \sum_{n=1}^{N-1} \frac{(-1)^{n} \polylog{-n}@{z}}{n!} \frac{\Pochhammersym{s}{n}}{a^{n+s}} + \bigO{a^{-N-s}}`
Mathematica	`\[CapitalPhi][z, s, a] == Divide[1,1 - z]*Divide[1,(a)^(s)]+ Sum[Divide[(-1)^(n)* PolyLog[-n, z],(n)!]*Divide[Pochhammer[s, n],(a)^(n + s)], {n, 1, N - 1}]+ O[a]^(- N - s)`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	-	✘
Explanation	Landau notations cannot be translated.					

Formula	$M(1,2,z) = (e^z-1)/z,$ $M(1,3,z) = 2!(e^z-1-z)/z^2$ **36:** Confluent hypergeometric function
LaTeX	`M(1,2,z)=(e^z-1)/z,\ \ M(1,3,z)=2!(e^z-1-z)/z^2`
Semantic LaTeX	`\KummerconfhyperM@{1}{2}{z} = (\expe^z - 1) / z , \KummerconfhyperM@{1}{3}{z} = 2! (\expe^z - 1 - z) / z^2`
Mathematica	`Hypergeometric1F1[1, 2, z] == (Exp[z]- 1)/z Hypergeometric1F1[1, 3, z] == (2)!*(Exp[z]- 1 - z)/(z)^(2)`
Maple	`KummerM(1, 2, z) = (exp(z)- 1)/z; KummerM(1, 3, z) = factorial(2)*(exp(z)- 1 - z)/(z)^(2)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-
Explanation	—					

Formula	$\sigma = \pm 1$ **37:** Mathieu function
LaTeX	`\sigma = \pm 1`
Semantic LaTeX	`\sigma = \pm 1`
Mathematica	`\[Sigma] == \[PlusMinus]1`
Maple	`sigma = &+- 1`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\frac{d^2 f}{dz^2} + \left(\tilde{a} z^2 + \tilde{b} z + \tilde{c}\right) f = 0$ **38:** Parabolic cylinder function
LaTeX	`\frac{d^2f}{dz^2} + \left(\tilde{a}z^2+\tilde{b}z+\tilde{c}\right)f=0`
Semantic LaTeX	`\deriv [2]{f}{z} +(\tilde{a} z^2 + \tilde{b} z + \tilde{c}) f = 0`
Mathematica	`D[f[z], {z, 2}] + (a*z^2 + b*z + c)*f[z] == 0`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	-	✘	-

Explanation ODEs are currently not translatable.
In addition, f hides the argument $f(z)$.

Formula	$c = \infty$ **39:** Painlevé transcendents
LaTeX	`c=\infty`
Semantic LaTeX	`c=\infty`
Mathematica	`c == Infinity`
Maple	`c = infinity`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$c = a + 1$ **40:** Hypergeometric function
LaTeX	`c = a + 1`
Semantic LaTeX	`c = a + 1`
Mathematica	`c == a + 1`
Maple	`c := a + 1`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\frac{1}{\Gamma(z)} = ze^{\gamma z}\prod_{k=1}^{\infty}\left\{\left(1+\frac{z}{k}\right)e^{-z/k}\right\}$ **41:** Barnes G-function
LaTeX	`\frac{1}{\Gamma(z)}= z e^{\gamma z} \prod_{k=1}^\infty \left\{ \left(1+\frac{z}{k}\right)e^{-z/k} \right\}`
Semantic LaTeX	`\frac{1}{\EulerGamma@{z}} = z \expe^{\EulerConstant z} \prod_{k=1}^\infty \{(1 + \frac{z}{k}) \expe^{-z/k} \}`
Mathematica	`Divide[1,Gamma[z]] == z*Exp[EulerGamma*z]*Product[(1 +Divide[z,k])*Exp[- z/k], {k, 1, Infinity}]`
Maple	`(1)/(GAMMA(z)) = z*exp(gamma*z)*product((1 +(z)/(k))*exp(- z/k), k = 1..infinity)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$192/24 = 8 = 2 \times 4$ **42:** Heun function
LaTeX	`192/24 = 8 = 2 \times 4`
Semantic LaTeX	`192/24 = 8 = 2 \times 4`
Mathematica	`192/24 == 8 == 2 * 4`
Maple	`192/24 = 8 = 2 * 4`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$= 2$	**43:** Gegenbauer polynomials
LaTeX	`=2`	
Semantic LaTeX	`\alpha = 2`	
Mathematica	`\[Alpha] = 2`	
Maple	`alpha = 2`	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	-	-
Explanation	The formula was not properly detected in a caption text of a figure because $\alpha = 2$ was given in plain text (UTF-8 encoded).					

Formula	$\lim_{q\to 1} {}_j\phi_k \left[\begin{matrix} q^{a_1} & q^{a_2} & \dots & q^{a_j} \\ q^{b_1} & q^{b_2} & \dots & q^{b_k} \end{matrix} ; q, (q-1)^{1+k-j} z \right] = {}_jF_k \left[\begin{matrix} a_1 & a_2 & \dots & a_j \\ b_1 & b_2 & \dots & b_k \end{matrix} ; z \right]$	**44:** Basic hypergeometric series
LaTeX	`\lim_{q\to 1}\;_{j}\phi_k \left[\begin{matrix} q^{a_1} & q^{a_2} & \ldots & q^{a_j} \\ q^{b_1} & q^{b_2} & \ldots & q^{b_k} \end{matrix} ; q,(q-1)^{1+k-j} z \right]=\;_{j}F_k \left[\begin{matrix} a_1 & a_2 & \ldots & a_j \\ b_1 & b_2 & \ldots & b_k \end{matrix} ;z \right]`	
Semantic LaTeX	`\lim_{q\to 1} \qgenhyperphi{j}{k}@{q^{a_1} , q^{a_2} , \ldots , q^{a_j}}{q^{b_1} , q^{b_2} , \ldots , q^{b_k}}{q}{(q - 1)^{1+k-j} z} = \genhyperF{j}{k}@{a_1 , a_2 , \ldots , a_j}{b_1 , b_2 , \ldots , b_k}{z}`	
Mathematica		
Maple		

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	✘
Explanation	An indefinite length of arguments is not translatable.					

Formula	$\frac{d^2w}{dz^2}+\left(-\frac{1}{4}+\frac{\kappa}{z}+\frac{1/4-\mu^2}{z^2}\right)w=0$ **45:** Whittaker function
LaTeX	`\frac{d^2w}{dz^2}+\left(-\frac{1}{4}+\frac{\kappa}{z}+\frac{1/4-\mu^2}{z^2}\right)w=0`
Semantic LaTeX	`\deriv [2]{w}{z} +(- \frac{1}{4} + \frac{\kappa}{z} + \frac{1/4-\mu^2}{z^2}) w = 0`
Mathematica	`D[w, {z, 2}]+(-Divide[1,4]+Divide[\[Kappa],z]+Divide[1/4 - \[Mu]^(2),(z)^(2)])*w == 0`
Maple	`diff(w, [z$(2)])+(-(1)/(4)+(kappa)/(z)+(1/4 - (mu)^(2))/((z)^(2)))*w = 0`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Unttranslatable
✔	✔	-	-	-	-	-
Explanation	—					

Formula	$e_1=\frac{1}{2},\ e_2=0,\ e_3=-\frac{1}{2}$ **46:** Lemniscatic elliptic function
LaTeX	`e_1=\tfrac12,\qquad e_2=0,\qquad e_3=-\tfrac12`
Semantic LaTeX	`e_1=\tfrac12,\qquad e_2=0,\qquad e_3=-\tfrac12`
Mathematica	`Subscript[e, 1] == Divide[1,2] Subscript[e, 2] = 0 Subscript[e, 3] = -Divide[1,2]`
Maple	`e[1] := (1)/(2); e[2] := 0; e[3] := -(1)/(2)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	-	-	-
Explanation	A bug in LaCAST causes issues when handling multiple equations in one formula. Here, only the first equation was properly translated. The other two were ignored by LaCAST.					

Formula	$\gamma > 0, n - p = m - q > 0$ **47:** Meijer G-function
LaTeX	`\gamma> 0,n-p=m-q> 0`
Semantic LaTeX	`\gamma> 0,n-p=m-q> 0`
Mathematica	`\[Gamma] > 0 n - p == m - q > 0`
Maple	`gamma > 0; n - p = m - q > 0`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\begin{pmatrix} j & \\ m & m' \end{pmatrix} := \sqrt{2j+1} \begin{pmatrix} j & 0 & j \\ m & 0 & m' \end{pmatrix} = (-1)^{j-m'} \delta_{m,-m'}$ **48:** 3-j symbol
LaTeX	`\begin{pmatrix} j \\ m \quad m'\end{pmatrix}:= \sqrt{2 j + 1}\begin{pmatrix} j & 0 & j \\ m & 0 & m'\end{pmatrix}= (-1)^{j - m'} \delta_{m, -m'}`
Semantic LaTeX	`\begin{pmatrix} j \\ m \quad m'\end{pmatrix}:= \sqrt{2 j + 1}\begin{pmatrix} j & 0 & j \\ m & 0 & m'\end{pmatrix}= (-1)^{j - m'} \delta_{m, -m'}`
Mathematica	`Wigner[j_, m_, m\[Prime]_] := Sqrt[2*j+1] * {{j, 0, j}, {m, 0, m\[Prime]}} = (-1)^(j-m\[Prime])*Subscript[\[Delta], m, -m\[Prime]]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	-	-	✘

Explanation LaCaST does not support matrix translations yet.

Formula	$\begin{Bmatrix} i & j & \ell \\ k & m & n \end{Bmatrix} = (\Phi_{i,j}^{k,m})_{\ell,n}$ **49:** 6-j symbol
LaTeX	`\begin{Bmatrix} i & j & \ell\\ k & m & n \end{Bmatrix}= (\Phi_{i,j}^{k,m})_{\ell,n}`
Semantic LaTeX	`\Wignersixjsym{i}{j}{\ell}{k}{m}{n} = (\Phi_{i,j}^{k,m})_{\ell,n}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	✘	-	-	✘	✘

Explanation This is a definition. Definition of matrix functions are not translatable.

Formula	$\sum_{j_7 j_8} (2j_7+1)(2j_8+1) \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3' \\ j_4 & j_5 & j_6' \\ j_7 & j_8 & j_9 \end{Bmatrix} = \frac{\delta_{j_3 j_3'} \delta_{j_6 j_6'} \begin{Bmatrix} j_1 & j_2 & j_3 \end{Bmatrix} \begin{Bmatrix} j_4 & j_5 & j_6 \end{Bmatrix} \begin{Bmatrix} j_3 & j_6 & j_9 \end{Bmatrix}}{(2j_3+1)(2j_6+1)}$ **50:** 9-j symbol
LaTeX	`\sum_{j_7 j_8} (2j_7+1)(2j_8+1) \begin{Bmatrix} j_1 & j_2 & j_3\\ j_4 & j_5 & j_6\\ j_7 & j_8 & j_9 \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3'\\ j_4 & j_5 & j_6'\\ j_7 & j_8 & j_9 \end{Bmatrix} = \frac{\delta_{j_3j_3'}\delta_{j_6j_6'} \begin{Bmatrix} j_1 & j_2 & j_3 \end{Bmatrix} \begin{Bmatrix} j_4 & j_5 & j_6\end{Bmatrix} \begin{Bmatrix} j_3 & j_6 & j_9 \end{Bmatrix}} {(2j_3+1)(2j_6+1)}`
Semantic LaTeX	`\sum_{j_7 j_8} (2j_7+1)(2j_8+1) \Wignerninejsym{j_1}{j_2}{j_3}{j_4}{j_5}{j_6}{j_7}{j_8}{j_9} \Wignerninejsym{j_1}{j_2}{j_3'}{j_4}{j_5}{j_6'}{j_7}{j_8}{j_9} = \frac{\delta_{j_3j_3'}\delta_{j_6j_6'}} \begin{Bmatrix} j_1 & j_2 & j_3 \end{Bmatrix} \begin{Bmatrix} j_4 & j_5 & j_6\end{Bmatrix} \begin{Bmatrix} j_3 & j_6 & j_9 \end{Bmatrix}}{(2j_3+1)(2j_6+1)}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	-

Explanation Mistakenly interpreted as Wigner's $6-j$ rather than Wigner's $9-j$ symbol.

Formula	$\mathcal{K}_k(x;n,q) = \sum_{j=0}^{k}(-q)^j(q-1)^{k-j}\binom{n-j}{k-j}\binom{x}{j}$ **51:** Kravchuk polynomials
LaTeX	`\mathcal{K}_k(x; n,q) = \sum_{j=0}^{k}(-q)^j (q-1)^{k-j} \binom {n-j}{k-j} \binom{x}{j}`
Semantic LaTeX	`\KrawtchoukpolyK{k}@{x}{n}{q} = \sum_{j=0}^{k}(-q)^j (q-1)^{k-j} \binom {n-j}{k-j} \binom{x}{j}`
Mathematica	`K[k_, x_, n_, q_] := Sum[(- q)^(j)*(q - 1)^(k - j)*Binomial[n - j,k - j]* Binomial[x,j], {j, 0, k}]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-
Explanation	Synonym issue. The DLMF spells the Mathematician *Krawtchouk* but Wikipedia uses *Kravchuk*.					

Formula	$g_1(x) = \sum_{k\geq 1}\frac{\sin(k\pi/4)}{k!(8x)^k}\prod_{l=1}^{k}(2l-1)^2$ **52:** Kelvin functions
LaTeX	`g_1(x) = \sum_{k \geq 1} \frac{\sin(k \pi / 4)}{k! (8x)^k} \prod_{l = 1}^k (2l - 1)^2`
Semantic LaTeX	`g_1(x) = \sum_{k \geq 1} \frac{\sin(k \cpi / 4)}{k! (8x)^k} \prod_{l = 1}^k (2 l - 1)^2`
Mathematica	`Subscript[g, 1][x_] := Sum[Divide[Sin[k*Pi/4],(k)!*(8*x)^(k)]*Product[(2*l - 1)^(2), {l, 1, k}], {k, 1, Infinity}]`
Maple	`g[1] := (x) -> sum((sin(k*Pi/4))/(factorial(k)*(8*x)^(k))*product((2*l - 1)^(2), l = 1..k), k = 1..infinity)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	-
Explanation	We did not detect it as a definition					

Formula	$S_{\mu,\nu}(z) = s_{\mu,\nu}(z) + 2^{\mu-1}\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right)\left(\sin\left[(\mu-\nu)\frac{\pi}{2}\right]J_\nu(z) - \cos\left[(\mu-\nu)\frac{\pi}{2}\right]Y_\nu(z)\right)$	**53:** Lommel function
LaTeX	`S_{\mu,\nu}(z) = s_{\mu,\nu}(z) + 2^{\mu-1} \Gamma\left(\frac{\mu + \nu + 1}{2}\right) \Gamma\left(\frac{\mu - \nu + 1}{2}\right)\left(\sin \left[(\mu - \nu)\frac{\pi}{2}\right] J_\nu(z) - \cos \left[(\mu - \nu)\frac{\pi}{2}\right] Y_\nu(z)\right)`	
Semantic LaTeX	`\LommelS{\mu}{\nu}@{z} = \Lommels{\mu}{\nu}@{z} + 2^{\mu-1} \EulerGamma@{\frac{\mu + \nu + 1}{2}} \EulerGamma@{\frac{\mu - \nu + 1}{2}}(\sin [(\mu - \nu) \frac{\cpi}{2}] \BesselJ{\nu}@{z} - \cos [(\mu - \nu) \frac{\cpi}{2}] \BesselY{\nu}@{z})`	
Mathematica	`S[\[Mu]_, \[Nu]_, z_] := Divide[Pi,2]*(BesselY[\[Nu], z]*Integrate[(x)^\[Mu]* BesselJ[\[Nu], x], {x, 0, z}]- BesselJ[\[Nu], z]*Integrate[(x)^\[Mu]* BesselY[\[Nu], x], {x, 0, z}]) + (2)^(\[Mu]- 1)* Gamma[Divide[\[Mu]+ \[Nu]+ 1,2]]*Gamma[Divide[\[Mu]- \[Nu]+ 1,2]]*(Sin[((\[Mu]- \[Nu])*Divide[Pi,2])*]* BesselJ[\[Nu], z]- Cos[((\[Mu]- \[Nu])*Divide[Pi,2])*]*BesselY[\[Nu], z])`	
Maple	`LommelS1(mu, nu, z) = (Pi)/(2)*(BesselY(nu, z)*int((x)^(mu)* BesselJ(nu, x), x = 0..z)- BesselJ(nu, z)*int((x)^(mu)* BesselY(nu, x), x = 0..z))`	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-

Explanation No information about the Gamma function in the context.

Formula	$\mathbf{H}_\alpha(z) = \frac{z^{\alpha+1}}{2^\alpha \sqrt{\pi}\Gamma\left(\alpha+\frac{3}{2}\right)} \cdot {}_1F_2\left(1, \frac{3}{2}, \alpha+\frac{3}{2}, -\frac{z^2}{4}\right)$ **54:** Struve function
LaTeX	`\mathbf{H}_{\alpha}(z) = \frac{z^{\alpha+1}}{2^{\alpha}\sqrt{\pi} \Gamma \left (\alpha+\tfrac{3}{2} \right )} {}_1F_2 \left (1,\tfrac{3}{2}, \alpha+\tfrac{3}{2},-\tfrac{z^2}{4} \right )`
Semantic LaTeX	`\StruveH{\alpha}@{z} = \frac{z^{\alpha+1}}{2^{\alpha} \sqrt{\cpi} \EulerGamma@{\alpha + \tfrac{3}{2}}} \genhyperF{1}{2}@{1}{\tfrac{3}{2}, \alpha + \tfrac{3}{2}}{- \tfrac{z^2}{4}}`
Mathematica	`StruveH[\[Alpha], z] == Divide[(z)^(\[Alpha]+ 1),(2)^\[Alpha]*Sqrt[Pi]*Gamma[\[Alpha]+Divide[3,2]]]*HypergeometricPFQ[{1}, {Divide[3,2], \[Alpha]+Divide[3,2]}, -Divide[(z)^(2),4]]`
Maple	`StruveH(alpha, z) = ((z)^(alpha + 1))/((2)^(alpha)*sqrt(Pi)*GAMMA(alpha +(3)/(2)))*hypergeom([1], [(3)/(2), alpha +(3)/(2)], -((z)^(2))/(4))`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-

Explanation The arguments of ${}_1F_2$ are split by commas which is wrong (ambiguous) in Wikipedia. One must use semicolon to split the arguments. Hence, our patterns did not match.

Formula	$f(t+p) = f(t)$ **55:** Hill differential equation
LaTeX	`f(t+p) = f(t)`
Semantic LaTeX	`f(t+p) = f(t)`
Mathematica	`f[t + p] == f[t]`
Maple	`f(t + p) = f(t)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$\mathbf{J}_\nu(z) = \cos\frac{\pi\nu}{2}\sum_{k=0}^{\infty}\frac{(-1)^k z^{2k}}{4^k\Gamma\left(k+\frac{\nu}{2}+1\right)\Gamma\left(k-\frac{\nu}{2}+1\right)} + \sin\frac{\pi\nu}{2}\sum_{k=0}^{\infty}\frac{(-1)^k z^{2k+1}}{2^{2k+1}\Gamma\left(k+\frac{\nu}{2}+\frac{3}{2}\right)\Gamma\left(k-\frac{\nu}{2}+\frac{3}{2}\right)}$ **56:** Anger function
LaTeX	`\mathbf{J}_\nu(z)=\cos\frac{\pi\nu}{2}\sum_{k=0}^\infty\frac{(-1)^kz^{2k}}{4^k\Gamma\left(k+\frac{\nu}{2}+1\right)\Gamma\left(k-\frac{\nu}{2}+1\right)}+\sin\frac{\pi\nu}{2}\sum_{k=0}^\infty\frac{(-1)^kz^{2k+1}}{2^{2k+1}\Gamma\left(k+\frac{\nu}{2}+\frac{3}{2}\right)\Gamma\left(k-\frac{\nu}{2}+\frac{3}{2}\right)}`
Semantic LaTeX	`\AngerJ{\nu}@{z}=\cos\frac{\cpi\nu}{2}\sum_{k=0}^\infty\frac{(-1)^k z^{2k}}{4^k\EulerGamma@{k+\frac{\nu}{2}+1}\EulerGamma@{k-\frac{\nu}{2}+1}}+\sin\frac{\cpi\nu}{2}\sum_{k=0}^\infty\frac{(-1)^k z^{2k+1}}{2^{2k+1}\EulerGamma@{k+\frac{\nu}{2}+\frac{3}{2}}\EulerGamma@{k-\frac{\nu}{2}+\frac{3}{2}}}`
Mathematica	`AngerJ[\[Nu],z]==Cos[Divide[Pi*\[Nu],2]]*Sum[Divide[(-1)^(k)*(z)^(2*k),(4)^(k)*Gamma[k+Divide[\[Nu],2]+1]*Gamma[k-Divide[\[Nu],2]+1]],{k,0,Infinity}]+Sin[Divide[Pi*\[Nu],2]]*Sum[Divide[(-1)^(k)*(z)^(2*k+1),(2)^(2*k+1)*Gamma[k+Divide[\[Nu],2]+Divide[3,2]]*Gamma[k-Divide[\[Nu],2]+Divide[3,2]]],{k,0,Infinity}]`
Maple	`AngerJ(nu, z) = cos((Pi*nu)/(2))*sum(((- 1)^(k)* (z)^(2*k))/((4)^(k)* GAMMA(k +(nu)/(2)+ 1)*GAMMA(k -(nu)/(2)+ 1)), k = 0..infinity)+ sin((Pi*nu)/(2))*sum(((- 1)^(k)* (z)^(2*k + 1))/((2)^(2*k + 1)* GAMMA(k +(nu)/(2)+(3)/(2))*GAMMA(k -(nu)/(2)+(3)/(2))), k = 0..infinity)`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-

Explanation No information about the Gamma function in the context.

Formula	$(\mathrm{Ec})'_{2K} = (\mathrm{Ec})'_0 = 0,$ $(\mathrm{Es})'_{2K} = (\mathrm{Es})'_0 = 0$ **57:** Lamé function
LaTeX	`(\operatorname{Ec})^'_{2K} = (\operatorname{Ec})^'_0 = 0, \;\; (\operatorname{Es})^'_{2K} = (\operatorname{Es})^'_0 = 0`
Semantic LaTeX	`(\operatorname{Ec})_{2K}^' =(\operatorname{Ec})_0^' = 0 ,(\operatorname{Es})_{2K}^' =(\operatorname{Es})_0^' = 0`
Mathematica	
Maple	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘

Explanation Not translatable for human annotator.

Formula	$\int_{-\infty}^{+\infty} e^{-x^2} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$ **58:** Gauss–Hermite quadrature
LaTeX	`\int_{-\infty}^{+\infty} e^{-x^2} f(x)\,dx \approx \sum_{i=1}^n w_i f(x_i)`
Semantic LaTeX	`\int_{-\infty}^{+\infty} \expe^{-x^2} f(x) \diff{x} \approx \sum_{i=1}^n w_i f(x_i)`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘

Explanation Not translatable for human annotator.

Formula	$p_n(x;a,b,c,d\vert q) = (ab,ac,ad;q)_n a^{-n}.$ $\quad {}_4\phi_3\left[\begin{matrix} q^{-n} & abcdq^{n-1} & ae^{i\theta} & ae^{-i\theta} \\ ab & ac & ad & \end{matrix}; q, q\right]$ **59:** Askey–Wilson polynomials
LaTeX	`p_n(x;a,b,c,d|q) =(ab,ac,ad;q)_na^{-n}\;_{4}\phi_3 \left[\begin{matrix} q^{-n}&abcdq^{n-1}&ae^{i\theta}&ae^{-i\theta} \\ ab&ac&ad \end{matrix} ; q,q \right]`
Semantic LaTeX	`\AskeyWilsonpolyp{n}@{x}{a}{b}{c}{d}{q} = \qmultiPochhammersym{ab , ac , ad}{q}{n} a^{-n} \qgenhyperphi{4}{3}@{q^{-n} , abcdq^{n-1} , a\expe^{\iunit\theta} , a\expe^{-\iunit\theta}}{ab , ac , ad}{q}{q}`
Mathematica	`p[n_, x_, a_, b_, c_, d_, q_] := Product[QPochhammer[Part[{a*b , a*c , a*d}, i],q,n],{i,1,Length[{a*b , a*c , a*d}]}]*(a)^(- n)* QHypergeometricPFQ[{(q)^(- n), a*b*c*d*(q)^(n - 1), a*Exp[I*\[Theta]], a*Exp[- I*\[Theta]]},{a*b , a*c , a*d},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	-	-

Explanation Unable to retrieve *Askey-Wilson poylnomial* from the context.

Formula	$Q_n(x;\alpha,\beta,N)$ $= {}_3F_2(-n,-x,n+\alpha+\beta+1;$ $\alpha+1,-N+1;1).$ **60:** Hahn polynomials
LaTeX	`Q_n(x;\alpha,\beta,N)= {}_3F_2(-n,-x,n+\alpha+\beta+1;\alpha+1,-N+1;1).`
Semantic LaTeX	`\HahnpolyQ{n}@{x}{\alpha}{\beta}{N} = \genhyperF{3}{2}@{- n , - x , n + \ alpha + \beta + 1}{\alpha + 1 , - N + 1}{1}`
Mathematica	`Q[n_, x_, \[Alpha]_, \[Beta]_, N_] := HypergeometricPFQ[{- n , - x , n + \[ Alpha]+ \[Beta]+ 1}, {\[Alpha]+ 1 , - N + 1}, 1]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	✘
Explanation	Hahn polynomials are not supported by Mathematica and this equation is a definition.					

Formula	$\sum_{x=0}^{\infty} \frac{\mu^x}{x!} C_n(x;\mu)C_m(x;\mu)$ $= \mu^{-n} e^{\mu} n! \delta_{nm},\ \mu > 0$ **61:** Charlier polynomials
LaTeX	`\sum_{x=0}^\infty \frac{\mu^x}{x!} C_n(x; \mu)C_m(x; \mu)=\mu^{-n} e^\mu n! \ delta_{nm}, \quad \mu>0`
Semantic LaTeX	`\sum_{x=0}^\infty \frac{\mu^x}{x!} \CharlierpolyC{n}@{x}{\mu} \CharlierpolyC{ m}@{x}{\mu} = \mu^{-n} \expe^\mu n! \delta_{nm} , \quad \mu > 0`
Mathematica	`Sum[Divide[\[Mu]^x, x!] * HypergeometricPFQ[{-n, -x}, {}, -Divide[1,\[Mu]]] * HypergeometricPFQ[{-m, -x}, {}, -Divide[1,\[Mu]]], {x, 0, Infinity}] == \[ Mu]^(-n)*Exp[\[Mu]]*n!*Subscript[\[Delta], n, m]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-
Explanation	Unable to retrieve *Charlier polynomial* from the context.					

Formula	$p_n(q^{-x}+q^{x+1}cd;a,b,c,d;q) = {}_4\phi_3\left[\begin{matrix} q^{-n} & abq^{n+1} & q^{-x} & q^{x+1}cd \\ aq & bdq & cq & \end{matrix};q;q\right]$ **62:** Q-Racah polynomials
LaTeX	`p_n(q^{-x}+q^{x+1}cd;a,b,c,d;q) = {}_4\phi_3\left[\begin{matrix} q^{-n} &abq^{n+1}&q^{-x}&q^{x+1}cd\\aq&bdq&cq\\ \end{matrix};q;q\right]`
Semantic LaTeX	`\qRacahpolyR{n}@{q^{-x} + q^{x+1} cd}{a}{b}{c}{d}{q} = \qgenhyperphi{4}{3}@{ q^{-n}, abq^{n+1}, q^{-x}, q^{x+1}cd}{aq , bdq , cq}{q}{q}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	-	-	-	✘
Explanation	Since this is not a definition and q-Recah polynomials are not supported by Mathematica, a translation is infeasible.					

Formula	$c_n(q^{-x};a;q) = {}_2\phi_1(q^{-n},q^{-x};0;q,-q^{n+1}/a)$ **63:** Q-Charlier polynomials
LaTeX	`\displaystyle c_n(q^{-x};a;q) = {}_2\phi_1(q^{-n},q^{-x};0;q,-q^{n+1}/a)`
Semantic LaTeX	`c_n(q^{-x} ; a ; q) = \qgenhyperphi{2}{1}@{q^{-n} , q^{-x}}{0}{q}{- q^{n+1} / a}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘
Explanation	Same as 62: q-Charlier polynomials are not supported by Mathematica.					

Formula	$M_n(x,\beta,\gamma) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \binom{x}{k} k!(x+\beta)_{n-k}\gamma^{-k}$ **64:** Meixner polynomials
LaTeX	`M_n(x,\beta,\gamma) = \sum_{k=0}^n (-1)^k{n \choose k}{x\choose k}k!(x+\beta)_{n-k}\gamma^{-k}`
Semantic LaTeX	`\MeixnerpolyM{n}@{x}{\beta}{\gamma} = \sum_{k=0}^n(- 1)^k{n \choose k}{x\choose k} k! \Pochhammersym{x + \beta}{n-k} \gamma^{-k}`
Mathematica	`M[n_, x_, \[Beta]_, \[Gamma]_] := Sum[(- 1)^(k)*Binomial[n,k]*Binomial[x,k]*(k)!*Pochhammer[x + \[Beta], n - k]*\[Gamma]^(- k), {k, 0, n}]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	✘	-

Explanation Unable to retrieve *Meixner polynomial* from the context.

Formula	$x(1-x)\frac{\partial^2 F_1(x,y)}{\partial x^2} + y(1-x)\frac{\partial^2 F_1(x,y)}{\partial x \partial y} + [c-(a+b_1+1)x]\frac{\partial F_1(x,y)}{\partial x} - b_1 y \frac{\partial F_1(x,y)}{\partial y} - ab_1 F_1(x,y) = 0$ **65:** Appell series
LaTeX	`x(1-x)\frac{\partial^2F_1(x,y)}{\partial x^2} + y(1-x)\frac{\partial^2F_1(x,y)}{\partial x\partial y} + [c-(a+b_1+1)x]\frac{\partial F_1(x,y)}{\partial x}-b_1 y\frac{\partial F_1(x,y)}{\partial y}-a b_1 F_1(x,y)=0`
Semantic LaTeX	`x(1-x) \deriv[2]{\AppellF{1}@{a}{b_1}{b_2}{\gamma}{x}{y}}{x} + y(1-x) \frac{\pdiff[2]{\AppellF{1}@{a}{b_1}{b_2}{\gamma}{x}{y}}}{\pdiff{x}\pdiff{y}} + [c - (a+b_1+1) x] \deriv[1]{\AppellF{1}@{a}{b_1}{b_2}{\gamma}{x}{y}}{x} - b_1 y \deriv[1]{\AppellF{1}@{a}{b_1}{b_2}{\gamma}{x}{y}}{y} - a b_1 \AppellF{1}@{a}{b_1}{b_2}{\gamma}{x}{y} = 0`
Mathematica	`x*(1-x) * D[AppellF[a, Subscript[b, 1], Subscript[b, 2], \[Gamma], x, y], {x,2}] + y*(1-x) * D[AppellF[a, Subscript[b, 1], Subscript[b, 2], \[Gamma], x, y], x, y] + (c - (a+Subscript[b, 1]+1)*x) * D[AppellF[a, Subscript[b, 1], Subscript[b, 2], \[Gamma], x, y], x] - Subscript[b,1] * y * D[AppellF[a, Subscript[b, 1], Subscript[b, 2], \[Gamma], x, y], y] - a*Subscript[b,1]* AppellF[a, Subscript[b, 1], Subscript[b, 2], \[Gamma], x, y] == 0`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	✘	-

Explanation We cannot match hidden arguments of Appell's F_1 function.

Formula	$\Theta_\Lambda(\tau) = \sum_{x\in\Lambda} e^{i\pi\tau\|x\|^2}\ \mathrm{Im}\,\tau > 0$
LaTeX	`\Theta_\Lambda(\tau) = \sum_{x\in\Lambda}e^{i\pi\tau|x|^2}\qquad\mathrm{Im}\,\tau > 0`
Semantic LaTeX	`\Theta_\Lambda(\tau) = \sum_{x\in\Lambda} \expe^{\iunit \cpi \tau |x |^2} \qquad \imagpart \tau > 0`
Mathematica	`\[CapitalTheta][\[CapitalLambda]_, \[Tau]_] := Sum[Exp[I*Pi*\[Tau]*(Norm[x])^(2)], {x, \[CapitalLambda]}]`
Maple	

66: Theta function of a lattice

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	-	-

Explanation Unable to detect equation as a definition.

Formula	$\frac{d^2S}{dz^2} + \left(\sum_{j=1}^{N} \frac{\gamma_j}{z-a_j}\right)\frac{dS}{dz} + \frac{V(z)}{\prod_{j=1}^{N}(z-a_j)}S = 0$
LaTeX	`\frac{d^2 S}{dz^2}+\left(\sum _{j=1}^N \frac{\gamma _j}{z - a_j} \right) \frac{dS}{dz} + \frac{V(z)}{\prod _{j=1}^N (z - a_j)}S = 0`
Semantic LaTeX	`\deriv [2]{S}{z} +(\sum_{j=1}^N \frac{\gamma _j}{z - a_j}) \deriv[]{S}{z} + \frac{V(z)}{\prod _{j=1}^N (z - a_j)} S = 0`
Mathematica	
Maple	

67: Heine–Stieltjes polynomials

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	-

Explanation Mistakenly detected Stieltjes constant.
There is no translation available for Heine-Stieltjes polynomials.

Formula	$w(x) = \frac{k}{\sqrt{\pi}} x^{-1/2} \exp(-k^2 \log^2 x)$
LaTeX	`w(x) = \frac{k}{\sqrt{\pi}} x^{-1/2} \exp(-k^2\log^2 x)`
Semantic LaTeX	`w(x) = \frac{k}{\sqrt{\cpi}} x^{-1/2} \exp(- k^2 \log^2 x)`
Mathematica	`w[x_] := Divide[k,Sqrt[Pi]]*(x)^(- 1/2)* Exp[- (k)^(2)* (Log[x])^(2)]`
Maple	`w := (x) -> (k)/(sqrt(Pi))*(x)^(- 1/2)* exp(- (k)^(2)* (log(x))^(2))`

68: Stieltjes–Wigert polynomials

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	-

Explanation Unable to detect equation as a definition.

Formula	$y^2 = x(x-1)(x-\lambda)$
LaTeX	`y^2=x(x-1)(x-\lambda)`
Semantic LaTeX	`y^2=x(x-1)(x-\lambda)`
Mathematica	`(y)^(2) == x*(x - 1)*(x - \[Lambda])`
Maple	`(y)^(2) = x*(x - 1)*(x - lambda)`

69: Modular lambda function

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$P_1^{(\lambda)}(x;\phi) = 2(\lambda \cos\phi + x \sin\phi)$
LaTeX	`P_1^{(\lambda)}(x;\phi)=2(\lambda\cos\phi + x\sin\phi)`
Semantic LaTeX	`\MeixnerPollaczekpolyP{\lambda}{1}@{x}{\phi} = 2(\lambda \cos \phi + x \sin \phi)`
Mathematica	
Maple	

70: Meixner–Pollaczek polynomials

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘

Explanation Meixner-Pollaczek polynomials are not supported by Mathematica.

Formula	$P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!} * {}_2F_1\left(-n, 1+\alpha+\beta+n; \alpha+1; \frac{1}{2}(1-z)\right)$ **71:** Jacobi polynomials
LaTeX	`P_n^{(\alpha,\beta)}(z)=\frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n,1+\alpha+\beta+n;\alpha+1;\tfrac{1}{2}(1-z)\right)`
Semantic LaTeX	`\JacobipolyP{\alpha}{\beta}{n}@{z} = \frac{\Pochhammersym{\alpha + 1}{n}}{n!} \genhyperF{2}{1}@{- n , 1 + \alpha + \beta + n}{\alpha + 1}{\tfrac{1}{2}(1 - z)}`
Mathematica	`JacobiP[n, \[Alpha], \[Beta], z] == Divide[Pochhammer[\[Alpha]+ 1, n],(n)!]* HypergeometricPFQ[{- n , 1 + \[Alpha]+ \[Beta]+ n}, {\[Alpha]+ 1}, Divide [1,2]*(1 - z)]`
Maple	`JacobiP(n, alpha, beta, z) = (pochhammer(alpha + 1, n))/(factorial(n))* hypergeom([- n , 1 + alpha + beta + n], [alpha + 1], (1)/(2)*(1 - z))`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✔	-	-	-	-	-

Explanation —

Formula	$S_n(x^2; a, b, c) = {}_3F_2(-n, a+ix, a-ix; a+b, a+c; 1).$ **72:** Continuous dual Hahn polynomials
LaTeX	`S_n(x^2;a,b,c)= {}_3F_2(-n,a+ix,a-ix;a+b,a+c;1).`
Semantic LaTeX	`\contdualHahnpolyS{n}@{x^2}{a}{b}{c} = \genhyperF{3}{2}@{- n , a + \iunit x , a - \iunit x}{a + b , a + c}{1}`
Mathematica	
Maple	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘

Explanation Continuous dual Hahn polynomials are not supported by Mathematica.

Formula	$P_n^{(\alpha,\beta)} = \lim_{t\to\infty} t^{-n} p_n\left(\frac{1}{2}xt; \frac{1}{2}(\alpha+1-it), \frac{1}{2}(\beta+1+it), \frac{1}{2}(\alpha+1+it), \frac{1}{2}(\beta+1-it)\right)$ **73:** Continuous Hahn polynomials
LaTeX	`P_n^{(\alpha,\beta)}=\lim_{t\to\infty}t^{-n}p_n\left(\tfrac12xt; \tfrac12(\alpha+1-it), \tfrac12(\beta+1+it), \tfrac12(\alpha+1+it), \tfrac12(\beta+1-it)\right)`
Semantic LaTeX	`\JacobipolyP{\alpha}{\beta}{n}@{x} = \lim_{t\to\infty} t^{-n} \contHahnpolyp{n}@{\tfrac12 xt}{\tfrac12(\alpha + 1 - \iunit t)}{\tfrac12(\beta + 1 + \iunit t)}{\tfrac12(\alpha + 1 + \iunit t)}{\tfrac12(\beta + 1 - \iunit t)}`
Mathematica	`JacobiP[n, \[Alpha], \[Beta], x] == Limit[(t)^(- n)* I^(n)*Divide[Pochhammer[Divide[1,2]*(\[Alpha]+ 1 - I*t) + Divide[1,2]*(\[Alpha]+ 1 + I*t), n]* Pochhammer[Divide[1,2]*(\[Alpha]+ 1 - I*t) + Divide[1,2]*(\[Beta]+ 1 - I*t), n], (n)!] * HypergeometricPFQ[{-(n), n + 2*Re[Divide[1,2]*(\[Alpha]+ 1 - I*t) + Divide[1,2]*(\[Beta]+ 1 + I*t)] - 1, Divide[1,2]*(\[Alpha]+ 1 - I*t) + I*(Divide[1,2]*x*t)}, {Divide[1,2]*(\[Alpha]+ 1 - I*t) + Divide[1,2]*(\[Alpha]+ 1 + I*t), Divide[1,2]*(\[Alpha]+ 1 - I*t) + Divide[1,2]*(\[Beta]+ 1 - I*t)}, 1], t -> Infinity]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	✘	-

Explanation Hidden arguments caused matching issues.

Formula	$\sum_{s=a}^{b-1} w_n^{(c)}(s,a,b) w_m^{(c)}(s,a,b)$ $\rho(s)[\Delta x(s-\frac{1}{2})] = \delta_{nm} d_n^2$ **74:** Dual Hahn polynomials
LaTeX	`\sum^{b-1}_{s=a}w_n^{(c)}(s,a,b)w_m^{(c)}(s,a,b)\rho(s)[\Delta x(s-\frac{1}{2}) ]=\delta_{nm}d_n^2`
Semantic LaTeX	`\sum_{s=a}^{b-1} \dualHahnpolyR{n}@{c}{s}{a}{b} \dualHahnpolyR{m}@{c}{s}{a}{b} \rho(s) [\Delta x(s - \frac{1}{2})] = \delta_{nm} d_n^2`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	✘

Explanation Non-standard notation for dual Hahn polynomials. The DLMF uses R. Further, dual Hahn polynomials are not supported by Mathematica.

Formula	$p_n(x;a,b,c\|q) = a^{-n}e^{-inu}(abe^{2iu},ac,ad;q)_n \, {}_{*4}\Phi_3\left(q^{-n}, abcdq^{n-1}, ae^{i(t+2u)}, ae^{-it}; abe^{2iu}, ac, ad; q; q\right)$ **75:** Continuous q-Hahn polynomials
LaTeX	`p_n(x;a,b,c|q)=a^{-n}e^{-inu}(abe^{2iu},ac,ad;q)_n*_4\Phi_3(q^{-n},abcdq^{n-1},ae^{i{(t+2u)}},ae^{-it};abe^{2iu},ac,ad;q;q)`
Semantic LaTeX	`p_n(x ; a , b , c|q) = a^{-n} \expe^{-\iunit nu} \qmultiPochhammersym{ab\ expe^{2\iunit u} , ac , ad}{q}{n} * \qgenhyperphi{4}{3}@{q^{-n} , abcdq^{n-1} , a\expe^{\iunit{(t+2u)}} , a\expe^{-\iunit t}}{ab\expe^{2\iunit u} , ac , ad}{q}{q}`
Mathematica	`p[n_, x_, a_, b_, c_, q_] := (a)^(- n)* Exp[- I*\[Nu]]*Product[QPochhammer[ Part[{a*b*Exp[2*I*u], a*c , a*d},i],q,n],{i,1,Length[{a*b*Exp[2*I*u], a*c , a*d}]}]* QHypergeometricPFQ[{(q)^(- n), a*b*c*d*(q)^(n - 1), a*Exp[I*(t + 2* u)], a*Exp[- I*t]},{a*b*Exp[2*I*u], a*c , a*d},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-
Explanation	Erroneous notation in Wikipedia. The asterix $*$ had an index $*_4$ which is supposed to be part of ${}_4\Phi_3$.					

Formula	$p_n(x;a,b,c \mid q) = \frac{(ab,ac;q)_n}{a^n} \cdot {}_3\Phi_2(q^{-}n, ae^{i\theta}, ae^{-i\theta}; ab, ac \mid q; q)$ **76:** Continuous dual q-Hahn polynomials
LaTeX	`p_n(x;a,b,c\mid q)=\frac{(ab,ac;q)_n}{a^n}\cdot {_3\Phi_2}(q^-n,ae^{i\theta}, ae^{-i\theta}; ab, ac \mid q;q)`
Semantic LaTeX	`p_n(x ; a , b , c \mid q) = \frac{\qmultiPochhammersym{ab , ac}{q}{n}}{a^n} \cdot \qgenhyperphi{3}{2}@{q^{- n} , a\expe^{\iunit \theta} , a\expe^{- \iunit \theta}}{ab , ac}{q}{q}`
Mathematica	`p[n_, x_, a_, b_, c_, q_] := Divide[Product[QPochhammer[Part[{a*b , a*c},i], q,n],{i,1,Length[{a*b , a*c}]}],(a)^(n)] * QHypergeometricPFQ[{(q)^(- n), a* Exp[I*\[Theta]], a*Exp[- I*\[Theta]]},{a*b , a*c},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-
Explanation	LaCaST is unable to parse a subscript without base `_3`. Additionally, the formula is wrong in Wikipedia because of missing curly brackets in: `q^-n`.					

Formula	$Q_n(x;a,b,N;q) = {}_3\phi_2\left[\begin{matrix} q^{-}n & abq^n+1 & x \\ aq & q^{-}N & \end{matrix};q,q\right]$ **77:** Q-Hahn polynomials
LaTeX	`Q_n(x;a,b,N;q)=\;_{3}\phi_2\left[\begin{matrix} q^-n & abq^n+1 & x \\ aq & q^-N \end{matrix} ; q,q \right]`
Semantic LaTeX	`\qHahnpolyQ{n}@{x}{a}{b}{N}{q} = \qgenhyperphi{3}{2}@{q^-n , abq^n+1 , x}{aq , q^-N}{q}{q}`
Mathematica	`Q[n_, x_, a_, b_, N_, q_] := QHypergeometricPFQ[{(q)^(-)* n , a*b*(q)^(n)+ 1 , x},{a*q , (q)^(-)* N},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	-	-	-	-
Explanation	Unable to detect *Q-Hahn polynomials* in the context					

Formula	$x =$ **78:** Al-Salam–Chihara polynomials
LaTeX	`x=`
Semantic LaTeX	`x=\cos@{\theta}`
Mathematica	`x = Cos[\[Theta]]`
Maple	`x = cos(theta)`

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	-	-	-	-
Explanation	Wrong math detection because `cos` was given in plain text: `"x" = cos(θ)`.					

Formula	$\Phi_n^*(z) = z^n\overline{\Phi_n(1/\overline{z})}$ **79:** Orthogonal polynomials on the unit circle
LaTeX	`\Phi_n^*(z)=z^n\overline{\Phi_n(1/\overline{z})}`
Semantic LaTeX	`\Phi_n^*(z) = z^n{\conj{\Phi_n(1 / \conj{z})}}`
Mathematica	`\[CapitalPhi]\[Prima][n_, z_] := z^n*Conjugate[\[CapitalPhi][n, Divide[1, Conjugate[z]]]]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-
Explanation	A bug in LaCaST prevented us to detect the nested overline as conjugation. Further, its an undetected definition.					

Formula	$P_n(x) = c_n \det \begin{bmatrix} m_0 & m_1 & m_2 & \cdots & m_n \\ m_1 & m_2 & m_3 & \cdots & m_{n+1} \\ & & \vdots & & \vdots \\ m_{n-1} & m_n & m_{n+1} & \cdots & m_{2n-1} \\ 1 & x & x^2 & \cdots & x^n \end{bmatrix}$ **80:** Orthogonal polynomials
LaTeX	`P_n(x) = c_n \, \det \begin{bmatrix}m_0 & m_1 & m_2 &\cdots & m_n \\m_1 & m_2 & m_3 &\cdots & m_{n+1} \\&&\vdots&& \vdots \\m_{n-1} &m_n& m_{n+1} &\cdots &m_{2n-1}\\1 & x & x^2 & \cdots & x^n\end{bmatrix}`
Semantic LaTeX	`P_n(x) = c_n \det \begin{bmatrix}m_0 & m_1 & m_2 &\cdots & m_n \\m_1 & m_2 & m_3 &\cdots & m_{n+1} \\&&\vdots&& \vdots \\m_{n-1} &m_n& m_{n+1} &\cdots &m_{2n-1}\\1 & x & x^2 & \cdots & x^n\end{bmatrix}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	-	✘
Explanation	Indefinite number of arguments cannot be translated.					

Formula	$p_n(x;a,b;q) = {}_2\phi_1(q^{-n}, abq^{n+1}; aq; q, xq)$ **81:** Little q-Jacobi polynomials
LaTeX	`\displaystyle p_n(x;a,b;q) = {}_2\phi_1(q^{-n},abq^{n+1};aq;q,xq)`
Semantic LaTeX	`\littleqJacobipolyp{n}@{x}{a}{b}{q} = \qgenhyperphi{2}{1}@{q^{-n} , abq^{n+1}}{aq}{q}{xq}`
Mathematica	`p[n_, x_, a_, b_, q_] := QHypergeometricPFQ[{(q)^(- n), a*b*(q)^(n + 1)},{a*q},q,x*q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	✘
Explanation	No translation possible for little q-Jacobi polynomials.					

Formula	$P_n(x;a,b,c;q) = {}_3\phi_2(q^{-n}, abq^{n+1}, x; aq, cq; q, q)$ **82:** Big q-Jacobi polynomials
LaTeX	`\displaystyle P_n(x;a,b,c;q)={}_3\phi_2(q^{-n},abq^{n+1},x;aq,cq;q,q)`
Semantic LaTeX	`\bigqJacobipolyP{n}@{x}{a}{b}{c}{q} = \qgenhyperphi{3}{2}@{q^{-n} , abq^{n+1} , x}{aq , cq}{q}{q}`
Mathematica	`p[n_, x_, a_, b_, c_, q_] := QHypergeometricPFQ[{(q)^(- n), a*b*(q)^(n + 1), x},{a*q , c*q},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	✘
Explanation	No translation possible for big q-Jacobi polynomials.					

Formula	$P_n(x;a,b;q) = \frac{1}{(b^{-1} * q^{-n};q,n)} *_2 \Phi_1(q^{-n}, aqx^{-1}; aq \mid q; \frac{x}{b})$
LaTeX	`P_n(x;a,b;q)=\frac{1}{(b^{-1}*q^{-n};q,n)}*_2\Phi_1(q^{-n},aqx^{-1};aq|q;\frac{x}{b})`
Semantic LaTeX	`P_n(x;a,b;q) =\frac{1}{\qmultiPochhammersym{b^{-1}*q^{-n}}{q}{n}} * \qgenhyperphi{2}{1}@{q^{-n},aqx^{-1}}{aq}{q}{\frac{x}{b}}`
Mathematica	`P[n_, x_, a_, b_, q_] := Divide[1,Product[QPochhammer[Part[{(b)^(- 1)* (q)^(- n)},i],q,n],{i,1,Length[{(b)^(- 1)* (q)^(- n)}]}]]* QHypergeometricPFQ[{(q)^(- n), a*q*(x)^(- 1)},{a*q},q,Divide[x,b]]`
Maple	

83: Big q-Laguerre polynomials

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	-	-

Explanation Invalid notation in which the asterix $*$ has an index $*_2$.

Formula	$K_n(\lambda(x);c,N \mid q) =_3 \Phi_2(q^{-n}, q^{-x}, cq^{x-N}; q^{-N}, 0 \mid q; q)$
LaTeX	`K_n(\lambda(x);c,N|q)=_3\Phi_2(q^{-n},q^{-x},cq^{x-N};q^{-N},0|q;q)`
Semantic LaTeX	`K_n(\lambda(x);c,N|q) = \qgenhyperphi{3}{2}@{q^{-n},q^{-x},cq^{x-N}}{q^{-N},0}{q}{q}`
Mathematica	
Maple	

84: Dual q-Krawtchouk polynomials

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	-

Explanation Similar to 83: the equal sign has an index $=_3$.

Formula	$P_n^{(\alpha)}(x\|q) = \frac{(q^\alpha+1;q)_n}{(q;q)_n}$	**85:** Continuous q-Laguerre polynomials
LaTeX	`P_{n}^{(\alpha)}(x|q)=\frac{(q^\alpha+1;q)_{n}}{(q;q)_{n}}`	
Semantic LaTeX	`P_{n}^{(\alpha)}(x|q) = \frac{\qmultiPochhammersym{q^\alpha+1}{q}{n}}{\qPochhammer{q}{q}{n}} \qgenhyperphi{3}{2}@{q^{-n},q^{\alpha/2+1/4}\expe^{\iunit\theta},q^{\alpha/2+1/4}*\expe^{-\iunit\theta}}{q^{\alpha+1},0}{q}{q}`	
Mathematica	`P[n_, \[Alpha]_, x_, q_] := Divide[Product[QPochhammer[Part[{(q)^\[Alpha]+ 1},i],q,n],{i,1,Length[{(q)^\[Alpha]+ 1}]}],QPochhammer[q, q, n]]* QHypergeometricPFQ[{(q)^(- n), (q)^(\[Alpha]/2 + 1/4)* Exp[I*\[Theta]], (q)^(\[Alpha]/2 + 1/4)* Exp[- I*\[Theta]]},{(q)^(\[Alpha]+ 1), 0},q,q]`	
Maple		

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-

Explanation Unable to detect q-multi Pochhammer symbol in the equation.

Formula	$p_n(x;a\|q) = {}_2\phi_1(q^{-n},0;aq;q,qx)$ $= \frac{1}{(a^{-1}q^{-n};q)_n}$ ${}_2\phi_0(q^{-n},x^{-1};;q,x/a)$	**86:** Little q-Laguerre polynomials
LaTeX	`\displaystyle p_n(x;a|q) = {}_2\phi_1(q^{-n},0;aq;q,qx) = \frac{1}{(a^{-1}q^{-n};q)_n}{}_2\phi_0(q^{-n},x^{-1};;q,x/a)`	
Semantic LaTeX	`p_n(x ; a|q) = \qgenhyperphi{2}{1}@{q^{-n} , 0}{aq}{q}{qx} = \frac{1}{\qmultiPochhammersym{a^{-1} q^{-n}}{q}{n}} \qgenhyperphi{2}{0}@{q^{-n} , x^{-1}}{}{q}{x/a}`	
Mathematica	`p[n_, x_, a_, q_] := QHypergeometricPFQ[{(q)^(- n), 0},{a*q},q,q*x] == Divide[1,Product[QPochhammer[Part[{(a)^(- 1)* (q)^(- n)},i],q,n],{i,1,Length[{(a)^(- 1)* (q)^(- n)}]}]]*QHypergeometricPFQ[{(q)^(- n), (x)^(- 1)},{},q,x/a]`	
Maple		

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-

Explanation We cannot match empty arguments yet (remains a bug).

Formula	$y_n(x;a;q) = {}_2\phi_1\left(\begin{matrix} q^{-N} & -aq^n \\ 0 & \end{matrix}; q, qx\right)$ **87:** Q-Bessel polynomials
LaTeX	`y_{n}(x;a;q)=\;_{2}\phi_1 \left(\begin{matrix} q^{-N} & -aq^{n} \\ 0 \end{matrix} ; q,qx \right)`
Semantic LaTeX	`y_{n}(x;a;q) = \qgenhyperphi{2}{1}@{q^{-N} , -aq^{n}}{0}{q}{qx}`
Mathematica	`y[n_, x_, a_, q_] := QHypergeometricPFQ[{(q)^(- N), - a*(q)^(n)},{0},q,q*x]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-
Explanation	Again, the equal sign has an index $=_2$.					

Formula	$h_n(ix;q^{-1}) = i^n\hat{h}_n(x;q)$ **88:** Discrete q-Hermite polynomials
LaTeX	`h_n(ix;q^{-1}) = i^n\hat h_n(x;q)`
Semantic LaTeX	`\discqHermitepolyhI{n}@{\iunit x}{q^{-1}} = \iunit^n \discqHermitepolyhII{n}@{x}{q}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	✘	
Explanation	We correctly identified `\discqHermitepolyhI` but were unable to distinguish it from `\discqHermitepolyhII` in the right-hand side. Neither of them were translatable though.					

Formula	$P_n(x;a \mid q) = a^{-n} e^{in\phi} \frac{a^2;q_n}{(q;q)_n} {}_3\Phi_2(q^{-}n, ae^{i(\theta+2\phi)}, ae^{-i\theta}; a^2, 0 \mid q;q)$ **89:** Q-Meixner–Pollaczek polynomials
LaTeX	`P_{n}(x;a\mid q) = a^{-n} e^{in\phi} \frac{a^2;q_n}{(q;q)_n} {}_3\Phi_2(q^-n, ae^{i(\theta+2\phi)}, ae^{-i\theta}; a^2, 0 \mid q; q)`
Semantic LaTeX	`P_{n}(x ; a \mid q) = a^{-n} \expe^{\iunit n\phi} \frac{\qmultiPochhammersym{a^2}{q}{n}}{\qmultiPochhammersym{q}{q}{n}} \qgenhyperphi{3}{2}@{q^- n , a\expe^{\iunit(\theta + 2 \phi)} , a\expe^{- \iunit \theta}}{a^2, 0}{q}{q}`
Mathematica	`P[n_, x_, a_, q_] := (a)^(- n)* Exp[I*n*\[Phi]]*Divide[Product[QPochhammer[Part[{(a)^(2)},i],q,n],{i,1,Length[{(a)^(2)}]}],Product[QPochhammer[Part[{q},i],q,n],{i,1,Length[{q}]}]]*QHypergeometricPFQ[{(q)^(-)* n , a*Exp[I*(\[Theta]+ 2*\[Phi])], a*Exp[- I*\[Theta]]},{(a)^(2), 0},q,q]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	✘	✘	-	-	-
Explanation	We were unable to match the underscore `{_3}` in ${}_3\Phi_2$.					

Formula	$L_n^{(\alpha)}(x;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n} {}_1\phi_1(q^{-n};q^{\alpha+1};q,-q^{n+\alpha+1}x)$ **90:** Q-Laguerre polynomials
LaTeX	`\displaystyle L_n^{(\alpha)}(x;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n} {}_1\phi_1(q^{-n};q^{\alpha+1};q,-q^{n+\alpha+1}x)`
Semantic LaTeX	`\qLaguerrepolyL{\alpha}{n}@{x}{q} = \frac{\qmultiPochhammersym{q^{\alpha+1}}{q}{n}}{\qmultiPochhammersym{q}{q}{n}} \qgenhyperphi{1}{1}@{q^{-n}}{q^{\alpha+1}}{q}{- q^{n+\alpha+1} x}`
Mathematica	`L[n_, \[Alpha]_, x_, q_] := Divide[Product[QPochhammer[Part[{(q)^(\[Alpha]+1)},i],q,n],{i,1,Length[{(q)^(\[Alpha]+ 1)}]}],Product[QPochhammer[Part[{q},i],q,n],{i,1,Length[{q}]}]]*QHypergeometricPFQ[{(q)^(- n)},{(q)^(\[Alpha]+1)},q,- (q)^(n + \[Alpha]+ 1)* x]`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	✘
Explanation	Q-Laguerre polynomials are not supported in Mathematica.					

Formula	$\sum_{n=0}^{\infty} H_n(x \mid q) \frac{t^n}{(q;q)_n} = \frac{1}{\left(te^{i\theta}, te^{-i\theta}; q\right)_\infty}$ **91:** Continuous q-Hermite polynomials
LaTeX	`\sum_{n=0}^\infty H_n(x \mid q) \frac{t^n}{(q;q)_n} = \frac{1}{\left( t e^{i \theta},t e^{-i \theta};q \right)_\infty}`
Semantic LaTeX	`\sum_{n=0}^\infty \contqHermitepolyH{n}@{x}{q} \frac{t^n}{\qmultiPochhammersym{q}{q}{n}} = \frac{1}{\qmultiPochhammersym{t \expe^{\iunit \theta} , t \expe^{- \iunit \theta}}{q}{\infty}}`
Mathematica	
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	-	-	✘	-	-	-

Explanation Mistakenly detect Hermite polynomial but it should have been continuous q-Hermite polynomial.

Formula	$w'' + \xi \sin(2z) w' + (\eta - p\xi \cos(2z)) w = 0.$ **92:** Ince equation
LaTeX	`w^{\prime\prime}+\xi\sin(2z)w^{\prime}+(\eta-p\xi\cos(2z))w=0.`
Semantic LaTeX	`w^{\prime\prime}+\xi\sin(2z)w^{\prime}+(\eta-p\xi\cos(2z))w=0`
Mathematica	`D[w[z], {z, 2}] + \[Xi]*Sin[2*z]*D[w[z], {z, 1}] + (\[Eta]-p*\[Xi]*Cos[2*z])*w[z] == 0`
Maple	

Translations		Reason for Failure				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	-	-	✘	✘	-

Explanation ODEs cannot be translated yet.

Formula	$Q_v^\mu(x) = \cos(\mu\pi)\left(\frac{1+x}{1-x}\right)^{\mu/2}\frac{F(v+1,-v;1-\mu;1/2-2/x)}{\Gamma(1-\mu)}$ **93:** Ferrers function
LaTeX	`Q_v^\mu(x)= \cos(\mu\pi)\left(\frac{1+x}{1-x}\right)^{\mu/2}\frac{F(v+1,-v;1-\mu;1/2-2/x)} {\Gamma(1-\mu ) }`
Semantic LaTeX	`\FerrersQ[\mu]{v}@{x} = \cos(\mu \cpi)(\frac{1+x}{1-x})^{\mu/2} \frac{\ hyperF@{v+1}{-v}{1-\mu}{1/2-2/x}}{\EulerGamma@{1-\mu}}`
Mathematica	`LegendreQ[v, \[Mu], x] == Cos[(\[Mu]*Pi)]*(Divide[1 + x,1 - x])^(\[Mu]/2)* Divide[Hypergeometric2F1[v + 1, - v, 1 - \[Mu], 1/2 - 2/x],Gamma[1 - \[Mu]]]`
Maple	`LegendreQ(v, mu, x) = cos((mu*Pi))*((1 + x)/(1 - x))^(mu/2)*(hypergeom([v + 1, - v], [1 - mu], 1/2 - 2/x))/(GAMMA(1 - mu))`

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✘	✘	-	✘	-	✘	-
Explanation	No information about the Gamma and hypergeometric functions in the context.					

Formula	$H_{-v}^{(1)}(z,w) = e^{v\pi i}H_v^{(1)}(z,w)$ **94:** Incomplete Bessel functions
LaTeX	`H_{-v}^{(1)}(z,w)=e^{v\pi i}H_v^{(1)}(z,w)`
Semantic LaTeX	`H_{-v}^{(1)}(z,w) = \expe^{v \cpi \iunit} H_v^{(1)}(z , w)`
Mathematica	
Maple	

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	-	-	-	-	✘	✘
Explanation	No information about $H_{-v}^{(1)}(z,w)$ in the context.					

Formula	$K_v(x,y) = \int_1^\infty \frac{e^{-xt-\frac{y}{t}}}{t^{v+1}} dt$
LaTeX	`K_v(x,y)=\int_1^\infty\frac{e^{-xt-\frac{y}{t}}}{t^{v+1}}dt`
Semantic LaTeX	`K_v(x , y) = \int_1^\infty \frac{\expe^{-xt-\frac{y}{t}}}{t^{v+1}} \diff{t}`
Mathematica	`K[v_, x_, y_] := Integrate[Divide[Exp[- x*t -Divide[y,t]],(t)^(v + 1)], {t, 1, Infinity}]`
Maple	

95: Incomplete Bessel K function/ generalized incomplete gamma function

Translations		**Reason for Failure**				
Semantic LaTeX	CAS	Definition / Substitution	Pattern Matching	Derivatives / Primes	Missing Information	Untranslatable
✔	✘	✘	-	-	-	-
Explanation	Unable to detect equation as a definition.					

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Bibliography of Publications, Submissions & Talks of the Appendix

[1] H. S. Cohl, **A. Greiner-Petter**, and M. Schubotz. "Automated Symbolic and Numerical Testing of DLMF Formulae Using Computer Algebra Systems". In: *Proc. Conf. Intelligent Computer Mathematics (CICM)*. Vol. 11006. Hagenberg, Austria: Springer International Publishing, 2018, pp. 39–52. DOI: 10/ggv8dn. URL: https://arxiv.org/abs/2109.08899 (visited on 2021-09-08) (cit. on pp. 6, 21).

[2] **A. Greiner-Petter**. "Automatic Mathematical Information Retrieval to Perform Translations up to Computer Algebra Systems". In: *Bulletin of IEEE Technical Committee on Digital Libraries (TCDL)* 15.1 (2019). URL: https://arxiv.org/pdf/2011.14616.pdf (cit. on pp. 1, 3).

[3] **A. Greiner-Petter**, H. S. Cohl, A. Youssef, M. Schubotz, A. Trost, R. Dey, A. Aizawa, and B. Gipp. "Comparative Verification of the Digital Library of Mathematical Functions and Computer Algebra Systems". In: *Tools and Algorithms for the Construction and Analysis of Systems - 28th International Conference, (TACAS)*. Munich, Germany: Springer, 2022-04, pp. 87–105. DOI: 10.1007/978-3-030-99524-9_5. URL: https://arxiv.org/abs/2201.09488 (cit. on pp. 1, 6, 18).

[4] **A. Greiner-Petter**, M. Schubotz, C. Breitinger, P. Scharpf, A. Aizawa, and B. Gipp. "Do the Math: Making Mathematics in Wikipedia Computable". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI)* (2022). In Press, pp. 1–12. ISSN: 0162-8828. DOI: 10.1109/TPAMI.2022.3195261 (cit. on pp. 1, 23, 27).

[5] **A. Greiner-Petter**, M. Schubotz, H. S. Cohl, and B. Gipp. "Semantic Preserving Bijective Mappings for Expressions Involving Special Functions between Computer Algebra Systems and Document Preparation Systems". In: *Aslib Journal of Information Management* 71.3 (2019-05-20), pp. 415–439. ISSN: 2050-3806. DOI: 10/ggv8gx. URL: https://arxiv.org/abs/1906.11485 (visited on 2021-09-06) (cit. on pp. 1, 2, 6, 7).

[6] **A. Greiner-Petter**, M. Schubotz, F. Müller, C. Breitinger, H. Cohl, A. Aizawa, and B. Gipp. "Discovering Mathematical Objects of Interest — A Study of Mathematical Notations". In: *Proceedings of The Web Conference (WWW)*. Taipei, Taiwan: ACM, 2020-04-20, pp. 1445–1456. DOI: 10/ggv8gw. URL: https://arxiv.org/abs/2002.02712 (visited on 2021-07-30) (cit. on p. 30).

[7] M. Schubotz, **A. Greiner-Petter**, N. Meuschke, O. Teschke, and B. Gipp. "Mathematical Formulae in Wikimedia Projects 2020". In: *Proceedings of the ACM/IEEE Joint Conference on Digital Libraries (JCDL)*. Virtual Event, China: ACM, 2020-08, pp. 447–448. DOI: 10/ghn2t2. URL: https://arxiv.org/abs/2003.09417 (visited on 2021-08-02) (cit. on pp. 1, 4).

BACK MATTER

Bibliography of the Appendix

[8] E. T. Bell. "Exponential Polynomials". In: *The Annals of Mathematics* 35.2 (1934-04), p. 258. ISSN: 0003-486X. DOI: 10/cqhz37 (cit. on p. 7).

[9] R. M. Corless and D. J. Jeffrey. "Graphing Elementary Riemann Surfaces". In: *ACM SIGSAM Bulletin* 32.1 (1998-03), pp. 11–17. ISSN: 0163-5824. DOI: 10/dcpb9j (cit. on p. 2).

[10] R. M. Corless, D. J. Jeffrey, S. M. Watt, and J. H. Davenport. ""According to Abramowitz and Stegun" or Arccoth Needn't Be Uncouth". In: *ACM SIGSAM Bulletin* 34.2 (2000-06), pp. 58–65. ISSN: 0163-5824. DOI: 10/cc53nd (cit. on p. 2).

[11] J. H. Davenport. "The Challenges of Multivalued "Functions"". In: *Proc. Conf. Intelligent Computer Mathematics (CICM)*. Vol. 6167. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 1–12. DOI: 10.1007/978-3-642-14128-7_1 (cit. on p. 2).

[12] *NIST Digital Library of Mathematical Functions*. http://dlmf.nist.gov/, Release 1.0.25 of 2019-12-15. F. W. J. Olver, et al. eds. (cit. on pp. 3, 6–10, 18, 19).

[13] M. England, E. Cheb-Terrab, R. Bradford, J. H. Davenport, and D. Wilson. "Branch Cuts in Maple 17". In: *ACM Communications in Computer Algebra* 48.1/2 (2014-07-10), pp. 24–27. ISSN: 1932-2240. DOI: 10/gknppb (cit. on p. 2).

[14] L. Espinosa Anke and S. Schockaert. "Syntactically Aware Neural Architectures for Definition Extraction". In: *Proc. Ann. Meeting Association for Computational Linguistics (ACL)*. New Orleans, Louisiana: Association for Computational Linguistics, 2018, pp. 378–385. DOI: 10/ggwczd (cit. on p. 29).

[15] D. Ginev and B. R. Miller. "Scientific Statement Classification over arXiv.Org". In: *Proceedings of The 12th Language Resources and Evaluation Conference, LREC 2020, Marseille, France, May 11-16, 2020*. European Language Resources Association, 2020, pp. 1219–1226. URL: https://aclanthology.org/2020.lrec-1.153/ (visited on 2021-08-15) (cit. on p. 29).

[16] A. Head, K. Lo, D. Kang, et al. "Augmenting Scientific Papers with Just-in-Time, Position-Sensitive Definitions of Terms and Symbols". In: *Proceedings of the 2021 CHI Conference on Human Factors in Computing Systems*. Yokohama Japan: ACM, 2021-05-06, pp. 1–18. DOI: 10/gksmrc (cit. on p. 29).

[17] D. J. Jeffrey and A. C. Norman. "Not Seeing the Roots for the Branches: Multivalued Functions in Computer Algebra". In: *ACM SIGSAM Bulletin* 38.3 (2004-09), pp. 57–66. ISSN: 0163-5824. DOI: 10/bzw56h (cit. on p. 2).

[18] D. J. Jeffrey. "Multivalued Elementary Functions in Computer-Algebra Systems". In: *Artificial Intelligence and Symbolic Computation*. Vol. 8884. Cham: Springer International Publishing, 2014, pp. 157–167. DOI: 10.1007/978-3-319-13770-4_14 (cit. on p. 2).

[19] D. Kang, A. Head, R. Sidhu, K. Lo, D. Weld, and M. A. Hearst. "Document-Level Definition Detection in Scholarly Documents: Existing Models, Error Analyses, and Future Directions". In: *Proceedings of the First Workshop on Scholarly Document Processing*. Online: Association for Computational Linguistics, 2020, pp. 196–206. DOI: 10/gjzg7z (cit. on p. 29).

[20] G. Y. Kristianto, G. Topić, and A. Aizawa. "Utilizing Dependency Relationships between Math Expressions in
10/ggv8gd (cit. on p. 29).

[21] C. Manning, M. Surdeanu, J. Bauer, J. Finkel, S. Bethard, and D. McClosky. "The Stanford CoreNLP Natural Language Processing Toolkit". In: *Proc. Ann. Meeting Association for Computational Linguistics (ACL)*. Baltimore, Maryland: Association for Computational Linguistics, 2014, pp. 55–60. DOI: 10/gf3xhp (cit. on p. 23).

[22] A. Meurer, C. P. Smith, M. Paprocki, et al. "SymPy: Symbolic Computing in Python". In: *PeerJ Computer Science* 3 (2017-01-02), e103. ISSN: 2376-5992. DOI: 10/gfb682 (cit. on p. 12).

[23] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark. *NIST Handbook of Mathematical Functions*. Cambridge New York: Cambridge University Press NIST, 2010. 968 pp. ISBN: 9780521192255 (cit. on pp. 3, 6).

[24] P. Scharpf, I. Mackerracher, M. Schubotz, J. Beel, C. Breitinger, and B. Gipp. "AnnoMathTeX - a Formula Identifier Annotation Recommender System for STEM Documents". In: *Proceedings of the 13th ACM Conference on Recommender Systems*. Copenhagen Denmark: ACM, 2019-09-10, pp. 532–533. DOI: 10/ggv8jt (cit. on p. 4).

[25] P. Scharpf, M. Schubotz, and B. Gipp. "Representing Mathematical Formulae in Content MathML Using Wikidata". In: *Proc. Workshop on Bibliometric-Enhanced Information Retrieval and Natural Language Processing (BIRNDL@SIGIR)*. Vol. 2132. CEUR-WS.org, 2018, pp. 46–59. URL: http://ceur-ws.org/Vol-2132/paper5.pdf (cit. on p. 4).

[26] M. Schubotz, A. Grigorev, M. Leich, et al. "Semantification of Identifiers in Mathematics for Better Math Information Retrieval". In: *Proceedings of the 39th International ACM SIGIR Conference on Research and Development in Information Retrieval*. Pisa Italy: ACM, 2016-07-07, pp. 135–144. DOI: 10/ggv8jm (cit. on pp. 23, 30).

[27] M. Schubotz, L. Krämer, N. Meuschke, F. Hamborg, and B. Gipp. "Evaluating and Improving the Extraction of Mathematical Identifier Definitions". In: *Experimental IR Meets Multilinguality, Multimodality, and Interaction*. Vol. 10456. Cham: Springer International Publishing, 2017, pp. 82–94. DOI: 10/gn3sxn (cit. on p. 23).

[28] M. Schubotz, P. Scharpf, K. Dudhat, Y. Nagar, F. Hamborg, and B. Gipp. "Introducing MathQA - a Math-Aware Question Answering System". In: *Proceedings of the Annual International ACM/IEEE Joint Conference on Digital Libraries (JCDL), Workshop on Knowledge Discovery*. Fort Worth, USA, 2018-06. DOI: 10.1108/IDD-06-2018-0022 (cit. on p. 4).

[29] M. Schubotz and A. P. Sexton. "A Smooth Transition to Modern Mathoid-Based Math Rendering in Wikipedia with Automatic Visual Regression Testing". In: *Proc. Conf. Intelligent Computer Mathematics (CICM)*. Vol. 1785. Bialystok, Poland: CEUR-WS.org, 2016, pp. 132–145. DOI: 10/ggv8mp (cit. on p. 4).

[30] M. Schubotz and G. Wicke. "Mathoid: Robust, Scalable, Fast and Accessible Math Rendering for Wikipedia". In: *Proc. Conf. Intelligent Computer Mathematics (CICM)*. Vol. 8543. Springer, 2014, pp. 224–235. DOI: 10.1007/978-3-319-08434-3_17 (cit. on p. 4).

[31] N. Vanetik, M. Litvak, S. Shevchuk, and L. Reznik. "Automated Discovery of Mathematical Definitions in Text". In: *Proceedings of The 12th Language Resources and Evaluation Conference, LREC 2020, Marseille, France, May 11-16, 2020*. European Language Resources Association, 2020, pp. 2086–2094. URL: https://aclanthology.org/2020.lrec-1.256/ (visited on 2021-08-15) (cit. on p. 29).

[32] Wolfram. *Wolfram Language & System: Importing and Exporting*. Importing and Exporting. URL: https://reference.wolfram.com/language/tutorial/ImportingAndExporting.html (visited on 2021-09-12) (cit. on p. 12).

[33] A. Youssef and B. R. Miller. "A Contextual and Labeled Math-Dataset Derived from NIST's DLMF". In: *Proc. Conf. Intelligent Computer Mathematics (CICM)*. Vol. 12236. Cham: Springer International Publishing, 2020, pp. 324–330. DOI: 10/gn3sv6 (cit. on pp. 18, 20).